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# Abstract

The first chapter of this dissertation, coauthored with Martin Eichenbaum and Riccardo Bianchi-Vimercati, addresses the question: how sensitive is the power of fiscal policy at the ZLB to the assumption of rational expectations? We do so through the lens of a standard NK model in which people are level-$k$ thinkers. Our analysis *weakens* the case for using government spending to stabilize the economy when the ZLB binds. The less sophisticated people are, the smaller the government-spending multiplier is. Our analysis *strengthens* the case for using tax policy to stabilize output when the ZLB is binding. The power of tax policy to stabilize the economy during the ZLB period is essentially undiminished when agents do not have rational expectations. Finally, we show that the way in which tax policy is communicated is critical to its effectiveness.

In the second chapter, coauthored with Bence Bardóczy, we study the power of state-dependent unemployment insurance (UI) to stabilize short-run fluctuations, allowing for arbitrary deviations from full information and rational expectations. Expectations are critical because higher UI generosity raises consumption partly by lowering precautionary savings. If UI generosity is indexed to the unemployment rate, households must forecast the unemployment rate to anticipate the policy stance. We estimate unemployment expectations in response to identified aggregate shocks. We quantify the consequences of these imperfect expectations through the lens of a Heterogeneous Agent New Keynesian model. First, we work directly with the estimated forecast errors. Our methodological contribution is to use the non-parametric history of forecast errors and forecast revisions to solve dynamic decisions of optimizing agents. By doing so, we sidestep the need to choose a particular model of belief formation (e.g., cognitive discounting or sticky expectations). The estimated model implies that imperfect anticipation substantially affects the stimulative power of UI extensions. Second, we compare alternative ways of implementing UI policies. To run counterfactuals,

we estimate a structural model of belief formation. We show that a combination of noisy information and diagnostic expectations fits the data best among a large set of popular alternatives. A UI extension that is announced directly is more stimulative in the very short run than one that is indexed to the unemployment rate.

The third chapter studies how belief disagreement across households affects aggregate demand. I develop a model in which households are heterogeneously exposed to business cycles and show that the impact of disagreement can be summarized by a simple statistic–*correlated disagreement*–which captures the correlation between beliefs and individual business-cycle exposure. I model disagreement as endogenously heterogeneous attention. In this model, attention increases with the exposure to business cycles. Then, I show that disagreement amplifies general-equilibrium effects and acts as a propagation mechanism amplifying business cycles. I also provide evidence of this positive correlation using survey data on expectations. To quantify the implications of disagreement, I extend the analysis to a Heterogeneous-agent New Keynesian model featuring multiple sources of heterogeneity. I show that belief disagreement can substantially amplify business-cycle fluctuations. Finally, I show that targeting spending to the most cyclical workers can significantly increase the spending multiplier.

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# Table of Contents

# List of Tables

# List of Figures

# Chapter 1

# Fiscal Policy at the ZLB without Rational Expectations

## 1.1 Introduction

The *Zero Lower Bound* (ZLB) on interest rate poses a significant constraint on conventional monetary policy.[1] A large literature emphasizes that fiscal policy is particularly useful for stabilizing the aggregate economy when the ZLB binds. According to this literature, the government-spending multiplier is significantly higher than under normal circumstances, see, e.g., Christiano et al. (2011), Eggertsson (2011), and Woodford (2011).[2] In addition, appropriately designed tax policy can mimic the effect of conventional monetary policy on aggregate demand, see Feldstein (2003) and Correia et al. (2013).

Much of the modern literature on alternatives to conventional monetary policy assumes that people have rational expectations. For example, it is well known that when the ZLB is binding, forward guidance is extremely powerful in standard New Keynesian (NK) models.[3] But the power of that policy is considerably diminished under reasonable deviations from rational expectations (see the literature summary below). This observation leads to the

[1] We understand that interest rates can be negative. But there is some effective lower bound on interest rates. To facilitate comparisons with the literature we work with the ZLB, with the understanding that our key results would obtain when the effective lower interest rate is binding.

[2] See also the analyses in Werning (2011) and Farhi and Werning (2016).

[3] See Eggertsson and Woodford (2003) and Werning (2011) for analyses of the power of forward guidance in standard NK models.

natural question: how sensitive is the power of fiscal policy at the ZLB to the assumption of rational expectations? According to our analysis, the efficacy of government spending is quite sensitive to that assumption. Moreover, under plausible assumptions, the less sophisticated people are, the smaller is the multiplier. In contrast, tax policy at the ZLB is less sensitive to deviations from rational expectations. Indeed, in our analysis, tax policy continues to be able to support the flexible-price allocation even when agents are boundedly rational and the ZLB is binding.

We reach these conclusions using a simple representative-agent NK model with sticky wages and without capital accumulation. As in Correia et al. (2013), we assume that there is an unanticipated shock to people's discount factor at time zero that lasts for $T$ periods. As a result, the subjective discount rate falls below zero, driving the nominal interest rate to the ZLB.

In our benchmark model, wages are fully rigid and the price level is constant. We depart from rational expectations by assuming that people form beliefs about future endogenous variables via *level-$k$ thinking*. Individuals understand the structure of the economy but are limited in their ability to predict the behavior of other economic agents and, as a result, the time path for the endogenous variables in the economy (e.g., aggregate output). Starting from an initial belief for the least sophisticated agents, individuals update their expectations about changes in the future based on a finite reasoning process about other people's behavior, involving $k$ iterations. We are interested in how the power of fiscal policy depends on agents' level of cognitive sophistication as indexed by $k$.

In Section 1.2, we use the benchmark model to evaluate the effects of increased government spending and time-varying consumption taxes when the ZLB is binding. Consistent with earlier work by Woodford and Xie (2019) and Farhi et al. (2020), we establish that the size of the government-spending multiplier depends on agents' level of cognitive sophistication (Proposition 1).[4] The intuition is as follows. Despite their cognitive limitations, individuals understand that higher government spending implies increased taxes. Other things equal, this negative wealth effect leads to a decrease in consumer demand. However, higher government spending implies an increase in the demand for labor and higher labor income. The latter effect implies an increase in consumer demand. Under reasonable conditions, the less sophisticated people are, the less they take into account the positive general-equilibrium effects of higher spending. So, the negative wealth effect of higher taxes receives relatively more weight in people's decisions, leading to a larger drop in consumer demand. The net effect is that lower levels of cognitive sophistication imply lower values for the government spending multiplier.

We then turn to an analysis of tax policy at the ZLB. Correia et al. (2013) show that tax policy is a powerful tool for stabilizing the economy when the ZLB binds and people have rational expectations. Following these authors, we consider a policy of lowering an *ad-valorem* tax on consumption as soon as the ZLB binds and then slowly raising that tax to its pre-shock level. This policy has the effect of putting consumption "on sale" while the ZLB binds. We show that there always exists a time path for consumption taxes that completely stabilizes the economy at its pre-shock level, i.e., it supports the flexible-price allocation. In general, this policy depends on people's level of sophistication, $k$. However, suppose that, as in Farhi and Werning (2019), the least sophisticated people ($k = 1$) think aggregate output will remain at its pre-shock level. Then, the path for consumption taxes that supports the flexible-price allocation is the same regardless of how cognitively sophisticated people are.

Critically, the flexible-price allocation is the same as the pre-shock steady state of the economy. So, under the tax policy that supports this allocation, people's initial beliefs are self-confirming, i.e., they do not make any expectational errors. In this sense, the efficacy

[4] As discussed in the related literature section below, Angeletos and Lian (2018) obtain a similar result stemming from the assumption that people do not share common information about future government actions.

of this policy does not exploit people's lack of sophistication. Taken together these results, summarized in Proposition 2, show that tax policy is a powerful and robust way to stabilize the economy when the ZLB binds.

The basic intuition for why tax policy is robustly powerful is as follows. Suppose that the government announces a time-path for current and future tax rates. Then, people incorporate these rates into their personal consumption-savings decision and substitute consumption to dates when the tax rate is lower. This basic force is operative regardless of any general-equilibrium (GE) considerations, i.e., people do not need to calculate the GE effects of the announced tax rate to adjust their personal consumption decision to the tax rates. So, the policy boosts consumption demand and supports flexible-price allocation when the ZLB binds, even if people are very unsophisticated.

It is useful to contrast the efficacy of tax rate and interest-rate policy. In our model, changing the announced path of tax rates and interest rates affects the equilibrium in the same way. But there is one crucial difference. The ZLB constrains the class of feasible announced paths for interest rates but not the paths of tax rates. This constraint means that monetary policy can only boost consumption demand by promising to lower interest rates in the future after the ZLB is no longer binding (forward guidance). Farhi and Werning (2019) and García-Schmidt and Woodford (2019) show that the effects of such a policy can be quite sensitive to deviations from rational expectations.[5] In contrast, fiscal policy can stimulate consumption demand by changing tax rates as soon as the ZLB binds. This flexibility means that fiscal policy can support the flexible-price allocation, an outcome that is not possible with interest rate policy (with or without rational expectations).

With bounded rationality, the way policy is communicated matters. Above, we assumed that the government announces a sequence of consumption-tax rates that will apply during

[5]See also Angeletos and Lian (2018) who show that the same conclusions hold in a model with informational frictions and imperfect common knowledge.

the ZLB. Suppose instead that the government announces a rule according to which tax rates are set as a function of the output gap. We show that this form of communication leads to a substantial deterioration in the efficacy of tax rate policy.

To make this argument, we proceed as follows. First, we consider a rule for setting tax rates at the ZLB and calculate the corresponding sequence of tax rates that would obtain under rational expectations. Then we compute the equilibria in the level-$k$ economy under the announced policy rule and the sequence of corresponding announced tax rates. We show that, for any $k$, the decline in output is larger when policy is communicated as a tax rule rather than a sequence of tax rates.

The intuition for this result is as follows. When policy is communicated as a rule, individuals must forecast the future level of output to predict what tax rates will be. When individuals are limited in their ability to compute general-equilibrium effects, they will also be limited in their ability to forecast future tax rates. This limitation translates into a lower efficacy of tax policy in stimulating demand.

A natural question is whether our results are robust to alternative ways of modeling bounded rationality. In appendix A.2, we redo our analysis of the benchmark model using two alternatives to the level-$k$ thinking approach. The first alternative is that people have *reflective expectations* as in García-Schmidt and Woodford (2019). The second alternative is that people display *shallow reasoning* as developed in Angeletos and Sastry (2021). We show that Propositions 1 and 2 continue to hold for both cases.

Recall that we assume that the price level is constant in our benchmark model. This assumption does not hold in more general versions of the NK models. In those models, the impact of government spending at the ZLB on inflation and the real interest rate plays an important role in magnifying the size of the government spending multiplier. When the ZLB is binding, increases in government spending lead to upwards pressure on prices, which lowers the real interest rate and boosts the demand for consumption. To the extent that people do not understand these equilibrium effects, the size of the government-spending multiplier should be smaller, as shown by Angeletos and Lian (2018) and Farhi et al. (2020). It is not obvious how a variable price level affects the efficacy of tax policy under bounded rationality.

To study these issues, we redo our analysis in a framework where prices and wages are not constant. Specifically, in section 1.3, we assume that nominal wages are set subject to Calvo-style frictions as in Erceg et al. (2000).[6] Since wages are not constant, neither is the price level. We show numerically that the key results of Proposition 1 continue to hold. Turning to tax policy, we suppose that the government can impose time-varying tax rates on consumption and labor income. With this proviso, we show that the analog to Proposition 2 holds for the extended model. As in the benchmark economy, the policy that supports the flexible-price allocation does not depend on $k$ if the least sophisticated agents expect the economy to remain in steady-state. Finally, we show through a series of numerical examples that our results regarding the advantage of communicating policy via targets rather than rules continue to hold.

In our model, Ricardian equivalence holds. We make this assumption to focus on people's limited ability to understand the general-equilibrium effects of government policy. An important question is whether our conclusions about the relative efficacy of tax and government spending depend on Ricardian equivalence. We use the extended model to briefly discuss this issue and point out that both policies can exploit the failure of Ricardian equivalence. Our analysis suggests that the effects of government spending would be more sensitive than tax policy to the failure of Ricardian equivalence. We leave a complete analysis of the non-Ricardian case to future research.

Taken together, our results *weaken* the case for using government spending to stabilize the economy when the ZLB binds. At the same time, our results *strengthen* the case for

[6] Appendix D redoes the analysis of section 1.3 under the assumption that nominal prices, rather than nominal wages, are subject to Calvo-style frictions.

using tax policy to stabilize output when the ZLB is binding. The power of tax policy to stabilize the economy during the ZLB period is essentially undiminished when agents do not have rational expectations.

**Supporting empirical evidence** There is a vast empirical literature on quantifying deviations from standard notions of rationality. Of direct relevance is experiment-based evidence on the level of people's sophistication. Crawford et al. (2013) review this literature and argue that the experimental evidence is consistent with the distribution of cognitive levels being very concentrated at low levels of $k$. For example, Camerer et al. (2004) concludes that a substantial fraction of people are well characterized as having levels of $k$ between 0 and 2 and that the median level $k$ is between 1 and 2.[7] In our model, these levels of $k$ generate very different behavior than rational expectations. In a non-experimental setting, Iovino and Sergeyev (2018) estimate the sophistication level of professional forecasters by looking at survey data about mortgage rates and their response to quantitative easing. They find that 86 percent of forecasters in their data are level-1 thinkers.

There is a large literature that characterizes people's expectations of macro variables based on survey evidence, see, e.g., Coibion and Gorodnichenko (2015), Bordalo et al. (2012), and Angeletos et al. (2021). A key conclusion from this literature is that on average people's beliefs about macroeconomic aggregates like inflation and real GDP growth tend to under react to changes in macro fundamentals relative to the rational expectations benchmark. Our model is consistent with this finding.

Our conclusions about the efficacy of tax policy receive strong support from recent empirical work. D'Acunto et al. (2020) estimate the impact of forward guidance and consumption tax policies on household inflation expectations and spending. They show that forward guidance policies had little effect on household expectations and behavior. However, con-

[7] See also Stahl and Wilson (1995), Ho et al. (1998), Bosch-Domenech et al. (2002), among others.

sumption tax policies like those that we describe are effective at raising household spending. These empirical results are consistent with our conclusion that tax policy can be a powerful stabilization tool, even if people are not as sophisticated as in the rational expectations paradigm. Bachmann et al. (2021) provide strong evidence on the efficacy of a temporary VAT cut in Germany when the ZLB was binding. They find that (1) most households were aware of the policy change and (2) that people with different degrees of financial literacy responded in roughly the same way to the tax cut. On this basis, they conclude that the tax cut successfully stimulated aggregate consumption spending because of its simplicity and salience.

**Related theoretical literature** This paper belongs to a growing literature that studies the implications of deviations from rational expectations for the effectiveness of macroeconomic policy. The form of bounded rationality that we consider is based on level-$k$ thinking models originally studied by Nagel (1995) and Stahl and Wilson (1995). Farhi and Werning (2019) use this approach to study how deviations from rational expectations impact the efficacy of forward guidance. García-Schmidt and Woodford (2019) develop a closely related form of deviation from rational expectations, which they refer to as reflective expectations. They apply this form of expectations to study the impact of forward guidance and interest rate pegs on economic activity. Under both level-$k$ thinking and reflective expectations, individuals have a limited ability to understand the general-equilibrium consequences of monetary policy.[8] García-Schmidt and Woodford (2019) and Farhi and Werning (2019) show that this effect limits the power of forward guidance and mitigates some anomalous implications of this policy under rational expectations.[9] Iovino and Sergeyev (2018) apply

[8]Similar ideas are captured by the *calculation equilibrium* and *internal rationality* approach to bounded rationality discussed in Evans and Ramey (1992) and Adam and Marcet (2011), respectively.

[9]Similar results are derived in Woodford (2018) in a model in which individuals can only make contingent plans up to a finite number of future periods, i.e., they have *limited foresight*, Gabaix (2020) in a model in which individuals are inattentive to the interest rate, Angeletos and Lian (2018) in a model with informational frictions and imperfect common knowledge, and in Wiederholt (2015) in a model with sticky expectations.

level-$k$ thinking and reflective expectations to analyze the effects of quantitative easing.

Angeletos and Lian (2023) initially developed the idea that the lack of common knowledge attenuates general-equilibrium effects. Angeletos and Lian (2018) study a rational-expectations environment in which people do not have common knowledge about the relevant news. They show that the absence of common knowledge dampens the general-equilibrium effects of news and the size of the government spending multiplier. We obtain a similar result about government spending when people have complete information about the shocks, but are limited in their ability to forecast the GE consequences of policies. While the mechanism is different, this limitation attenuates the general-equilibrium effects of those shocks as in Angeletos and Lian (2018).

Woodford and Xie (2019) and Farhi et al. (2020) analyzed the consequences of bounded rationality for the size of fiscal multipliers. Following the approach developed by Woodford (2018), Woodford and Xie (2019) assumes that individuals can only plan for a finite number of periods but are fully rational within the planning horizon. They show that this behavioral bias may limit the size of the government-spending multiplier at the ZLB because the stimulus effect of future government spending on current output is zero if it occurs after the relevant planning horizon. Instead, we work with a model in which individuals have an infinite planning horizon but have a limited capacity to understand the GE effects of different policies.

Our analysis is closest to Farhi et al. (2020), who also assume that individuals are level-$k$ thinkers. Their main focus is on the *fiscal-multiplier puzzle* discussed in Farhi and Werning (2016), who note that, in standard representative-agent NK economies, the government-spending multiplier grow explosively as government spending is back-loaded. At the heart of this result is that back-loaded spending generates more inflation, which lowers the real interest rate when the ZLB is binding. Farhi et al. (2020) examine the fiscal multiplier puzzle in both representative-agent and heterogeneous agents NK models with level-$k$ thinking.

They show that the government-spending multiplier is generally lower, the lower is the level of cognitive sophistication in the economy and that models with level-$k$ thinking do not exhibit the fiscal-multiplier puzzle.

An important distinction between our paper and the literature just cited is that we study how deviations from rational expectations affect the efficacy of tax policy versus government spending when the ZLB is binding. In addition, we analyze how communication affects the power of tax policy at the ZLB.

Angeletos and Sastry (2021) analyze the implications of policy communication when agents have a particular form of bounded rationality. They analyze whether policy communication should focus on instruments (interest rates) or targets (unemployment). They show that the answer to this question depends on the relative importance of partial versus general-equilibrium effects of a given policy. Their substantive application is forward guidance, while we focus on tax policy. In addition, we look at rules versus instrument settings rather than their main focus of instruments versus targets.

Because our model features a continuum of identical households and Ricardian equivalence, there is no role for countercyclical fiscal transfers, e.g., unemployment benefits. McKay and Reis (2016, 2021) and Kekre (2021) study the role of tax and transfer programs in stimulating demand in heterogeneous-agent incomplete markets economies with rational expectations. Woodford and Xie (2022) shows that uniform lump-sum transfers can be a powerful stabilization tool in a model in which Ricardian equivalence fails due to bounded rationality.[10] Because our analysis focuses on people's limited ability to understand the general-equilibrium effects of government policy, we abstract from the failure of Ricardian equivalence.

The paper is organized as follows. Section 1.2 describes our benchmark NK model with

[10] Wolf (2021) also considers a general model in which Ricardian Equivalence fails and shows that aggregate allocations that are implementable with interest rate policy can be equivalently implemented with uniform cash transfers.

level-$k$ thinking. Section 1.2.1 analyzes the effects of government spending and the implications of bounded rationality for the government spending multiplier in the benchmark model. Section 1.2.2 presents our results on consumption-tax policy in the benchmark model. Section 1.3 considers the extended model with time-varying wages and prices. Finally, section 1.4 contains concluding remarks. The proofs for all propositions are contained in the appendix.

## 1.2 A benchmark model

In this section, we describe our benchmark model. Sections 1.2.1 and 1.2.2 analyze the effect of government spending and tax policy, respectively.

Consider a simple NK economy with fully rigid wages. Without loss of generality, we normalize nominal wages to one, $W_t = 1$. There is a continuum of identical households, each of which has preferences over sequences of consumption, $C_t$, and labor, $N_t$, are given by:

$$\sum_{t=0}^{\infty} \beta^t \xi_t \left[ u\left(C_t\right) - v\left(N_t\right) \right], \tag{1.1}$$

where $u(C) = C^{1-\sigma^{-1}} / \left(1 - \sigma^{-1}\right)$ and $v(N) = N^{1+\varphi} / (1 + \varphi)$. As in Correia et al. (2013), we assume that the steady state subjective discount factor $\beta \in (0, 1)$ is perturbed by a *discount-factor shock:*

$$\xi_t = e^{-\chi(T-t)}, \tag{1.2}$$

for $t = 0, 1, ..., T$ and $\xi_t = 1$ for $t \geq T$. This assumption implies that the household's subjective discount rate between periods $t$ and $t+1$ is

$$\log \frac{\xi_t}{\beta \xi_{t+1}} = \rho - \chi, \quad t \leq T - 1,$$

where $\rho \equiv \log \beta^{-1}$. We assume that the shock satisfies $\chi > \rho$, so that the subjective discount rate is negative for $t \leq T-1$.

For simplicity, we assume that the production function is linear in labor, $Y_t = N_t$. The goods market clearing condition is

$$C_t + G_t = Y_t, \tag{1.3}$$

where $G_t$ denotes government spending. Our baseline specification, assumes that government spending is zero, $G_t = 0$.

In this simple economy, the first-best (flexible-price) allocation is

$$Y_t = C_t = N_t = 1.$$

Note that the discount-rate shock does not affect aggregate consumption or production in this allocation. However, implementing this allocation requires a negative real interest rate. So that allocation cannot be achieved using only conventional monetary policy.

**Firms** Firms are perfectly competitive and maximize profits. An interior solution for the firms' problem requires that $W_t = P_t$. Because wages are fully rigid, there is no inflation:

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} = 1. \tag{1.4}$$

**Monetary and fiscal policies** The monetary authority controls the nominal interest rate, $R_t$. During $t \leq T-1$ the nominal interest rate is at the *ZLB*,

$$R_t = 1, \tag{1.5}$$

and then goes back to its pre-shock level: $R_t = \beta^{-1}$ for $t = T, T+1, ...$

The fiscal authority sets government spending $G_t$, consumption taxes $\tau_t^c$, and lump-sum taxes $T_t$. The government's intertemporal budget constraint is given by:

$$\sum_{s=0}^{\infty} Q_{t,t+s} G_{t+s} + R_{t-1} B_t = \sum_{s=0}^{\infty} Q_{t,t+s} \left[ \tau^c_{t+s} C_{t+s} + T_{t+s} \right], \quad \forall t \geq 0. \tag{1.6}$$

Here $Q_{t,t+s}$ is the discount factor between $t$ and $t+s$,

$$Q_{t,t+s} \equiv \prod_{m=t}^{t+s-1} R_m^{-1}$$

for $s \geq 1$, $Q_{t,t} \equiv 1$.

**Households and expectations** The household has perfect foresight regarding exogenous variables so that it correctly anticipates the path for the discount rate shock, $\xi_t$. For now, we assume that the government announces sequences of nominal interest rates, $R_t$, government spending, $G_t$, and consumption taxes, $\tau^c_t$. The fact that the household correctly anticipates the path for these policy variables is consistent with the idea that they see and understand policy announcements.[11] However, the household is limited in its ability to fully predict the equilibrium changes that occur due to these policies. We denote by $Y^e_t$ and $T^e_t$ the household's beliefs about the time $t$ values of output and lump-sum taxes, respectively. There is no uncertainty in this economy so these beliefs do not correspond to expectations over possible realizations of $Y_t$ and $T_t$. Instead, they are what households think those variables will be with probability one.

Our goal is to transparently highlight the consequences of failures in predicting the general-equilibrium implications of fiscal policies for their effectiveness. We isolate this particular form of bounded rational behavior from other potential sources of non-rational expectations. So, we assume that given their beliefs for output, the household's expectations

[11] Bachmann et al. (2021) study an unexpected and temporary VAT cut in Germany that occurred in the second half of 2020. They find that most households were aware of the tax cut, which supports our assumptions.

for lump-sum taxes are consistent with the government's intertemporal budget. Formally, we assume that household beliefs for $T_t^e$ satisfy:

$$\sum_{s\geq 0} Q_{t,t+s} T_{t+s}^e = \sum_{s\geq 0} Q_{t,t+s} \left[G_{t+s} - \tau_{t+s}^c \left(Y_{t+s}^e - G_{t+s}\right)\right] + R_{t-1} B_t. \tag{1.7}$$

This expression implies that Ricardian equivalence holds in our model.[12]

The household enters period $t$ with financial assets $B_t$ earning the interest rate $R_{t-1}$. As in Farhi and Werning (2019), we assume that the household knows its contemporaneous income $Y_t$ and taxes $T_t$.[13] When solving its dynamic consumption-savings problem, the household maximizes its perceived utility which is evaluated based on today's consumption, $C_t$, and on its plans for future consumption $\tilde{C}_{t+s}$ for $s = 1, 2, ...$ To the extent that the household makes mistakes in predicting its future disposable income, actual consumption will deviate from planned consumption.

The household solves the problem:

$$\max_{\tilde{C}_{t+s}} \sum_{s\geq 0} \beta^s \xi_{t+s} \frac{\tilde{C}_{t+s}^{1-\sigma^{-1}}}{1-\sigma^{-1}}, \quad \text{subject to}$$

$$\sum_{s\geq 0} Q_{t,t+s} \left(1+\tau_{t+s}^c\right) \tilde{C}_{t+s} = \sum_{s\geq 0} Q_{t,t+s} \left[Y_{t+s}^e - T_{t+s}^e\right] + R_{t-1} B_t.$$

Since wages are rigid, equilibrium output and labor are demand determined. The solution to the household's problem implies that $C_t$ satisfies

$$C_t = \frac{Y_t - T_t + \sum_{s\geq 1} Q_{t,t+s} \left[Y_{t+s}^e - T_{t+s}^e\right] + R_{t-1} B_t}{\left(1+\tau_t^c\right) \left[1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^\sigma \left[Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right]^{1-\sigma}\right]}.$$

[12]Iovino and Sergeyev (2018) analyze the impact of central bank balance sheet policy on the economy. They do so assuming that people are level-$k$ thinkers who do not fully understand the intertemporal nature of the government's budget constraint. So in their model economy Ricardian equivalence does not hold.

[13]Our results go through if we assume that the household does not see contemporaneous $Y_t$ and $C_t$.

Replacing the present value of lump-sum taxes using equation (1.7), we obtain:

$$C_t = \frac{(Y_t - G_t) + \sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t} \left[Y^e_{t+s} - G_{t+s}\right]}{1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^\sigma \left[Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}}. \tag{1.8}$$

**Temporary and rational-expectations equilibria** We start by defining a *temporary equilibrium*. Because this general equilibrium concept does not impose any restrictions on agents' expectations, it serves as a good starting point for our analysis. Formally, for given beliefs $\{Y^e_t\}$, a temporary equilibrium is a sequence of allocations that satisfy private optimality for households and firms and the budget constraint of the government. In addition, markets clear. Using equation (1.8) and imposing market clearing $Y_t = C_t + G_t$, the temporary equilibrium output is given by

$$Y_t = \mathcal{Y}_t\left(\left\{Y^e_{t+s}\right\}_{s\geq 1}\right) = G_t + \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t} \left[Y^e_{t+s} - G_{t+s}\right]}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^\sigma \left[Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}}, \tag{1.9}$$

for all $t$.

A *rational-expectations equilibrium* is a temporary equilibrium in which expectations are consistent with the equilibrium path for these variables: $Y^e_t = Y_t$. The RE equilibrium, $Y^*_t$, solves the fixed-point problem

$$Y^*_t = \mathcal{Y}_t\left(\left\{Y^*_{t+s}\right\}_{s\geq 1}\right),$$

for all $t$.

**Level-$k$ equilibria** We now describe the concept of a level-$k$ equilibrium for our model economy. Let $Y^k_t$ denote the time-$t$ output level in an economy where all agents are level $k$. Also, $Y^{e,k}_t$ denotes the household's beliefs about output.

To compute the level-$k$ equilibrium, we must ascribe to people views about the level-

$(k-1)$ equilibrium is. The recursion takes as given what people in a level-1 economy would believe (see Farhi and Werning 2019). We denote these beliefs by $\{Y_t^{e,1}\}$. For convenience, we refer to these beliefs as belonging to level-1 people, understanding that such people don't exist in a level $k \geq 2$ economy. These beliefs are essentially free parameters. For example, one could assume that level-1 people believe that output will stay at its pre-shock level, i.e., $Y_t^{e,1} = 1$. This assumption is consistent with the approach in Farhi and Werning (2019). It captures the intuitive idea that level-1 people don't take into account how the shocks and policy will affect the future state of the economy.

Given these beliefs, a level-1 equilibrium is given by

$$Y_t^1 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,1}\right\}_{s\geq 1}\right).$$

In the standard level-$k$ thinking model, individuals believe that all other agents are exactly one level below them in terms of cognitive ability. So level-2 people believe the economy is entirely populated by level-1 people. Moreover, level-2 people can calculate the market equilibrium in an economy populated entirely by level-1 people. So, level-2 people think that equilibrium output is given by $Y_t^{e,2} = Y_t^1$. The level-2 equilibrium is therefore given by

$$Y_t^2 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,2}\right\}_{s\geq 1}\right) = \mathcal{Y}_t\left(\left\{Y_{t+s}^{1}\right\}_{s\geq 1}\right).$$

Level-3 thinkers can work through the reasoning process of both level-1 and level-2 individuals. So they think that equilibrium output is given by $Y_t^{e,3} = Y_t^2$. The level-3 equilibrium is given by

$$Y_t^3 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{2}\right\}_{s\geq 1}\right).$$

More generally, level-$k$ people think that equilibrium output is given by $Y_t^{e,k} = Y_t^{k-1}$ so that level-$k$ equilibrium is

$$Y_t^k = \mathcal{Y}_t\left(\left\{Y_{t+s}^{k-1}\right\}_{s\geq 1}\right). \tag{1.10}$$

Note that, in this model, people do not update expectations over time. This property is a well-known shortcoming of the level-$k$ approach to modeling bounded rationality. See, for example, Farhi and Werning (2019) and García-Schmidt and Woodford (2019). Like these authors, we think of our model as appropriate for analyzing people's behavior in the wake of rare or unprecedented events. Our propositions derive results for the full dynamic path of our model economy. We understand that the more time people spend in a new environment, like a binding ZLB episode, the more likely they will begin to learn about the equilibrium mapping from shocks and policies to economy-wide variables. But, as long as people do not learn about that mapping instantly, they are likely to underplay the importance of general-equilibrium effects. Because this feature is the crucial one underlying our results, the qualitative insights of our analysis would continue to hold even if beliefs were updated over time.

### 1.2.1 Government spending multipliers

This section assumes that consumption taxes are kept at their steady-state level $\tau_t^c = \tau^c$ for all periods and consider an increase in government spending, $\Delta G_t$, during the ZLB periods, i.e., for $t \leq T-1$.

**Rational expectations** In this model, the monetary authority pegs the real interest rate. It is widely understood that, under such a policy, there are multiple equilibria in the standard rational expectations NK model. As in Farhi and Werning (2019), we focus on rational expectations equilibria for which $Y_t \to 1$ as $t \to \infty$, i.e., the equilibrium converges to the pre-shock steady state. The household's Euler equation then implies that

$$C_t = C_{t+1} = C_{t+2} = \lim_{s \to \infty} C_{t+s} = 1$$

for all $t \geq T$.

During the ZLB period, the real interest rate is higher than the subjective discount rate. So consumption is lower than in the pre-shock steady-state:

$$C_t = (\beta e^{\chi})^{-\sigma} C_{t+1} = ... = e^{-\sigma(T-t)(\chi-\rho)}. \tag{1.11}$$

Here, $\rho \equiv -\log(\beta)$. The rational expectation equilibrium level of output is given by

$$Y_t^* = G_t + e^{-\sigma(T-t)(\chi-\rho)}.$$

Consistent with Bilbiie (2011) and Woodford (2011), equation (1.11) implies that government spending does not affect consumption in the rational expectations equilibrium. So, the government-spending multiplier is exactly equal to one

$$\frac{\Delta Y_t^*}{\Delta G_t} = 1, \tag{1.12}$$

where $\Delta Y_t$ denotes the difference in output relative to the output level in the equilibrium without government spending.

Note that in this simple model, the multiplier does not depend on the path of government spending. As it turns out, this result depends on the assumption of rational expectations.[14] To show this formally, we now turn to the temporary equilibrium.

[14] The multiplier would not be independent of the path of $G_t$ in more general versions of the NK model or a neo-classical growth model with savings, flexible hours worked and/or time-varying prices.

**With bounded rationality** Relation (1.9) implies that the temporary equilibrium is given by

$$\mathcal{Y}_t\left(\left\{Y^e_{t+s}\right\}\right) = G_t + \frac{\sum_{s\geq 1} Q_{t,t+s}\left[Y^e_{t+s} - G_{t+s}\right]}{\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} Q^{1-\sigma}_{t,t+s}}.$$

It seems natural to assume that level-1 people believe the economy goes back to its steady-state after the shock reverts to its pre-shock value, i.e., $Y^{e,1}_t = 1$ for $t \geq T$. This assumption implies that $Y_t$ is equal to its steady-state level for $t \geq T$. It follows that $Y^{e,k}_t = 1$ for all $k$ and $t \geq T$ . So we can write the equilibrium level of output for $t \leq T-1$ as follows

$$Y_t = G_t + \Omega_t \left\{ \sum_{s=1}^{T-t-1} \left[Y^e_{t+s} - G_{t+s}\right] + \frac{1}{1-\beta} \right\},$$

where $\Omega_t \equiv \left[e^{\sigma(\chi-\rho)}\left[\frac{1-e^{\sigma(\chi-\rho)(T-t-1)}}{1-e^{\sigma(\chi-\rho)}} + \frac{e^{\sigma(\chi-\rho)(T-t-1)}}{1-\beta}\right]\right]^{-1} \in (0,1]$.

**Lemma 1.** *In a temporary equilibrium, the government spending multiplier is given by*

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1}\left[\frac{\Delta Y^e_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t}. \tag{1.13}$$

Note that in a temporary equilibrium, the time $t$ government spending multiplier depends on people's beliefs regarding future income. Recall that this dependency is not a feature of the rational expectations equilibrium for our simple model.

The intuition about how beliefs about future government spending affect the time $t$ multiplier is as follows. First, if expectations for future incomes do not change with the policy ($\Delta Y^e_{t+s} = 0$) then the effect of future spending on current output is negative,

$$-\Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t}.$$

We refer to this effect as the *partial-equilibrium effect* of government spending: higher taxes associated with higher current and future expenditures lead to a negative wealth effect that causes people to reduce consumption.

The *general-equilibrium effect* of government spending is given by

$$\Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta Y^e_{t+s}}{\Delta G_{t+s}} \frac{\Delta G_{t+s}}{\Delta G_t}. \tag{1.14}$$

Higher future spending leads people to believe that their future incomes will be higher. The associated positive wealth effect leads to an increase in current consumption. Other things equal, this increase leads to a rise in actual current output. The fact that the government spending multiplier is one under rational expectations reflects that the partial and general-equilibrium effects exactly offset each other in this model.

We now consider the level-$k$ economy and show that, under plausible conditions, the less sophisticated people are, the less they take GE effects into account. This effect leads to a lower government spending multiplier.

For now, assume that level-1 people believe that aggregate output does not change in response to higher government spending, $\Delta Y_t^{e,1}/\Delta G_t = 0$ (below we relax this assumption). Then the government spending multiplier in a level-1 equilibrium is given by:

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t}.$$

The previous formula shows that the multiplier, $\Delta Y_t^1/\Delta G_t$, is less than one because level-1 agents only consider the partial-equilibrium effect of a change in government spending. In this case, level-1 people believe that their labor income will not be affected by higher spending but correctly anticipate that higher spending leads to higher taxes. So they think that their after-tax permanent income will fall. As a result, level-1 people react to the fiscal policy announcement by cutting back their consumption, leading to a lower spending multiplier.

More generally, the government spending multiplier for a level-$k$ economy can be computed using the recursive equation:

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t}, \quad t = 0, 1, ..., T-1, \tag{1.15}$$

where $\Delta Y_{t+s}^{k-1}/\Delta G_{t+s} = \Delta Y_{t+s}^{e,k}/\Delta G_{t+s}$ denotes the household's belief about future government spending multipliers. Note that if $\Delta Y_{t+s}^{k-1}/\Delta G_{t+s} \leq 1$ for all $t$ and $s$, then $\Delta Y_t^k/\Delta G_t \leq 1$ for all $t$. It follows that if level-1 people do not expect their incomes to change, then the government spending multiplier for a level-$k$ economy is always lower than the multiplier under rational expectations.

Furthermore, suppose that $\Delta Y_t^1/\Delta G_t > 0$ for all $t$, i.e., $1 - \Omega_t \sum_{s=1}^{T-t-1} \Delta G_{t+s}/\Delta G_t > 0$, then the spending multiplier in a level-2 economy is strictly higher than the multiplier in a level-1 economy:

$$\frac{\Delta Y_t^2}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^1}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} > 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^1}{\Delta G_t}.$$

More generally, as long as $\Delta Y_{t+s}^{k-1}/\Delta G_{t+s} \geq \Delta Y_{t+s}^{k-2}/\Delta G_{t+s}$ for all $t$, then (1.15) implies that $\Delta Y_{t+s}^k/\Delta G_{t+s} \geq \Delta Y_{t+s}^{k-1}/\Delta G_{t+s}$. It follows that the level-$k$ multiplier increases with cognitive ability $k$. Intuitively, the higher is $k$, the more people understand the general-equilibrium consequences of spending policy, and the lower is the contraction in consumption demand. So the larger is the spending multiplier.

In the discussion above, we assumed that level-1 individuals believe that their labor incomes and GDP are unaffected by the change in spending policy. To generalize the results above, suppose now that level-1 people think that $\Delta Y_t^{e,1}/\Delta G_t = \eta$ for all $t$ and $0 \leq \eta \leq 1$. A value of $\eta > 0$ corresponds to the assumption that level-1 people expect aggregate output will rise in response to higher government spending. A value of $\eta = 1$ corresponds to people's beliefs in the rational expectations equilibrium.

**Proposition 1.** *Suppose that $\Delta Y_t^{e,1}/\Delta G_t = \eta$ for all $t \leq T-1$.*

1. *If $0 \leq \eta < 1$, then the level-k government spending multiplier is lower than one, i.e., $\Delta Y_t^k/\Delta G_t \leq 1$ for all $t$. Furthermore, if $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \geq 0$ for all $t$, then the government spending multiplier is increasing in $k$.*

2. *If $\eta = 1$, then the level-k government spending multiplier is exactly one for all $k$, i.e.,* $\Delta Y_t^k/\Delta G_t = 1$ for all $t$.

According to this Proposition, the more sophisticated people are (higher $k$), the higher is the value of the multiplier. For finite $k$ and $\eta < 1$, the government spending multiplier is lower than under rational expectations. When $\eta = 0$, level-1 people believe that pre-tax labor income is unaffected by government spending. In this case, the multiplier is at its lowest. When $\eta = 1$, level-1 people believe that their *after-tax* income is unaffected by government spending, i.e., changes in government spending map one-to-one to changes in *pre-tax* income. In this case, the government spending multiplier is unaffected by the level of cognitive reasoning $k$. This result follows trivially from the fact that level-1 individuals expect the multiplier to be the same as in the rational expectations equilibrium.

With $\Delta Y_t^{e,1}/\Delta G_t = \eta$, the GE effect in the government spending multiplier, (1.14) is given by

$$\eta \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t}.$$

It follows from the Proposition that the multiplier is increasing in $\eta$ because the GE effect of an increase in government spending is larger.

Note that $\eta$ could be larger than 1, i.e., people believe their after-tax income will rise due to increased government spending. In this case, the multiplier is larger than one. In effect, the increase in government spending acts as a large, direct, exogenous increase in expectations about future income. This effect leads to a rise in current aggregate demand and output. We do not pursue this case because it seems inconsistent with the view that with bounded rationality, people place less emphasis on general-equilibrium effects than when they have rational expectations.

Recall that, based on survey evidence, authors like Coibion and Gorodnichenko (2015), Bordalo et al. (2012), and Angeletos et al. (2021) find that, on average, people's beliefs about macroeconomic aggregates like inflation and real GDP growth tend to *underreact* to changes in macro fundamentals relative to the rational expectations benchmark. These findings support the notion that $\eta$ is a relatively small number, strictly less than one.

In sum, proposition 1 shows that when $0 \leq \eta < 1$, departing from rational expectations by introducing level-$k$ thinking implies a decline in the size of the government spending multiplier. As discussed above, all households internalize the effects of higher taxes associated with higher government spending. However, understanding the expansionary impact of government spending requires that people compute how, in equilibrium, higher government spending leads to higher labor income. The less sophisticated people are, the less weight they give to the expansionary effect, the lower their expected future disposable income and the lower their current consumption is. In this way, lower levels of sophistication lead to lower values of government spending multipliers.

### 1.2.2 Consumption-tax policy

This section discusses the efficacy of consumption-tax policy when the ZLB is binding. Following Correia et al. (2013), we show that consumption-tax policy can implement the flexible-price allocation under rational expectations. We then evaluate the efficacy of consumption-tax policy under level-$k$ thinking and show that there always exists a policy that supports that allocation. Moreover, under plausible assumptions, that policy does not depend on the value of $k$ and its success does not depend on people making systematic errors in their beliefs about economy-wide variables.

Assume that government spending does not respond to the discount rate shock so that $G_t$ remains at its steady-state value of zero. Consumption taxes change during the ZLB period and converge back to their pre-shock level, $\tau^c$, once the economy exits the ZLB ($t = T$).

**Rational expectations** With time-varying consumption taxes, the household's Euler equation for $t \leq T - 1$ can be written as as

$$Y_t = Y_{t+1} \left( \beta \frac{\xi_{t+1}}{\xi_t} R_t \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right)^{-\sigma}$$

where we have set $C_t = Y_t$. This expression makes clear that the relevant relative price of consumption at time $t$ versus time $t+1$ is the real interest rate times the ratio of consumption taxes, $R_t \left(1 + \tau_t^c\right) / \left(1 + \tau_{t+1}^c\right)$.

We write this Euler equation in log terms,

$$y_t = y_{t+1} - \sigma \left( r_t + \log \left( \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) - (\rho - \chi) \right), \tag{1.16}$$

where $r_t = \log R_t = 0$. Note that, for $t \geq T$, the real interest rate returns to its pre-shock level, $r_t = \rho$, and $y_t = 0$ (or $Y_t = 1$).

Suppose that, at time 0, the government announces that taxes will follow the path $\tau_t^c = \tau_t^{c,*}$, where

$$\tau_t^{c,*} = (1 + \tau^c)\, e^{-(T-t)(\chi - \rho)} - 1 \tag{1.17}$$

for $t \leq T$. With this specification, the consumption tax falls at time 0 and then slowly converges back to its pre-shock value. Also, note that:

$$\log\left(\frac{1+\tau_t^{c,*}}{1+\tau_{t+1}^{c,*}}\right) = \rho - \chi.$$

Under this assumption, the relative price of consumption is equal to the subjective discount rate even if the nominal interest rate is at the ZLB.

Equation (1.16) implies that under this policy $y_t = y_{t+1}$ for all $t$. Since $y_t \to 0$ in the limit, it follows that this tax policy implements the flexible-price allocation, i.e., $y_t^* = 0$ for all t. The conclusion that tax policy can effectively circumvent the ZLB and achieve the flexible-price allocation is the key result in Correia et al. (2013).[15] We assumed that the government has access to lump-sum taxes. However, Correia et al. (2013) show that even if lump-sum taxes are unavailable, consumption taxes can still be used to fully offset the ZLB restriction and support the flexible-price allocation.

As emphasized by Correia et al. (2013), consumption taxes affect the relative price of leisure. So, in general, the government must change labor income taxes to compensate for the effects of changes in consumption taxes on labor supply. In our simple model, hours worked are demand determined so that labor-income taxes are equivalent to lump-sum taxes. We return to this point in section 1.3.

**Bounded rationality** Suppose that the government announces a path for consumption taxes, $\tau_t^c$, such that taxes go back to their pre-shock level as soon as the economy exits the ZLB, i.e., $\tau_t^c = \tau^c$ for $t \geq T$. In addition, suppose that everyone expects the economy to return to its pre-shock steady state once the ZLB is no longer binding. Then the temporary

[15] In a more general setting, Correia et al. (2008) show that fiscal policy can be used to neutralize the effects of price stickiness in standard NK models.

equilibrium level of output is given by:

$$Y_t = \left(\frac{1+\tau^c}{1+\tau_t^c}\right)^{\sigma} \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)Y_{t+s}^e + 1}{(1-\beta)\sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}}. \tag{1.18}$$

Equation (1.18) highlights the effect of time-varying consumption taxes on consumption and equilibrium output. For $t = T-1$, we can write this equation as

$$Y_{T-1} = \left(\frac{1+\tau^c}{1+\tau_{T-1}^c}\right)^{\sigma} e^{-\sigma(\chi-\rho)}.$$

This expression makes clear that setting $\tau_{T-1}^c = (1+\tau^c)\, e^{-(\chi-\rho)} - 1$ implements $Y_{T-1} = 1$.

It follows directly from (1.18) that, for given beliefs $Y_t^e$, there always exists an appropriate choice of $\tau_t^c$ for which $Y_t = 1$ for all $t$. However, in models of belief revision like level-$k$ thinking, beliefs are endogenous to the policy that is implemented. Still, Proposition 2 shows that for every level of cognitive ability $k$, there is an appropriately chosen path for consumption taxes that implements flexible-price allocation. As agents become more sophisticated, this policy approaches the rational expectations optimal policy, $\tau_t^{c,*}$. In general, the path of consumption taxes which implements the flexible-price allocation depends on $k$. However, if the expectations of unsophisticated agents about aggregate output are anchored at the initial steady state, then the policy that achieves full stabilization is the same regardless of $k$. Moreover, that policy coincides with the optimal policy under rational expectations.

**Proposition 2.** *Suppose that level-*1 *people believe that the economy goes back to steady state after the ZLB period, i.e.,* $Y_t^{e,1} = 1$ for $t \geq T$.

1. *For each $k$, there exists a policy announcement* $\left\{\tau_t^{c,k}\right\}$ *which implements the flexible-price allocation.*

2. *Suppose that* $Y_t^{e,1} = 1$ *for all* $t \geq 0$*, then the policy announcement* $\{\tau_t^{c,*}\}$ *implements*

*the flexible-price allocation for all $k$.*

In the appendix, we prove the first result in the Proposition. Specifically, we show how to construct the path for consumption taxes that implements the flexible-price allocation for a given level of cognitive sophistication. In general, this policy is a function of $k$, which means that its correct design would require the government to know the people's cognitive sophistication.

A simple proof of the second result in the Proposition is as follows. Recall that under the tax policy $\{\tau_t^{c,*}\}$, the rational expectations equilibrium is $Y_t^* = 1$. By definition, this equilibrium is a fixed point of the temporary equilibrium relation (1.18). Suppose level- 1 individuals expect the aggregate output to remain at its steady-state level. In that case, they will adjust their behavior so that it is the same equilibrium outcome, i.e., $Y_t^1 = 1$. Since level-2 individuals believe that the equilibrium is $Y_t^{e,2} = Y_t^1 = 1$, then the level-2 equilibrium is the same as the level-1 equilibrium. The same logic applies for any $k$. We conclude that the belief $Y_t^{e,1} = 1$ is self-confirming under the proposed tax policy. It immediately follows that the proposed tax policy does not rely on people making mistakes. On the contrary, the tax policy leads to an equilibrium in which people's beliefs coincide with actual outcomes.

It is useful to contrast the efficacy of tax rate and interest-rate policy. In our model, changing the announced path of tax rates and interest rates affects the equilibrium in the same way. However, there is one crucial difference. The ZLB constrains the class of feasible announced paths for interest rates. So, monetary policy can only boost consumption demand via forward guidance, i.e., a promise to lower interest rates in the future after the ZLB is no longer binding. Farhi and Werning (2019) and García-Schmidt and Woodford (2019) show that the strong stimulative power of forward guidance relies heavily on general-equilibrium effects. Those effects become muted when people are boundedly rational. Instead, consumption taxes can be changed as soon as the ZLB becomes binding. So, tax policy can effectively counteract the effects of the discount factor shock and support the flexible-price allocation. In our analysis, this flexibility implies that consumption-tax rates have an important advantage relative to interest rate policy in circumstances where the ZLB is binding.

**Rules versus targets**

Proposition 2 provides a strong rationale for using tax policy to fight recessions at the ZLB. In this section, we highlight that the efficacy of the policy depends crucially on how it is communicated. We consider two communication strategies. First, tax policy is communicated and implemented as a sequence of *targets* for consumption taxes. Second, tax policy is communicated and implemented as a *rule* involving endogenous objects like the output gap. We refer to these two strategies as target-based and rule-based communication policies. The reason that communication matters in our setting is straightforward. Under target-based communication, individuals immediately know what tax rates will be in the future and incorporate those rates into their decisions. But under rule-based communication, individuals must work out the future general-equilibrium effects of the policy to understand what current and future tax rates will be. In a world populated by level-$k$ thinkers, this difference matters.

Assume that monetary policy is given by a Taylor rule subject to a ZLB constraint

$$R_t = \max\left\{\beta^{-1} Y_t^{\phi_y}, 1\right\} \Leftrightarrow r_t = \max\left\{\rho + \phi_y y_t, 0\right\} \tag{1.19}$$

where $\phi_y$ denotes the elasticity of $R_t$ to the output gap.[16] As in the quantitative analysis of Correia et al. (2013), we assume that consumption taxes are set as:

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\beta^{-1} Y_t^{\phi_y}, 1\right\} \Leftrightarrow \log \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\rho + \phi_y y_t, 0\right\}. \tag{1.20}$$

[16] We do not include inflation in the Taylor rule because inflation is always zero for our simple economy.

Under this policy, consumption-tax rates do not change when the ZLB does not bind. But if the ZLB binds, then consumption-tax rates do change. Regardless of whether ZLB binds, the relative price of consumption is given by:

$$R_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \beta^{-1} Y_t^{\phi_y}.$$

Critically, under this announced policy, agents must predict current and future output values to forecast future tax rates, a calculation that involves general-equilibrium effects.

The temporary equilibrium is given by

$$\mathcal{Y}_t\left(\left\{Y_{t+s}^e\right\}\right) = \left[\beta^\sigma \frac{\sum_{s=1}^\infty Q_{t+1,t+s}^e \left(\frac{1+\tau_{t+s}^{c,e}}{1+\tau_{t+1}^{c,e}}\right) Y_{t+s}^e}{\sum_{s=1}^\infty \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^\sigma \left[Q_{t+1,t+s}^e \frac{1+\tau_{t+s}^{c,e}}{1+\tau_{t+1}^{c,e}}\right]^{1-\sigma}}\right]^{\frac{1}{1+\sigma\pi}}. \tag{1.21}$$

where $Q_{t+1,t+s}^e \left(\frac{1+\tau_{t+s}^{c,e}}{1+\tau_{t+1}^{c,e}}\right) \equiv \beta^{s-1} \prod_{\tau=t+1}^{t+s-1} \left(Y_t^e\right)^{-\phi_y}$.

**Rational expectations** As before, once the economy exits the ZLB, output returns to its pre-shock steady state, so that $y_t = 0$ for $t \geq T$. For earlier dates, we can find the equilibrium using the individual's Euler equation:

$$y_t = y_{t+1} - \sigma\left(\rho + \phi_y y_t - \rho + \chi\right) \Leftrightarrow y_t = \frac{y_{t+1} - \sigma\chi}{1+\sigma\phi_y}.$$

Iterating forward, we obtain the rational-expectations level of log-output

$$y_t^* = -\frac{\chi}{\phi_y}\left[1 - \frac{1}{\left(1+\sigma\phi_y\right)^{T-t}}\right]. \tag{1.22}$$

As long as the policy is not infinitely reactive ($\phi_y \to \infty$), then the rules-based policy will not achieve the flexible-price allocation.

The equilibrium path for consumption taxes under this policy is:

$$\log\left(\frac{1+\tau_t^{c,r}}{1+\tau_{t+1}^{c,r}}\right) = \rho - \chi\left[1-(1+\sigma\phi_y)^{-(T-t)}\right], \tag{1.23}$$

and $r_t = 0$ for $t \leq T-1$.

To evaluate the relative power of rules- versus targets-based policy under bounded rationality, we compute the level-$k$ equilibrium under a rules-based policy and the policy that announces consumption tax targets that satisfy (1.23). This comparison preserves the underlying rational expectations equilibrium under each type of policy communication.

**Bounded rationality** We now describe the implications of bounded rationality for the efficacy of rules-based policy. It is convenient to consider the benchmark case in which $y_t^{e,1} = 0$. So, in this case, as in Farhi and Werning (2019), level-1 people's expectations are anchored at the initial steady state. For our purpose, what's important is that people expect output to fall less when the ZLB binds than they do under rational expectations.

We begin by describing the equilibrium for level-1 individuals. In the appendix, we show that, under rule-based communication, level-1 equilibrium log-output is given by

$$y_t^1 = -\frac{\sigma\chi + \varphi_t}{1+\sigma\phi_y}, \tag{1.24}$$

where $\varphi_t$ is a function of structural parameters.

When policy is communicated as the target path which satisfies (1.23), the equilibrium can be computed using (1.18). Our next proposition summarizes our main result.

**Proposition 3.** *Suppose that $y_t^{e,1} = 0$ for all $t$. If policy is announced as a target for consumption-tax rates, then $y_t^1 \geq y_t^*$ with equality only if $t = T-1$. Suppose that $\beta > (1+\sigma\phi_y)^{-1}$. If policy is announced as a rule, then $y_t^1 \leq y_t^*$ with equality only if $t = T-1$.*

The condition that $\beta > (1+\sigma\phi_y)^{-1}$ is easily satisfied in standard calibrations. For example, the calibration for the medium-scale DSGE model in Christiano et al. (2011) features $\sigma = 0.5$ and $\phi_y = 0.25$, which implies that $(1+\sigma\phi_y)^{-1} = 0.89$, which is lower than the value of $\beta$ that they assume.

According to Proposition 3, consumption-tax policy is less powerful under rule-based communication than when policy is communicated via targets. The intuition is as follows. Under a rules (and targets) based policy, level-1 people don't understand that future output will be lower after the discount-rate shock. Other things equal, this error implies that their consumption will be higher than under rational expectations. Under a rules-based policy, level-1 people don't think output will change. So, they don't believe that future consumption-tax rates will change. Other things equal, this error implies that their consumption will be lower than under rational expectations. If $\beta > (1+\sigma\phi_y)^{-1}$, then the effect of the second error dominates the effect of the first error, and output is *lower* in the level-1 equilibrium than in the rational expectations equilibrium.

Under target-based communication, level-1 people internalize the exact path of future consumption-tax rates. So, the expansionary effects of the tax rate change become operative even if people are not very sophisticated. This effect is as strong as it would be under rational expectations. But level-1 people still underestimate the decline in their future income. So, consumption and output are *higher* than under rational expectations.

As it turns out, a version of the proposition extends to the case where $y_t^* \leq y_t^{e,1} \leq 0$, i.e., level-1 people expect output to fall, but by less than it would under rational expectations. To simplify, consider a log-linear version of the economy in which case log-output is given by

$$y_t = -\left[\beta - \frac{1}{1+\sigma\phi_y}\right] \sum_{s=1}^{T-t-1} \beta^{s-1} y_{t+s}^e - \frac{\sigma\chi}{1+\sigma\phi_y} \frac{1-\beta^{T-t}}{1-\beta}. \tag{1.25}$$

As before, we assume that people believe output goes back to steady-state after $t = T$.

The extended proposition immediately follows from the assumption that $y_t^{e,1} \geq y_t^*$ and $\beta > (1+\sigma\phi_y)^{-1}$.

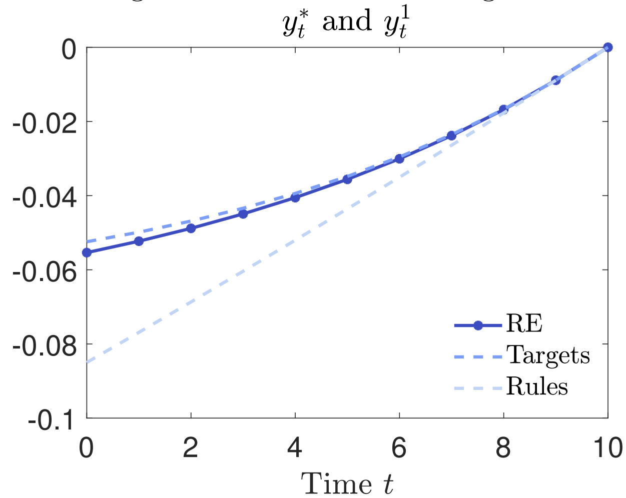
Figure 1.1 illustrates the properties of the rational expectations and level-1 equilibria under rules- and targets-based communication. We set the discount factor $\beta$ to 0.99, the intertemporal elasticity of substitution $\sigma$ to 0.5, and the coefficient on output in the Taylor rule, $\phi_y$, equal to 0.25.[17] We assume that the ZLB lasts for ten periods, $T = 10$, and we choose the discount rate shock so that $\beta e^{\chi} = 1.01$,and $\chi = 0.02$. Finally, we assume that $y_t^{e,1} = 0$. Four findings emerge from Figure 1.1. First, equilibrium output under target-based communication is close to the rational expectations equilibrium output level. Second, equilibrium output under rule-based communication is much lower than the rational expectations equilibrium output. Third, the poor performance of rule-based communication is more pronounced the earlier we are in the ZLB episode, i.e., the longer the episode is expected to last. Finally, Figure 1.1 shows that, with targets-based communication, output is higher in the level-1 equilibrium than in the rational expectations equilibrium. In that sense, the same policy is more potent at stabilizing output when people are not very sophisticated. As it turns out, this result holds for all levels of $k$.

The following proposition summarizes how the efficacy of targets-based communication policy depends on $k$. In line with the discussion above, we derive the results for the general case in which level-1 people believe that output falls by less than it would under rational expectations.

**Proposition 4.** *Suppose that the government announces the target for tax policy $\tau_t^r$, given by (1.23), and suppose that $y_t^* \leq y_t^{e,1} \leq 0$ for all $t$. Suppose, furthermore, that level-1 people believe that the economy goes back to steady state after the ZLB period, i.e., $Y_t^{e,1} = 1$ for $t \geq T$. Then, for any $k$, output in the level-$k$ equilibrium is higher than under rational expectations, i.e., $y_t^k \geq y_t^*$. Furthermore, $y_t^k$ converges monotonically to $y_t^*$ as $k \to \infty$.*

[17] These parameters satisfy the condition in Proposition 3.

Figure 1.1: Rules versus targets




This proposition shows that under target-based communication, the consumption-tax policy under consideration becomes more powerful the less sophisticated people are. The intuition follows from the discussion after proposition 3. As $k$ increases, people expect an increasingly large recession after the discount rate shock. So, equilibrium consumption and output drop by more as $k$ increases, eventually converging to the rational expectations equilibrium.

To extend the previous analysis of rule-based communication when $k > 1$, we must confront the following well-known problem. Under rules-based communication, the level-$k$ model under consideration exhibits a peculiar type of oscillatory behavior as a function of $k$. The equilibrium level of output lies below the rational expectations equilibrium level for odd levels of $k$ but is above it when $k$ is even. The log-linearized version of the temporary equilibrium is given by (1.25). Since output in the level-1 equilibrium is lower than under rational expectations, level-2 people believe that $y_t^{e,2} = y_t^1 < y_t^*$. Since $\beta - (1 + \sigma\phi_y)^{-1} > 0$ it follows that the level-2 equilibrium level of output is higher than the rational expectations equilibrium level of output, $y_t^2 > y_t^*$. This oscillatory pattern emerges more generally as a function of $k$.

This peculiar oscillatory feature reflects a more general oscillatory behavior in standard level-$k$ thinking models discussed in García-Schmidt and Woodford (2019) and Angeletos and Sastry (2021). They argue that this feature is a “bug” of the standard level-$k$ thinking approach, which is not present in other similar models of bounded rationality.

A key question is whether our key conclusions are robust to other models of bounded rationality which do not feature this bug. To address this question, we proceed as follows. First, in the main text, we redo the analysis in the previous figure for various levels of $k$ in a generalized level-$k$ thinking model. Second, in appendix A.2, we redo the analysis of this section for (i) a *generalized level-k thinking* model based on Camerer et al. (2004), (ii) a *reflective expectations* model based on García-Schmidt and Woodford (2019), and (iii) a *shallow reasoning* model based on Angeletos and Sastry (2021). All of our previous results go through for these alternative models of bounded rationality.

**Generalized level-$k$ thinking** This section considers the effects of rules-based policy in a generalized level-$k$ economy for the log-linearized economy. Following Camerer et al. (2004), we assume that level-$k$ individuals think that other people are distributed over lower levels of cognitive ability according to the distribution $f_k(j)$ for $0 \leq j \leq k-1$. The reasoning process underlying the generalized level-$k$ model is analogous to the standard level-$k$ model process. As in Farhi and Werning (2019), we assume that contemporaneous output, $y_t$, is observed.

To analyze this economy, we must introduce the concept of a level-0 person. This type of person continues to act as they did before the discount rate shock, i.e., their consumption decisions are such that $y_t^0 = 0$.

Level-1 individuals believe that the economy is populated by level-0 people so $y_t^{e,1} = y_t^0 =$

0. Given current output $y_t$,

$$c_t^1(y_t) = -(\beta(1+\sigma\phi_y)-1)\,y_t - (\beta(1+\sigma\phi_y)-1)\sum_{s=1}^{\infty}\beta^s y_{t+s}^{e,1} - \sigma\beta\chi\frac{1-\beta^{T-t}}{1-\beta}. \tag{1.26}$$

Suppose that the economy is populated entirely by level-1 individuals. Solving (1.26) for $y_t^1$ yields,

$$y_t^1 = -\left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{\infty}\beta^{s-1}y_{t+s}^{e,1} - \frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}.$$

Level-2 individuals believe that a fraction $f_2(j)$ of the population is level $j=0,1$ and work through the problem of level-0 and level-1 people. So they believe that $y_t^2$ is the solution to

$$y_t^{e,2} = \sum_{j=0}^{1} f_2(j)\,c_t^j\left(y_t^{e,2}\right).$$

More generally, level-$k$ people believe that output is the solution to

$$y_t^{e,k} \equiv \sum_{j=0}^{k-1} f_k(j)\,c_t^j\left(y_t^{e,k}\right). \tag{1.27}$$

Since contemporaneous output is observed, people with different cognitive levels expect different consumption levels for less sophisticated people than themselves. Technically, this means that level-$k$ people think that level-$j$ people's consumption is given by:

$$c_t^j(y_t) = -(\beta(1+\sigma\phi_y)-1)\,y_t - (\beta(1+\sigma\phi_y)-1)\sum_{s=1}^{\infty}\beta^s y_{t+s}^{e,j} - \sigma\beta\chi\frac{1-\beta^{T-t}}{1-\beta}, \tag{1.28}$$

for $j \geq 1$.

Using conditions (1.27) and (1.28), the beliefs of level-$k$ individuals can be written as

$$y_t^{e,k} = \sum_{j=0}^{k-1} f_k(j)\,y_t^j,$$

where

$$y_t^j \equiv -\left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{\infty}\beta^{s-1}y_{t+s}^{e,j} - \frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta},$$

for $j \geq 1$ and $y_t^0 = 0$.

Camerer et al. (2004) assume that the distributions $f_k(\cdot)$ are consistent with the physical distribution of cognitive levels in the economy. In contrast, we maintain the representative agent assumption so that everyone shares the same level $k$. We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. The distributions $f_k(\cdot)$ are such that for any $k_1 < k_2$ and $s, s' < k_1$

$$\frac{f_{k_1}(s)}{f_{k_1}(s')} = \frac{f_{k_2}(s)}{f_{k_2}(s')}. \tag{1.29}$$

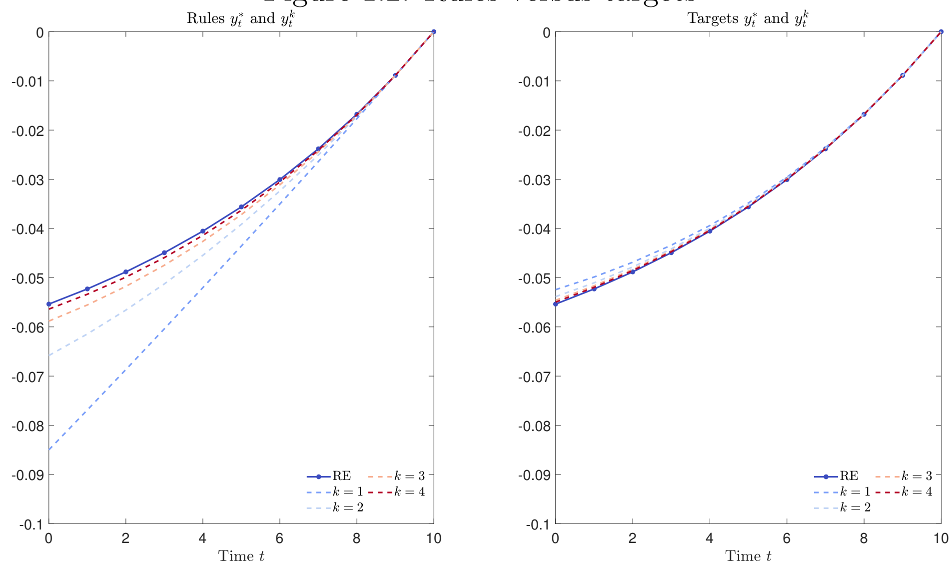
Let $\gamma_k \equiv f_k(k-1)$ for all $k$. Then assumption (1.29) implies that $f_k(j) = (1-\gamma_k) f_{k-1}(j)$ for $j \leq k-2$. We can write the expectation of level-$k$ individuals as follows:

$$y_t^{e,k} = (1-\gamma_k)\sum_{j=0}^{k-2} f_{k-1}(j)\, y_t^j + \gamma_k y_t^{k-1} = (1-\gamma_k)\, y_t^{e,k-1} + \gamma_k y_t^{k-1}. \tag{1.30}$$

Intuitively, the beliefs of a level-$k$ thinker are given by a weighted average of the beliefs of level-$(k-1)$ agents and the equilibrium that would arise if everyone in the economy was a level-$(k-1)$ thinker. Standard level-$k$ thinking corresponds to the case of $\gamma_k = 1$. By varying $\gamma_k$, we can control the intensity of updating across level-$k$ iterations.

Figure 1.2 displays the numerical solution for this economy under rational expectations as well as the four lowest levels of cognitive sophistication. The parameter values are the same as those used for Figure 1.1. For illustrative purposes, we assume that $\gamma_k = 0.5$ so that level-$k$ people think that half of the population is level $k-1$. In practice, we find that our qualitative results are robust to moderate perturbations of $\gamma_k$. The left and right panels show the equilibrium for the case in which policy is communicated as a rule and as a sequence of targets, respectively.

Figure 1.2: Rules versus targets




A number of key results emerge from Figure 1.2. First, rule-based communication does not lead to oscillatory behavior in this model economy as people become more sophisticated. The reason is that expectations about income are updated more smoothly than under standard level-$k$ thinking. Second, target-based communication does better than rules-based communication in stabilizing output. For any given $k$, target-based communication results in a higher level of output than under rational expectations. But the opposite is true of rule-based communication. The intuition for these results follows from our discussion of the level-1 economy. Third, under rules-based communication, the level of people's sophistication is an important determinant of the size of the recession. Indeed, if people are not very sophisticated, output can be two to three percentage points lower than under rational expectations. In contrast, the level of sophistication is quantitatively less relevant under target-based communication. Finally, as was the case under standard level-$k$ thinking, under rules-based communication, the differential impact of $k$ on output is larger the longer the ZLB period is expected to last.

## 1.3 A model with Calvo-style wage rigidities

This section extends the baseline model to allow for time-varying prices and wages. We do so by introducing Calvo-style wage rigidities as in Erceg et al. (2000) and Schmitt-Grohé and Uribe (2005). In Appendix A.4, we show that our results are robust to assuming Calvo-style price rigidities.

The model economy is populated by a continuum of households, unions, goods producers, and the government. Each household has a continuum of workers who have different labor skills. Output can be used for private or government consumption so that the aggregate resource constraint is still given by (1.3).

**Goods producer** The final good is produced by a representative firm using a Cobb-Douglas technology from a fixed stock of capital, $\bar{K}$, and a composite labor input, $N_t$:

$$Y_t = A\overline{K}^{\alpha} N_t^{1-\alpha}, \tag{1.31}$$

where $A > 0$ denotes total-factor productivity, and $\alpha \in [0, 1]$ denotes the capital share of output. We assume that capital is fixed for simplicity and to avoid complications in modeling investment decisions when agents have bounded rationality. This assumption can be rationalized for business cycle dynamic analysis if there are large capital adjustment costs (see for example Rotemberg and Woodford, 1997 and Farhi and Werning, 2019).

The composite labor input $N_t$ is generated using a continuum of labor varieties according to the technology:

$$N_t = \left[ \int_0^1 n_{u,t}^{\frac{\theta-1}{\theta}} \, du \right]^{\frac{\theta}{\theta-1}}, \tag{1.32}$$

where $\theta > 1$ captures the elasticity of substitution across the labor varieties. The firm, which is perfectly competitive in both the goods and the labor market, produces final output using the technology given by (1.31) and (1.32). The firm maximizes

$$P_t Y_t - \int_0^1 w_{u,t} n_{u,t} du$$

subject to (1.31) and (1.32). Here $P_t$ denotes the price of the consumption good and $w_{u,t}$ denotes the wage of $n_{u,t}$. The solution to this problem is given by:

$$n_{u,t} = \left( \frac{w_{u,t}}{W_t} \right)^{-\theta} N_t, \tag{1.33}$$

where

$$W_t = \left[ \int_0^1 w_{u,t}^{1-\theta} du \right]^{\frac{1}{1-\theta}}, \tag{1.34}$$

and

$$\frac{W_t}{P_t} = (1-\alpha) A \left( \frac{\overline{K}}{N_t} \right)^{\alpha}. \tag{1.35}$$

**Households** The household enters period $t$ with financial assets $B_t$ which earn the interest rate $R_{t-1}$. As in section 1.2, we assume that the household knows its time-$t$ income $Y_t$ and taxes $T_t$. When solving its dynamic consumption-savings problem, the household maximizes its perceived utility which is evaluated based on today's consumption, $C_t$, and on its plans for future consumption, $\tilde{C}_{t+s}$ for $s = 1, 2, ....$ Labor supply is determined by labor unions as described below. We denote by $L_t$ the total hours worked by the household,

$$L_t = \int_0^1 n_{u,t}.$$

With wage dispersion induced by nominal rigidities, $L_t$ is not to equal $N_t$.

The representative household maximizes (1.1) subject to

$$\left(1+\tau_{t+s}^{c}\right) P_{t+s}^{e} \tilde{C}_{t+s}+\tilde{B}_{t+s+1}=\left(1-\tau_{t+s}^{n}\right) W_{t+s}^{e} N_{t+s}^{e}+\Omega_{t+s}^{e}+R_{t+s-1} \tilde{B}_{t+s}-T_{t+s}^{e},$$

where $\Omega_{t+s}^{e}$ denotes lump-sum profits from firms and $\tau_t^n$ denotes the time $t$ tax rate on labor income.

The household has perfect foresight with respect to exogenous variables, including the discount rate shock, $\xi_t$. For now, we assume that the government announces sequences of nominal interest rates, $\{R_t\}$, government spending, $\{G_t\}$, and taxes $\{\tau_t^c, \tau_t^n\}$. Household beliefs for $T_t^e$ satisfy:

$$\sum_{s\geq 0} Q_{t,t+s} T_{t+s}^{e}=\sum_{s\geq 0} Q_{t,t+s}\left[P_{t+s}^{e} G_{t+s}-\tau_{t+s}^{c} P_{t+s}^{e} C_{t+s}^{e}-\tau_{t+s}^{n} W_{t+s}^{e} N_{t+s}^{e}\right]+R_{t-1} B_{t}. \tag{1.36}$$

Along with our other assumptions, (1.36) implies that Ricardian equivalence holds in our model.

As shown in appendix A.3.1, the solution to the household's problem implies

$$C_{t}=\frac{\sum_{s\geq 0} Q_{t,t+s} \frac{P_{t+s}^{e}\left(1+\tau_{t+s}^{c}\right)}{P_{t}\left(1+\tau_{t}\right)}\left[Y_{t+s}^{e}-G_{t+s}\right]}{1+\sum_{s\geq 1}\left(\beta^{s} \frac{\xi_{t+s}}{\xi_{t}}\right)^{\sigma}\left[Q_{t,t+s} \frac{P_{t+s}^{e}\left(1+\tau_{t+s}^{c}\right)}{P_{t}\left(1+\tau_{t}\right)}\right]^{1-\sigma}}. \tag{1.37}$$

**Labor market and unions** Unions decide wages. In the presence of sticky wages, actual employment is demand determined. Each household supplies $n_{u,t}$ units of type $u$ labor to a union indexed by $u \in [0,1]$. Union $u$ faces labor demand given (1.33).

The union sets wages subject to Calvo-style frictions. At each date, $1-\lambda$ unions are randomly selected to adjust their wage, $w_{u,t}$. For the other $\lambda$ unions, $w_{u,t}=w_{u,t-1}$. Unions act on behalf of households and choose wages and labor hours to maximize the expected household's valuation of labor income.

In a symmetric equilibrium, unions that can reset their wages choose the same value. We denote the common new reset wage by $W_t^*$. In appendix A.3.2, we show that $W_t^*$ satisfies

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s \xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e v'\left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty}(\beta\lambda)^s \xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}. \tag{1.38}$$

The union has perfect foresight with respect to exogenous variables but is boundedly rational with respect to endogenous variables. In particular, we assume that the union forms beliefs about future aggregate prices, $P_t^e$, wages, $W_t^e$, consumption, $C_t^e$, the labor composite, $N_t^e$, and labor input, $L_t^e$, using level-$k$ thinking.

**Monetary and fiscal policies** Nominal interest rates during and after the ZLB period are as described in the benchmark model. The fiscal authority sets government spending $G_t$, consumption taxes $\tau_t^c$, labor income taxes $\tau_t^n$, and lump-sum taxes $T_t$, subject to the intertemporal budget constraint:

$$\sum_{s\geq 0} Q_{t,t+s}P_{t+s}G_{t+s} + R_{t-1}B_t = \sum_{s\geq 0} Q_{t,t+s}\left[\tau_{t+s}^c P_{t+s}C_{t+s} + \tau_{t+s}^n W_{t+s}N_{t+s} + T_{t+s}\right]. \tag{1.39}$$

**Temporary Equilibrium** As in Farhi and Werning (2019), we assume that people's beliefs regarding future nominal prices and wages are scaled by $P_t/P_t^e$. This assumption allows people to incorporate current and past surprise inflation into their beliefs, leaving beliefs about future inflation and real wages unchanged.

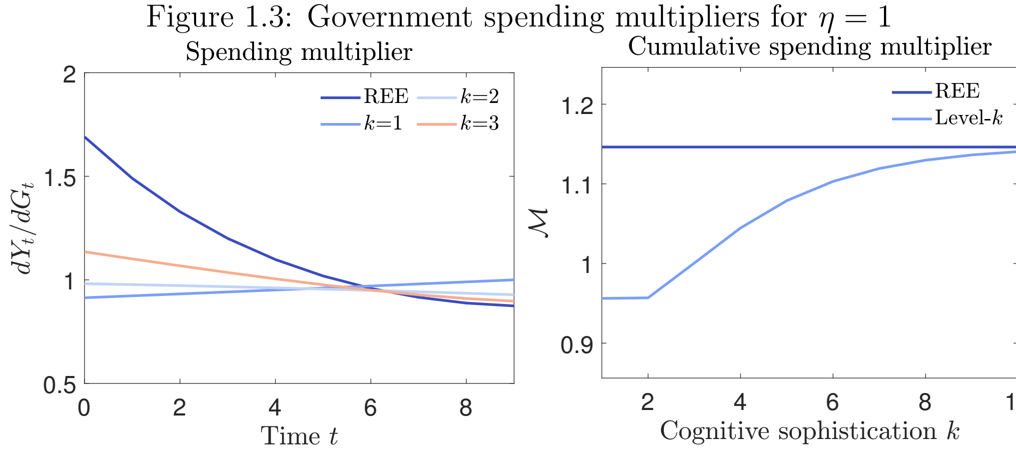
For each date $t$, given beliefs $\mathcal{A}_t^e = \left\{Y_t^e, C_t^e, N_t^e, L_t^e, P_t^e/P_{t-1}^e, W_t^e/P_t^e\right\}$, a temporary equilibrium is a sequence of allocations and prices $\mathcal{A}_t = \{Y_t, C_t, N_t, L_t, P_t/P_{t-1}, W_t/P_t\}$ in which households, firms, and unions solve their optimization problem, and goods markets clear. In appendix A.3, we summarize the equations whose solution defines an equilibrium for this economy. In addition, we present the log-linearized system and show how to compute generalized level-$k$ equilibria in which beliefs evolve analogously to those in equation (1.30).

**Calibration** As in section 1.2, we assume that the elasticity of intertemporal substitution is $\sigma = 0.5$, $\beta = 0.99$, $\chi = 0.02$, and $G/Y = 0.2$. Consistent with the evidence in Chetty et al. (2011) we set the Frisch elasticity is $\varphi^{-1} = 0.75$. We normalize $\overline{K} = 1$ and set the capital share, $\alpha$, to 0.33. In addition, we set total factor productivity, $A$, so that steady state output is equal to one. Following Correia et al. (2013), we assume that the elasticity of substitution across labor types $\theta$ is equal to 3, and the Calvo parameter $\lambda$ is 0.85. We set the steady-state tax rates $\tau^c$ and $\tau^n$ equal to 0.05 and 0.28, respectively. Finally, we assume that level-1 beliefs about aggregate output are anchored at the initial steady state, i.e., $Y_t^{e,1} = 1$.

### 1.3.1 Government spending multipliers

This section briefly illustrates the analog to Proposition 1 for the case in which tax rates are constant and government spending rises by $\Delta G$ during the ZLB period.

Figure 1.3: Government spending multipliers for $\eta = 1$




The left panel of Figure 1.3 displays the government spending multiplier, $\Delta Y_t/\Delta G_t$, computed under the assumption of rational expectations and for various levels of $k$. Under rational expectations, this multiplier is initially close to 1.5. Consistent with results in the NK literature, the large size of this multiplier reflects the fact that government spending induces inflation, which lowers the real interest rate during the ZLB period. Because of intertemporal substitution effects, this fall induces households to raise their demand for consumption which raises output. Other things equal, perfectly rational agents understand that these intertemporal substitution effects increase current and future output. In a virtuous cycle, the rise in future income raises people's permanent income, raising current spending and inflation. The latter effect lowers the real interest rate, strengthening the intertemporal substitution effect. The net effect is a sequence of large multipliers, exceeding one in value.

To assess the impact of level-$k$ thinking, it is useful to define the cumulative spending multiplier as[18]

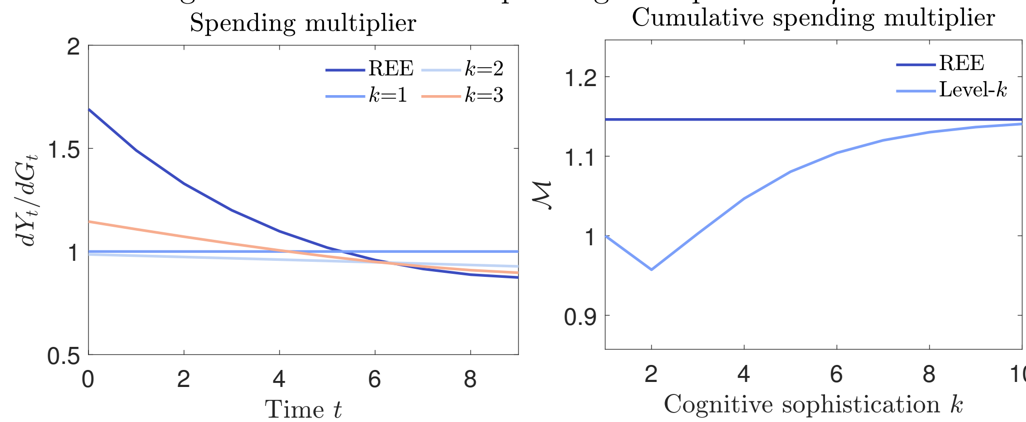
$$\mathcal{M} \equiv \frac{\sum_t \Delta Y_t}{\sum_t \Delta G_t} = \sum_t \frac{\Delta G_t}{\sum_t \Delta G_t} \frac{\Delta Y_t}{\Delta G_t}.$$

The right panel of Figure 1.3 shows that the cumulative multiplier increases with $k$. The intuition is as follows. The lower the cognitive level of individuals, the less they understand the general-equilibrium effects of spending on total GDP and inflation. So lower level-$k$ people predict a relatively small rise in their income and inflation in response to the increase in government spending. The result is that the lower is $k$, the smaller is the rise in consumption induced by government spending. Indeed level-1 and level-2 people cut their spending because the tax effects of an increase in government spending outweigh the income effects.

Taken together, the results in this section reinforce the message from the benchmark model: bounded rationality weakens the case for the efficacy of government spending as a tool for stabilizing output in the face of a shock that causes the ZLB to bind.

[18]Since the cumulative multiplier can be decomposed into a weighted sum of the time $t$ multipliers, the results in Proposition 1 for the benchmark model also hold for the cumulative multiplier.

Figure 1.4: Government spending multipliers for $\eta = 1$




## 1.3.2 Consumption-tax policy

This section considers the efficacy of tax policy in the extended version of our benchmark economy. Our key result is that Proposition (2) continues to hold so that tax policy can support the flexible-price allocation even when prices and wages are not fully rigid.

Under rational expectations, the requisite tax policy sets consumption taxes according to

$$\tau_t^{c,*} = (1 + \tau^c)\, e^{-(T-t)(\chi-\rho)} - 1.$$

Recall that in the benchmark economy, wages are fully rigid. Employment is determined entirely by the demand for labor. In the extended model, consumption taxes, $\tau_t^{c,*}$, induce distortions in labor supply which affect the equilibrium because wages aren't perfectly rigid. To support the flexible-price allocation, the government must adjust labor taxes to undo these distortions:

$$\frac{1 - \tau_t^{n,*}}{1 + \tau_t^{c,*}} = \frac{1 - \tau^n}{1 + \tau^c}.$$

Under this policy, the tax wedge on labor supply is constant over time. Critically, the government announces its policy for $\tau_t^{c,*}$ and $\tau_t^{n,*}$ as a sequence of tax rate *targets*.

We now state the analog to Proposition (2) for the extended model.

**Proposition 5.** *Suppose that level-*1 *people believe that the economy goes back to steady state after the ZLB period, i.e.,* $\mathcal{A}_t^{e,1} = \mathcal{A} \equiv \{Y, C, N, L, 1, W/P\}$ for $t \geq T$. *Consider the log-linearized version of the model economy. Then,*

1. *For each* $k$*, there exists a policy* $\left\{\tau_t^{c,k}, \tau_t^{n,k}\right\}$ *which implements the flexible-price allocation.*

2. *Suppose that* $\mathcal{A}_t^{e,1} = \mathcal{A}$ *for all* $t \geq 0$*, then the policy* $\{\tau_t^{c,*}, \tau_t^{n,*}\}$ *implements the flexible-price allocation for all* $k$*.*

Here $\mathcal{A}_t^{e,k}$ denotes the beliefs of level-$k$ people. This proposition generalizes Proposition 2 to the extended model and demonstrates that tax policy is still very powerful even under bounded rationality in the presence of time-varying wages and prices.

### Rules versus targets

This section revisits the effectiveness of rules-based communication in the extended model. As in Correia et al. (2013), we assume that the interest rate is given by a Taylor rule subject to a ZLB constraint,

$$R_t = \max\left\{\beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}, 1\right\}. \tag{1.40}$$

Here $\phi_\pi$ is the coefficient on realized inflation and $\phi_y$ is the elasticity of the interest rate with respect to the output gap. The rule for consumption taxes and labor-income taxes is

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}, 1\right\}, \tag{1.41}$$

and

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}. \tag{1.42}$$

Critically, the government announces tax policy in the form of the *rules*, (1.40)-(1.42).

Proposition 3 follows trivially for the extended model with $k = 1$ because everyone expects inflation to be zero and output to remain at its steady-state level. However, in general, it is not possible to prove the analog proposition for $k > 1$. However we can show numerically that the basic results in that Proposition continue to hold. We follow Christiano et al. (2011) and set $\phi_\pi = 1.5$ and $\phi_y = 0.25$.

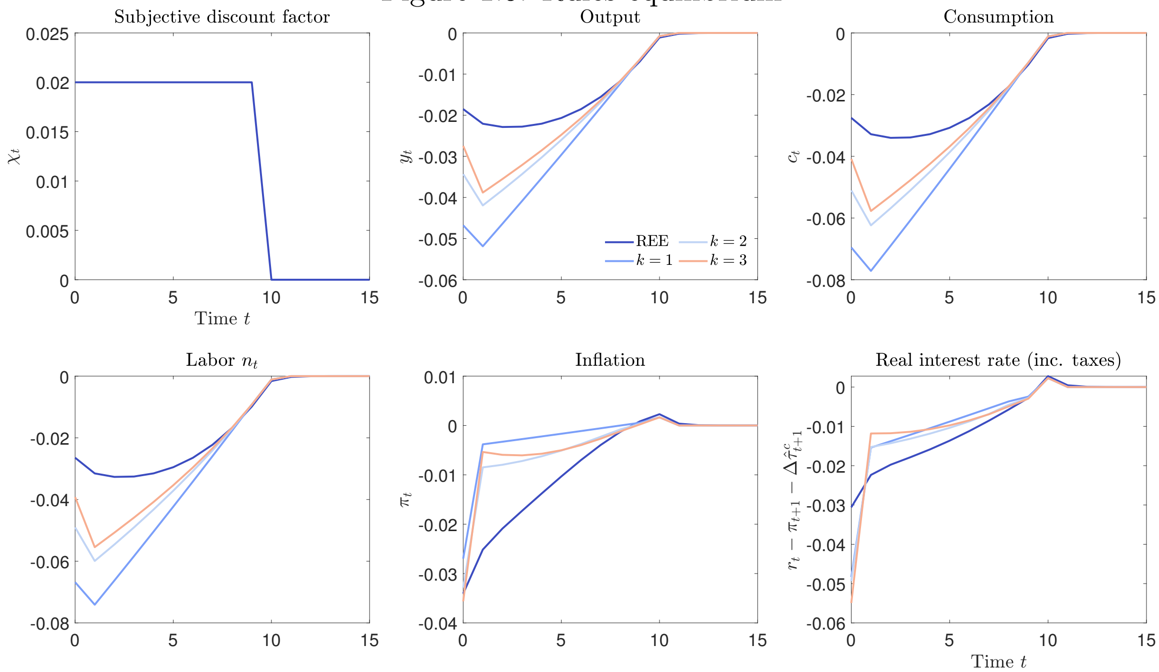
Figure 1.5 displays our results under rational expectations and level-$k$ thinking, assuming that $Y_t^{e,1} = 1$. The (1,1) element of Figure 1.5 displays the shock to the subjective discount factor $\chi_t$. The (1,2), (1,3), and (2,1) elements show the log deviation of output ($y_t$), consumption ($c_t$), and labor ($n_t$), from their steady-state levels, respectively. Finally, the (2,2) and (2,3) elements show inflation, $\pi_t$, and the after-tax real interest rate, $r_t - \pi_{t+1} - \Delta\hat{\tau}^c_{t+1}$.

Recall that in the flexible-price allocation, all quantities remain at their pre-shock steady-state values. The solid blue lines depict the equilibrium under the rules-based monetary and fiscal policies (1.40)-(1.42). Correia et al. (2013) show that, under rational expectations, the proposed fiscal policy has a powerful stabilizing influence on the economy. For example, if tax rates are kept constant in our model economy, the maximal drop in output exceeds seven percent. Under the proposed fiscal policy, the maximal decline in output would be roughly two percent (see Figure 1.5).

With level-$k$ thinking, rules-based fiscal policy is much less powerful than under rational expectations. For example, when $k = 1$, the maximal decline in output is slightly over five percent. As $k$ rises, the efficacy of rules-based fiscal policy increases as people are better able to understand the evolution of future tax rates. Finally, as $k$ goes to infinity, the response of the model economy converges to the rational-expectations equilibrium.

Taken together, the results in this subsection reinforce the message from the benchmark model. When agents are level-$k$ thinkers, target-based communication is more effective than rules-based communication when the ZLB is binding.

Figure 1.5: Rules equilibrium



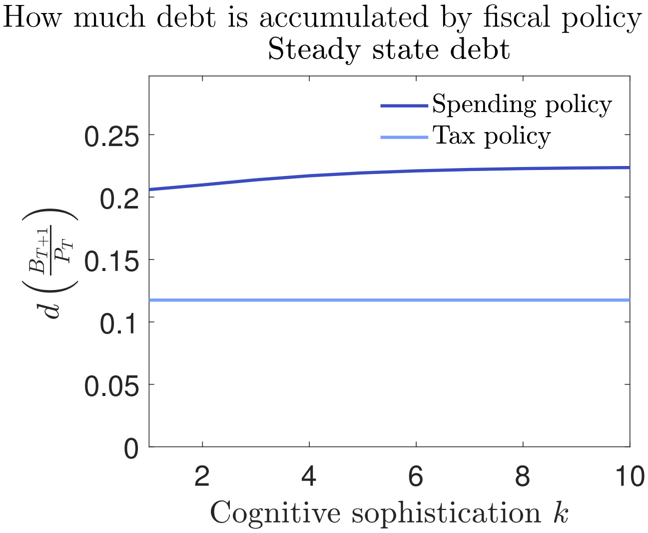

## 1.3.3 On Ricardian Equivalence

In our analysis, we assume that people understand the government-budget constraint. If we relaxed this assumption, Ricardian equivalence would not hold, and we would have to take a stand on a variety of issues. Most importantly, we would have to fully specify the timing of lump-sum taxes and how that path is communicated to people. Our results about the efficacy of fiscal policy would convolve the impact of those assumptions with those of people's limited understanding of GE effects. The mechanisms that we stress in our analysis would continue to operate in the more complicated environment, but their effects would be less transparent.

Still, it is of interest to shed light on the relative sensitivity of our tax and spending results to the failure of Ricardian equivalence. To this end, we suppose that the government decides not to change lump-sum taxes from their steady-state level during the ZLB episode. We then compare how much debt the government accumulates under different policies during the ZLB period. To the extent that more debt is accumulated under one policy, conclusions about that policy are more likely to be affected by departures from Ricardian equivalence.

Here we consider two policies. The first is a policy in which consumption and labor-income taxes are set to achieve the flexible-price allocation. The second is a policy in which the government raises spending by a constant amount at all dates during which the ZLB binds. The constant is chosen so that the cumulative deviation of output from its steady-state level is equal to zero. In Figure 1.6 we display the increase in real debt incurred under the two policies by the end of the ZLB period. We do so for different levels of $k$. Since steady-state output is equal to one, these debt levels can also be interpreted as changes in the debt-to-GDP ratio.

Figure 1.6: How much debt is accumulated by fiscal policy alternatives?




Several key results emerge from Figure 1.6. First, the government-spending policy is associated with an increase in the debt-to-GDP ratio to 21 percentage points for $k = 1$. As we increase $k$, that ratio converges monotonically to 22 percentage points.[19] Second, the

[19] The monotonicity result may be puzzling in light of the fact that higher levels of $k$ are associated with higher government spending multipliers. But note that, other things equal, the size of the recession is increasing in $k$. So to eliminate the cumulative output gap, government spending must be an increasing function of $k$. As it turns out, this effect dominates the multiplier effect so that total debt is slightly increasing in $k$.

tax policy is associated with only a 12 percentage points increase in the debt-to-GDP ratio. Since this policy is independent of $k$, so too is the amount of debt that the government incurs. The fact that tax policy is associated with less debt than the government-spending policy reflects that even though the consumption-tax rate is lower during the ZLB period than its steady-state value, the tax rate on labor is higher. The latter provides a partial offset to the lost revenue from the lower tax on consumption.

Our results show that the total amount of debt incurred under the spending policy is larger than under the tax policy. To the extent that Ricardian equivalence fails because of bounded rationality, this suggests that results regarding the government-spending policy will be more sensitive than results regarding tax policy if only because there is more to be financed under the spending policy. We leave a complete analysis of the non-Ricardian case to future research.

## 1.4 Conclusions

This paper addresses the question: how sensitive is the power of fiscal policy at the ZLB to the assumption of rational expectations? We do so using a standard NK model in which people have a limited understanding of the general-equilibrium effects of fiscal policy.

Our analysis *weakens* the case for using government spending to stabilize the economy when the ZLB binds. The reason is that the efficacy of government spending is quite sensitive to how sophisticated people are. Using a variant of the standard NK model, we find that the less sophisticated people are, the smaller the government-spending multiplier is. The basic intuition is that the power of government spending depends on people's ability to compute and internalize the general-equilibrium effects of spending on their own incomes. The less sophisticated people are, the less they understand these general-equilibrium effects, the more they cut their consumption and the more output falls during the ZLB period.

Our analysis *strengthens* the case for using tax policy to stabilize output when the ZLB is binding. Correia et al. (2013) argues that tax policy is a powerful way to stabilize the economy when the ZLB binds, and people have rational expectations. We show that the power of tax policy during the ZLB period is essentially undiminished when agents do not have rational expectations. Indeed, even when people have low levels of sophistication, it is always possible to achieve the flexible-price allocation during a binding ZLB period. Suppose that the least sophisticated people think that the economy will remain at its pre-shock level. Then, the path for consumption taxes that supports the flexible-price allocation is the same regardless of how cognitively sophisticated people are. Critically, under this tax policy, people's initial beliefs are self-confirming so that the efficacy of the policy does not exploit people's lack of sophistication. Taken together, these results show that tax policy for stabilizing the economy when the ZLB binds is powerful *and* robust to how sophisticated people are.

We also show that when people have limited cognitive abilities, the way in which tax policy is communicated becomes critical to its effectiveness. Tax policy is more effective when it is communicated as a sequence of tax rates instead of a rule involving equilibrium objects like the output gap. The reason is simple: when policy is communicated as a sequence of tax rates, people immediately incorporate those rates into their decisions. When policy is communicated via a tax rule, people must deduce the implications of the rule for the variables that they care about, like consumption-tax rates. In our model, unsophisticated people underestimate how stimulative future policy will be, so tax policy will be less powerful at stabilizing output. Communication matters in a world where people are boundedly rational.

We conclude by noting that a well-known shortcoming of the standard level-$k$ approach to modeling bounded rationality is that people do not update their expectations over time. So we think that this approach is best suited for analyzing people's behavior in the aftermath of unprecedented events. How people actually learn about the structure of the economy when such events do occur is an open and important question. But as long as they do not learn about that structure instantly, they are likely to underplay the importance of general-equilibrium effects. Because this feature is the crucial one underlying our results, the qualitative insights of our analysis would continue to hold even if we assumed that beliefs were updated over time.

# Chapter 2

# Unemployment Insurance in Macroeconomic Stabilization with Imperfect Expectations

## 2.1 Introduction

Unemployment insurance (UI) is an essential component of the social safety net. Temporary UI duration extensions are among the most commonly used fiscal-policy instruments to fight recessions. In the U.S., legislators have passed additional extensions on five separate occasions in the last 40 years. For example, during the Great Recession, the maximum duration of UI benefits increased from 26 weeks to 99 weeks. More recently, during the 2020 recession, unemployment benefits were again extended by 13 weeks. Despite the ubiquitous nature of UI extensions, their benefits and costs remain a main subject of debate.

A recent literature emphasizes that a central channel by which UI operates is the households' *precautionary saving* motive (e.g., McKay and Reis 2016 and Kekre 2021). An increase in the safety net's generosity boosts aggregate demand by reducing households' incentives to save in anticipation of unemployment spells. However, this modern literature assumes that people have full-information and rational expectations (FIRE). The FIRE assumption is critical in these environments since precautionary saving is governed by people's expectations regarding risk and income upon unemployment.

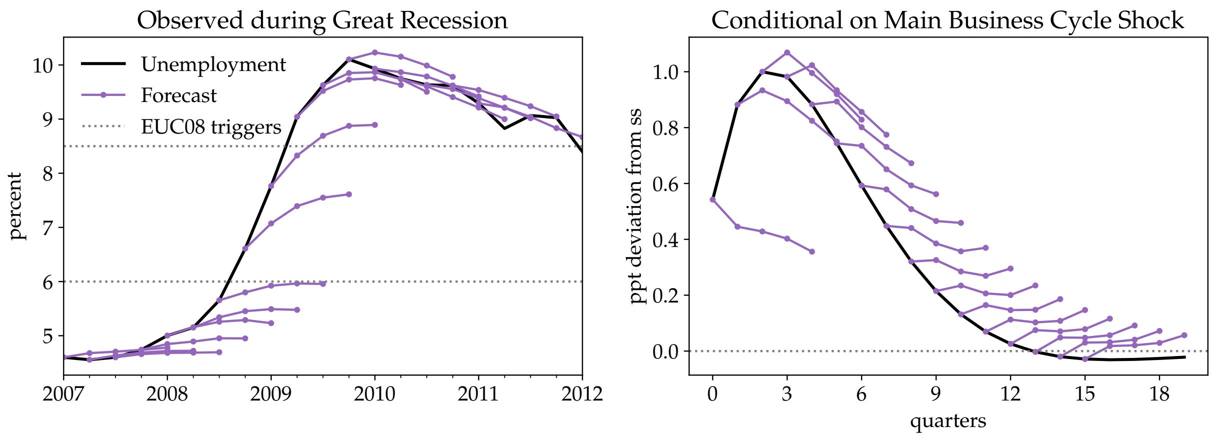
It is now well documented that survey data on beliefs show large deviations from full information and rational expectations (e.g., Coibion and Gorodnichenko 2012, 2015, Bordalo, Gennaioli, Ma, and Shleifer 2020 and Angeletos, Huo, and Sastry 2021). For illustration, the left panel in Figure 2.1 shows the unemployment rate during the Great Recession (in black) alongside the consensus forecast for this variable at multiple horizons in the Survey of Professional Forecasters (in purple).[1] We highlight two main facts. First, we find that SPF beliefs systematically under-forecasted the increase in the unemployment rate during the buildup phase. One observation is worthy of mention. The UI extensions legislation of 2008 stipulated that benefits would be increased by an additional 13 weeks in case the unemployment rate increased above 6 percent. At the national level, this unemployment rate is reached in the third quarter of 2008. Interestingly, right until the quarter just before that, professional forecasters did not anticipate that the unemployment rate would ever cross the 6 percent threshold. This suggests that people may not have expected that the Tier 3 would be activated. Second, following the peak of unemployment, forecasts lagged behind the decline in actual unemployment, with people systematically over forecasting the future unemployment rate.

Because they affect what people think about activating these automatic triggers, these forecast failures also change how people respond to the policy announcement. In this paper, we are interested in understanding the performance of UI extensions in stimulating demand when people's expectations are not full-information and rational.

**Illustrative model.** We begin our analysis with an illustrative model that isolates the role of expectations in determining the aggregate demand response to increases in UI benefits. We work with a two-period setting and fully rigid prices, allowing for a simple analytical

[1] We understand that this figure does not represent definite proof of the failure of the FIRE model because new shocks could be realized at every point in time. The right panel shows that the same pattern is also present in the impulse responses of beliefs to an identified shock–the main business cycle shock of Angeletos, Collard, and Dellas (2020).

Figure 2.1: Consensus Forecast of Unemployment Rate




*Notes.* On both panels, the black line is based on the seasonally-adjusted civilian unemployment rate from the U.S. Bureau of Labor Statistics, and the purple line is based on the median unemployment forecast from the Survey of Professional Forcasters. On the left panel, we plot the level of the unemployment rate and forecasts between 2007Q1 and 2012Q1. On the right panel, we plot the impulse responses of these variables to the main business cycle shock (targeting unemployment) from Angeletos, Collard, and Dellas (2020). We estimate the impulse responses using an ARMA-IV specification as in Angeletos, Huo, and Sastry (2021). We scale the shock such that peak response of unemployment is 1 percentage point.

solution. In both periods, workers can be either employed, earning labor income, or unemployed, earning UI benefits. The individual's unemployment shock is independent across periods. We consider a demand-induced recession in the second period, increasing households' incentives to take precautionary savings in the first period. We evaluate the effects of two scenarios in shaping the anticipatory response of aggregate demand. The first is a policy in which unemployment benefits are a function of the unemployment rate, i.e., a policy rule as considered in other settings by Angeletos and Sastry (2021) and Bianchi-Vimercati, Eichenbaum, and Guerreiro (2021). The second is a policy of directly communicating the generosity of unemployment benefits. We show that the difference in response of output in these two economies is given by the product of four terms:

$$dY_0^{\text{rule}} - dY_0^* = -\mathcal{M} \cdot M_b \cdot \zeta_b \cdot \underbrace{(1-\lambda)dU_1}_{\text{forecast error}}, \tag{2.1}$$

where each term is: (1) the Keynesian-cross multiplier is $\mathcal{M} > 0$, (2) the partial-equilibrium response of aggregate demand to higher unemployment benefits, $M_b > 0$, (3) the elasticity of unemployment benefits to the unemployment rate, $\zeta_b > 0$, and finally (4) the forecast error in predicting the unemployment rate $(1-\lambda)dU_1$ where $1-\lambda$ denotes the *cognitive bias* and it is such that $dU_1^e = \lambda dU_1$, and $dU_1 > 0$ the increase in unemployment at time 1.

The relative performance of rules-based policy depends on whether beliefs underreact relative to FIRE ($\lambda < 1$) or overreact relative to FIRE ($\lambda > 1$). If individuals hold full-information and rational expectations, $\lambda = 1$, the output response is the same both scenarios. Instead, if beliefs underreact with respect to FIRE, individuals under forecast the increase in the generosity of UI benefits. It follows that the stabilization power of the policy is weaker. Instead, if beliefs overreact, the opposite happens. Individuals over forecast the change in the generosity of future UI benefits, leading to a larger cut in precautionary savings and thus a milder recession. This model emphasizes that the anticipation of UB extensions is an important margin by which these policies transmit to consumption.

**General framework.** The simple model emphasizes the importance of getting expectations right in assessing the effects of UI extensions. With the goal of quantifying the consequences of the empirical patterns of expectations, we then describe a general framework that allows a more complete description of the economy and its actors and a more general description of their beliefs.

In section 2.3, we discuss a general method that allows us to solve an economy under arbitrary assumptions on beliefs. This flexible method is based on the Sequence-space Jacobian framework developed in Auclert, Bardóczy, Rognlie, and Straub (2021) and further extended to models deviating from FIRE by Auclert, Rognlie, and Straub (2020).[2] Building on their main insight, we show that under general beliefs, it suffices to describe the response of each agent block to two additional objects: the *forecast errors* and the *forecast revisions*.

In this framework, FIRE is equivalent to perfect foresight. So, people make no forecast errors or revisions. It suffices to describe how people respond to the time-0 innovation, which forces the economy to deviate from a steady state. The Jacobians are sufficient statistics mapping changes in the path of endogenous and exogenous variables into the path of aggregate decisions of the agent block. For example, the consumption-real-interest-rate Jacobian $\mathcal{J}^{C,r}$ maps the change in real interest rates to the change in aggregate consumption of a household block. But, with general beliefs, people make mistakes in forecasting and may revise their expectations in the future. How individuals respond to these new objects can be computed directly from the FIRE Jacobian. The intuition for this result follows from the fact that because forecast errors and revisions are entirely unanticipated by the agents, then their response to these forecast updates is the same as their response to an unanticipated time-0 change. For example, the response of the household block to a forecast error in the time 1 real interest rate $r_1 - r_1^e$ is precisely the same as the agent would respond to a time 0 real interest rate shock $r_0$ under perfect foresight, $\mathcal{J}^{C,r}_{0,0}$.

Because it only uses the FIRE Jacobians, this method is very fast and easy to implement. We discuss how to implement a variety of popular models of deviations from FIRE.[3] More

[2]Auclert, Rognlie, and Straub (2020) show how to implement the method for sticky expectations (Mankiw and Reis 2002, Carroll, Crawley, Slacalek, Tokuoka, and White 2018), cognitive discounting (Gabaix 2020), and dispersed information (Angeletos and Huo 2021). They also discuss how this framework can be extended to other models.

[3]Including incomplete and dispersed information (Lucas 1972, Woodford 2001, Carroll 2003, Angeletos and Huo 2021), sticky expectations (Mankiw and Reis 2002, Carroll, Crawley, Slacalek, Tokuoka, and White 2018), cognitive discounting (Gabaix 2020), finite planning horizons (Woodford 2018), diagnostic expectations (Bordalo, Gennaioli, and Shleifer 2018, Bordalo, Gennaioli, Ma, and Shleifer 2020, Bianchi, Ilut, and Saijo 2021), shallow reasoning (Angeletos and Sastry 2021), adaptive expectations (Cagan 1956; Friedman 1957), among others.

importantly, the framework allows us to work with arbitrary expectations. We leverage this fact by working directly with empirically measured expectations. As we discuss below, this allows us to quantify the impact of imperfect expectations on the power of UI extensions in stimulating aggregate demand without imposing any extra assumptions on the belief formation model. Effectively, this method allows us to sidestep the issue of choosing among the "wilderness" of alternative models of belief formation, a common criticism of this literature going back to Sims (1980) and Sargent (1999).

**Quantitative framework and results.** Equipped with a framework to solve and analyze dynamic models with arbitrary deviations from FIRE, we refine the analytical results in (2.1). We need four objects. First, the dynamics of forecast errors about the unemployment rate. Second, a UI extension policy that indexes duration to the unemployment rate. Third, a model of households that maps beliefs about UI duration into aggregate demand. Fourth, a model of the macroeconomy that maps shocks into equilibrium output, unemployment rate, and so on.

To obtain empirically relevant forecast errors, we estimate the impulse responses of the unemployment rate and its forecasts at different horizons to an identified aggregate demand shock. We measure expectations as the median forecast from the Survey of Professional Forecasters. Our identified shock is the main business cycle shock of Angeletos, Collard, and Dellas (2020). Combining these estimates allows us to compute the dynamics of forecast errors and revisions conditional on a relevant aggregate demand shock.

We implement automatic UI extensions via a policy rule that indexes the UI expiration probability to the equilibrium unemployment rate. We calibrate the semi-elasticity in the rule, $\zeta_b$, to match the ratio of UI extensions in the EUC08 policy to the rise in the unemployment rate during the Great Recession. The calibrated rule implies that a one percentage point increase in the unemployment rate triggers about a one-quarter increase in average UI duration.

We embed this policy rule in a New Keynesian model with incomplete markets, heterogeneous households, and search and matching frictions. Our model incorporates many features that have been emphasized in modern models of social insurance (McKay and Reis, 2016; Kekre, 2021), and aggregate demand (Auclert, Rognlie, and Straub, 2018). Notably, it features intertemporal optimization by risk-averse, liquidity-constrained households; heterogeneity in marginal propensities to consume (MPC); endogenous unemployment risk; and nominal rigidities. We estimate our model following similarly to the procedure popularized in Christiano, Eichenbaum, and Evans (2005) and recently extended to an heterogeneous-agent environment by Auclert, Rognlie, and Straub (2020). First, we calibrate the model's steady state to deliver realistic MPCs, and estimate transition-specific parameters to match the impulse responses to the identified shock. Second, we estimate the remaining parameters by matching the empirical impulse. Importantly, this estimation exercise can be performed using directly the expectations observed in the data in response to the identified shocks.

Our estimated model implies that perceived UI duration is more important for aggregate stabilization than actual UI duration. A UI extension raises incomes only for those workers who experience a job loss and stay eligible thanks to the extension. Most households remain employed even in deep recessions; for them, only perceived UI duration matters, affecting their precautionary saving. So expectations are crucial in assessing the effectiveness of the policy. We show that the policy is less effective in the short run than under FIRE. This finding is a direct consequence of the pattern of initial belief underreaction observed in Figure 2.1. However, after the peak of the recession, expectations turn overly optimistic relative to FIRE. This pattern of delayed overreaction implies that the rules-based policy becomes even more effective under the estimated beliefs than under FIRE.

We use our model to quantify the impact of UI extensions on equilibrium unemployment and consumption relative to a counterfactual scenario in which UI duration was constant.

In order to run counterfactuals, we describe and estimate a model of belief formation which combines noisy information with diagnostic expectations and long-memory. We show that, at the onset of the recession, the policy reduces the unemployment rate by 0.4 percentage points and increases aggregate consumption by 0.6 percentage points, while in FIRE the same policy would have reduced the unemployment rate by 0.7 percentage points and increased consumption by 1 percentage point. It follows that the initial belief underreaction makes this type of policy almost half as effective as would be predicted by models with full-information and rational expectations. However, due to the pattern of delayed overreaction, the impact of the policy on aggregates is hump-shaped in our model (instead, with FIRE, the peak effectiveness happens immediately). In our model, the peak effectiveness of the policy leads to a reduction of 0.5 percentage points in unemployment and an increase in consumption of almost 1 percentage point.

Finally, we use our model to assess the relative efficacy of different forms of policy communication. In particular, as in Bianchi-Vimercati, Eichenbaum, and Guerreiro (2021), we evaluate the stabilization power of announcing the UI duration directly to people rather than implementing as a contingent rule. We conclude that announcing the policy directly can be very stimulative in the very short run, but may lack efficacy later in the recession as expectations turn overly pessimistic.

**Relationship to the literature.** Our paper contributes to an extensive literature analyzing the consequences of macroeconomic shocks and policies without the FIRE assumption and exploiting survey data to calibrate the expectational components of macro models, see Angeletos and Lian (2023) and Milani (2023) for recent reviews. We share the interest in analyzing these questions in the context of Heterogeneous-agent New Keynesian models (HANK) with the recent contributions by Farhi and Werning (2019), Farhi, Petri, and Werning (2020), Auclert, Rognlie, and Straub (2020), Pappa, Ravn, and Sterk (2023), Dobrew, Gerke, Giesen, and Röttger (2023), and Guerreiro (2022). These papers consider parametric models of bounded rationality and study the impact of these particular deviations. We deviate from their contributions in two ways. First, we study the effects of UI extensions on the economy. Second, we discuss a method that allows us to quantify the impact of deviations from FIRE in a fully non-parametric way, directly exploiting the data coming from surveys of expectations. This allows us to sidestep the discussion of choosing a particular model of deviation from FIRE.

Our model includes incomplete markets, nominal rigidities, and suboptimal monetary policy, which has been found important in addressing these questions. Nominal rigidities and frictions to monetary policy adjustment can reverse the contractionary effects of UI extensions in Krusell, Mukoyama, and Şahin (2010), Nakajima (2012), and Mitman and Rabinovich (2015, 2019), see Christiano, Eichenbaum, and Trabandt (2016). Furthermore, Kekre (2021) emphasizes how these mechanisms can be complemented by the stimulus effect arising from the direct redistribution across workers with different marginal propensities to consume and the impact of reducing precautionary savings motives. However, this literature has evaluated these policies working exclusively with full information and rational expectations. Our paper contributes a new perspective on the quantitative relevance of the different mechanisms when beliefs accord to the survey evidence.

In a closely related paper, Fernandes and Rigato (2022) study UI in a model where households have present-biased preferences. Present bias reduces the responsiveness of precautionary saving to UI extensions. However, they maintain the assumption of full-information rational expectations, making their contribution complementary to ours.

There is a large empirical literature on the effects of UI extensions on unemployment, particularly focusing on the Great Recession. This literature includes Chodorow-Reich, Coglianese, and Karabarbounis (2019), Boone, Dube, Goodman, and Kaplan (2021), Dieterle, Bartalotti, and Brummet (2020), Hagedorn, Karahan, Manovskii, and Mitman (2013), and Hagedorn, Manovskii, and Mitman (2015), obtaining conflicting results.

**Outline.** The structure of the paper is as follows. Section 2, analytical model. Section 3, general framework with propositions. Section 4, quantitative model and results. Section 5 concludes.

## 2.2 Illustrative model

We start with an analytical demonstration that imperfect expectations interfere with the power of unemployment insurance (UI) extensions to stabilize business cycles. We consider a simple two-period environment. We engineer a recession in period 1, which triggers precautionary responses in period 0. Then, we analyze how equilibrium output at time 0 depends on households' expectations and the implementation of UI. Appendix B.1 contains detailed derivations and proofs.

### 2.2.1 Setup

Consider a two-period model, $t = 0, 1$. The economy is populated by a measure one of households, a representative firm, and a government. The sequence of events within the two periods is the same. First, the representative firm randomly hires a fraction of households. Second, production takes place and households make a consumption-saving decision.

**Firm.** A competitive firm produces a final good $Y_t$ from labor $N_t$ according to the production function

$$Y_t = N_t \tag{2.2}$$

The only cost of production is the real wage bill $w_t N_t$ paid to workers. In equilibrium, $w_t = 1$ and the firm hires just enough workers to meet aggregate demand while making zero profit.

**Households.** In period $t$, a fraction $N_t \in [0,1]$ of households is employed. The remaining $1-N_t$ households are unemployed. The probability that an individual household is employed is the same for all workers and equal to the employment rate $N_t$. Employed workers earn real wage $w_t = 1$. Unemployed workers receive real benefits $b_t \in (0,1)$, financed by a lump-sum tax $\tau_t$ levied on all households. Once their employment status for the current period ($e_t \in \{0,1\}$) is determined, households choose consumption $c_t$ and savings $a_t$ in a non-contingent bond with real return $r$ to maximize their anticipated life-time utility

$$u(c_0) + \beta u(c_1) \tag{2.3}$$

subject to period budget constraints

$$c_t + a_t = (1+r)a_{t-1} + e_t w_t + (1-e_t)b_t - \tau_t \tag{2.4}$$

and borrowing constraints $a_t \geq 0$ for $t = 0, 1$. Let the felicity function $u(\cdot)$ be smooth, increasing, concave, and have a positive third derivative, i.e. households are prudent in the sense of Kimball (1990).

At time 0, households may not have perfect foresight of the endogenous variables $N_1, b_1, \tau_1$, and hence even of their own consumption $c_1^e$. Let $N_1^e, b_1^e, \tau_1^e$, and $c_1^e$ denote the beliefs for each variable. We assume that all households have the same beliefs and do not consider uncertainty regarding the beliefs. We focus on the first-order response of this economy around a non-stochastic equilibrium. Adding uncertainty would not change our results.

Prudence and market incompleteness implies that households have precautionary saving motive in period 0 against unemployment risk in period 1. We can see this from the Euler equation

$$u'(c_0) \geq \beta(1+r)\left[N_1^e \cdot u'\big(\underbrace{1-\tau_1^e+(1+r)a_0}_{c_1^e \text{ if employed}}\big) + (1-N_1^e)\cdot u'\big(\underbrace{b_1^e-\tau_1^e+(1+r)a_0}_{c_1^e \text{ if unemployed}}\big)\right] \quad (2.5)$$

As is standard in models at the zero liquidity limit (e.g., Werning 2015), we assume that at least one Euler equation holds with equality. As we show in appendix B.1.1, this will be the employed workers' Euler equation, because they have a stronger incentive to save in period 0. Then, (2.5) implies that the consumption of employed workers in period 0, $c_0(E)$, is increasing in the expectations for employment in period 1, $N_1^e$.

**Policy.** The government runs a balanced budget

$$\tau_t = (1-N_t)b_t \quad (2.6)$$

We specify the different implementations of unemployment benefits $b_t$ below in the context of the business cycle stabilization experiment.

We impose a cash-in-advance constraint

$$P_t C_t = M_t \quad (2.7)$$

where $P_t$ is the price level, $C_t$ is aggregate consumption, and $M_t$ is money supply. We assume that prices are fully rigid and normalize the price level to one, $P_t \equiv 1$. The monetary authority sets the money supply, $M_1$, and the real rate between periods 0 and 1, $r$. Let $M_0$ adjust to support the equilibrium given exogenous monetary policy $(r, M_1)$.

In this simple model, we consider an exogenous shock to time-1 money supply. The combination of sticky prices and the cash-in-advance constraint (2.7) implies that these shocks also affect aggregate quantities. As a result, these assumptions allow us to consider demand shocks in this simple two-period model.

**Equilibrium.** Given initial assets $a_{-1}$, exogenous variables $\{b_t, r, M_1\}$, and beliefs $\{N_1^e, b_1^e, \tau_1^e\}$, a temporary equilibrium is a collection of prices $\{w_t\}$ and allocations $\{c_t^E, c_t^U, N_t, \tau_t, M_0\}$ such that the representative firms optimizes, households optimize, government budget is balanced, the cash in advance constraint is satisfied, goods market clears

$$Y_t = C_t = N_t c_t^E + (1 - N_t) c_t^U \tag{2.8}$$

and asset market clears

$$0 = A_t = N_t a_t^E + (1 - N_t) a_t^U \tag{2.9}$$

The formal derivation of the model solution is relegated to appendix B.1.1. In the zero liquidity limit, the model is purely forward-looking. So the time-1 equilibrium is independent of time-0 outcomes, including the expectations that households hold in period 0. However, since employed workers are on the Euler equation, their expectations are relevant for equilibrium in period 0. As such, the model isolates the effect of imperfect anticipation of benefits in general equilibrium (GE).

### 2.2.2 Beliefs

For the purposes of this section, we impose a simple model of beliefs, based on the idea of belief distortion about future deviations from steady state. Formally, we assume that, for a given variable $x_1$, households' beliefs are given by

$$dx_1^e = \lambda dx_1 \tag{2.10}$$

where $\lambda$ is a cognitive bias. Note that $\lambda = 1$ corresponds to full-information and rational expectations (FIRE). If $\lambda < 1$, then beliefs underreact relative to FIRE. If $\lambda > 1$, then beliefs overreact relative to FIRE. As we discuss next, whether beliefs underreact or overreact with respect to the FIRE benchmark is essential in addressing our questions.

Do beliefs underreact or overreact to innovations in fundamentals? This question has been the focus of an extensive empirical literature looking at survey evidence, but a consensus has not been reached. For instance, Coibion and Gorodnichenko (2012, 2015) find evidence of belief underreaction. This finding is consistent with models of rational inattention or information rigidities, as in Sims (2003), Woodford (2001), Carroll (2003), Mankiw and Reis (2002), or Gabaix (2020). Instead, Bordalo, Gennaioli, Ma, and Shleifer (2020) find evidence of belief overreaction, which is consistent with models of diagnostic expectations and overextrapolation as in Bordalo, Gennaioli, and Shleifer (2018). More recently, Angeletos, Huo, and Sastry (2021) find evidence of initial underreaction and a pattern of delayed overreaction. Given the central importance of expectations in our analysis, in Section 2.3, we present a framework that can accommodate arbitrary deviations from FIRE and, in Section 2.4, we use this framework to directly match the empirical behavior of beliefs in surveys of expectations.

### 2.2.3 Macroeconomic stabilization

We demonstrate that deviations from FIRE affect the power of unemployment benefit extensions to stabilize aggregate demand. To this end, we induce a recession at time $t = 1$ and characterize the first-order change in equilibrium at time $t = 0$ from anticipating the recession.

The recession originates in a decrease in money supply, $dM_1 < 0$, that translates one-to-one into lower employment, $dN_1 = dM_1$. In response, employed households will try to save more in period 0 according to the Euler equation (2.5). Since they cannot save in equilibrium, their time-0 consumption has to fall to dissuade them from saving. Thus a recession arises endogenously in period 0. The recession's severity depends on the strength of households' precautionary saving motive which depends on expected unemployment benefits. We consider two implementations of countercyclical UI benefits $b_1$. In both cases, the other policy instruments $\{r, b_0\}$ remain constant.

1. **Instrument rule.** The government announces that unemployment benefits are indexed to the unemployment rate according to a rule

$$db_1^{\text{rule}} = -\zeta_b \cdot dN_1 \tag{2.11}$$

with semi-elasticity $\zeta_b > 0$. We assume that households understand the policy announcement and has first-order knowledge of the rule that determines unemployment benefits. It follows that their expectations of unemployment benefits are given by:

$$db_1^e = -\zeta_b dN_1^e \tag{2.12}$$

2. **Instrument announcement.** We also consider a counterfactual scenario in which the government announces the change in unemployment benefits, $db_1^*$, directly. We assume that households understand the policy announcement and update their expectations accordingly

$$db_1^e = db_1^* \tag{2.13}$$

To make the two policies comparable, we assume that they implement the same transfers, i.e. $db_1^* = db_1^{\text{rule}}$. This implies that the time-1 equilibrium $\{dN_1, d\tau_1, dc_1(E), dc_1(U)\}$ is the same under both policies. By extension, beliefs $dN_1^e = \lambda dN_1$ and $d\tau_1^e = \lambda\tau_1$ are also the same under the two policies. However, under a rule, the inability to predict the unemployment rate also translates to an inability of predicting the policy stance, and so $db_1^{e,rule} = \lambda db_1^{e,*}$. Proposition 6, the main result of this section, shows the implications for the endogenous recession in period 0.

**Proposition 6.** *Consider a shock $dM_1$ to the money supply in period 1. Let's assume that the government responds by announcing an unemployment benefit extension $db_1$ in one of two ways: either according to the rule* (2.11) *or a direct announcement. The first-order impact of the shock on time-0 output under these two regimes are*

$$dY_0^{rule} - dY_0^{*} = \mathcal{M} \cdot M_b \cdot \zeta_b \cdot (1-\lambda) dU_1 \tag{2.14}$$

*where the first term is a standard **Keynesian multiplier***

$$\mathcal{M} = \frac{1}{1 - \frac{\partial C_0}{\partial Y_0}} = \frac{1}{b_0} \tag{2.15}$$

*the second term is the **marginal propensity to consume** out of anticipated unemployment benefits*

$$M_b = \frac{\partial c_0(E)}{\partial b_1} = \frac{\beta(1+r)(1-N_1^e) \cdot u''(b_1^e - \tau_1^e)}{u''(1-\tau_0)} \tag{2.16}$$

*and the third term* $(1-\lambda)$ *is a **cognitive bias**.*

Equation (2.14) shows that implementing the same UI extension, $db_1$, as a rule or a direct announcement can affect the severity of the recession in period 0. Under FIRE ($\lambda = 1$), households forecast the unemployment rate perfectly and infer the correct level of benefits, leading to $dY_0^{\text{rule}} - dY_0^{*} = 0$. So, deviating from FIRE is a necessary condition for the implementation of UI extension to make a difference.

If $\lambda \neq 1$, households erroneously forecast the increase in the unemployment rate at time 1. Under the rules-based policy, this also means that they make mistakes in forecasting the increase in generosity of unemployment benefits. In other words, their misperception of tomorrow's unemployment rate also translates into a misperception of the future policy stance. Their forecast error is given by their cognitive bias $1 - \lambda$ multiplied by the change in benefits $\zeta_b dM_1$. Instead, under the instrument-announcement policy, the household understands the policy announcement and so their expectations of unemployment benefits will always be correct.

If household beliefs underreact relative to FIRE, $\lambda < 1$, forecast mistakes make the rules-based policy less effective than the instrument-announcement policy. The efficiency loss is characterized by two sufficient statistics. First, the MPC out of anticipated benefits, $M_b$. This captures the partial equilibrium effect of underestimating UI benefits on aggregate demand. Second, the multiplier, $\mathcal{M}$, which captures the general equilibrium feedback from a contemporaneous change in aggregate demand. Instead, if household beliefs overreact relative to FIRE, $\lambda > 1$, forecast mistakes make the rules-based policy more effective than the instrument-announcement policy.

## 2.3 A framework for dynamic models with imperfect expectations

Next we lay out a framework of dynamic decision making with imperfect expectations. Our framework has two components. First, a model of how actions evolve given any expectations. Second, a model of how expectations are formed from observations. In appendix B.2, we map many popular models of bounded rationality and information frictions into our framework.

### 2.3.1 Dynamic decisions with general deviations from FIRE

Consider a forward-looking agent who chooses an output $Y_t$ over periods $t = 0, 1 \ldots, T-1$. Let the vector $\mathbf{Y} \in \mathbb{R}^T$ denote the path of the output. For ease of exposition, let every object (parameters, initial-, and terminal conditions) that matters for the decision be fixed and known to the agent except the path of a single univariate input $\mathbf{X} \in \mathbb{R}^T$. The extension to multiple time-varying inputs is straightforward.

Auclert, Bardóczy, Rognlie, and Straub (2021) cast such dynamic decision problems as a mapping between sequences

$$\mathbf{Y} = f(\mathbf{X}) \tag{2.17}$$

Their sequence-space Jacobian (SSJ) method computes the Jacobian $\mathcal{J} \in \mathbb{R}^{T\times T}$ then computes impulse responses to any shock $d\mathbf{X}$ via matrix multiplication, $d\mathbf{Y} = \mathcal{J} d\mathbf{X}$.[4] The representation (2.17) is valid under two assumptions. First, certainty equivalence with respect to $\mathbf{X}$. When the agent chooses $Y_t$, she considers only her time-$t$ expectations $\mathbf{X}^{e,t} \in \mathbb{R}^T$, not the entire distribution of $\mathbf{X}$. Second, perfect foresight (FIRE) with respect to $\mathbf{X}$. The agent's expectations are correct, $\mathbf{X}^{e,t} = \mathbf{X}$.

We are interested in a generalization of this setup which relaxes the assumption of FIRE. We retain certainty equivalence, so only the mean expectation matters. However, expectations may not be correct and may evolve over time. In period 0, the agent expects a path $\mathbf{X}^{e,0}$; in period 1, she expects a path $\mathbf{X}^{e,1}$; and so on. Each vector $\mathbf{X}^{e,\tau} = \begin{bmatrix} X_0^{e,\tau} & X_1^{e,\tau} & \ldots & X_T^{e,\tau} \end{bmatrix}'$ captures the beliefs that the agent holds at time $\tau$ about the variable $X$ at every other date. We assume that the agent observes current and past realizations (or, alternatively, all current realizations and the sufficient state variables for their individual decision making), and also assume that the agent does not foresee their future forecast errors (i.e., they are naive). So

[4]The Jacobian is computed at a baseline path $\bar{\mathbf{X}}$, typically a constant path corresponding to the steady state $\bar{\mathbf{Y}} = f(\bar{\mathbf{X}})$. So the shock $d\mathbf{X} = \mathbf{X} - \bar{\mathbf{X}}$ and the impulse response $d\mathbf{Y} = \mathbf{Y} - \bar{\mathbf{Y}}$ are both deviations from the baseline path.

$X_t^{e,\tau} = X_t$ for all $t \leq \tau$. This ensures that the agent does not violate any constraints. In sum, relaxing FIRE implies that we have to keep track of the entire history of expectations, $\mathbf{X}^{e,t}$ for all $t$. Formally,

$$\mathbf{Y} = g\left(\mathbf{X}, \{\mathbf{X}^{e,t}\}_t\right) \tag{2.18}$$

Conceptually it is clear that if we could compute all the Jacobians of $g(\bullet)$, we could compute linearized impulse responses. But the domain of $g(\bullet)$ is $\mathbb{R}^{T+T\times T}$, a much larger space than the domain of $f(\bullet)$ which is just $\mathbb{R}^T$. Is this approach viable in practice? Propositions 7 and 8 show that it is. The key idea is to manipulate the FIRE Jacobian $\mathcal{J}$ to capture the responses to forecast errors. This insight appears in Auclert, Rognlie, and Straub (2020), who implemented specific deviations from FIRE via Jacobian manipulation.[5] Propositions 7 and 8 do the same for general deviations from FIRE, using the familiar concepts of forecast errors and forecast revisions.

Proposition 7 handles the special case of non-rational but time-invariant expectations $d\mathbf{X}^e \neq d\mathbf{X}$. An example of this is level-k thinking. The total response $d\mathbf{Y}$ is the sum of two effects. First, the response to the expected part of the shock. Second, the responses to the forecast errors that the agent observes along the way. The key new object is the forecast-error Jacobian, $\mathcal{E}$, that captures the second effect. Column $s$ of $\mathcal{E}$ can interpreted as the impulse response to the forecast error in $dX_s$ which the agent learns in period $s$. Constructing $\mathcal{E}$ is straightforward. It is a lower diagonal matrix whose columns are shifted versions of the first column of $\mathcal{J}$. The intuition is that observing a forecast error in period $t$ is equivalent to observing an unexpected shock in period 0. The formal proof is in appendix B.2.1.

**Proposition 7.** *Assuming constant beliefs* $X_{t+h}^{e,t} = X_{t+h}^{e,0}$ *for all* $t, h > 0$*, the linearized*

[5] Appendix D.3 of Auclert, Rognlie, and Straub (2020) provides recipes to implement sticky expectations, cognitive discounting, and dispersed information.

*impulse response* $d\boldsymbol{Y}$ *to an arbitrary shock* $d\boldsymbol{X}$ *is given by*

$$d\boldsymbol{Y} = \mathcal{J} \underbrace{d\boldsymbol{X}^{e,0}}_{forecast} + \mathcal{E} \underbrace{\left(d\boldsymbol{X} - d\boldsymbol{X}^{e,0}\right)}_{forecast\ error} \tag{2.19}$$

*where the* ***forecast-error Jacobian*** $\mathcal{E}$ *is given by*

$$\mathcal{E} = \begin{bmatrix} \mathcal{J}_{0,0} & 0 & \dots & 0 \\ \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{J}_{t,0} & \mathcal{J}_{t-1,0} & \dots & \mathcal{J}_{0,0} \end{bmatrix} \tag{2.20}$$

Proposition 8 handles the general case of time-variant expectations. The new element is that observing the forecast error $dX_t - dX_t^{e,t-1}$ may cause the agent to update her expectations for all future periods. Capturing this effect is most straightforward if we work with forecast revisions $d\boldsymbol{X}^{e,h} - d\boldsymbol{X}^{e,h-1}$ instead of forecast errors $d\boldsymbol{X} - d\boldsymbol{X}^{e,h}$. The Jacobians that act on forecast revision vectors are simply shifted versions of the FIRE Jacobian $\mathcal{J}$. The intuition is that a forecast revision for periods $t, \dots, T-1$ is equivalent to observing an unanticipated shock for periods $0, \dots, T-t$. The formal proof is in appendix B.2.2.

**Proposition 8.** *Assuming time-variant beliefs* $\boldsymbol{X}^{e,t}$*, the linearized impulse response* $d\boldsymbol{Y}$ *to an arbitrary shock* $d\boldsymbol{X}$ *is given by*

$$d\boldsymbol{Y} = \mathcal{J} \underbrace{d\boldsymbol{X}^{e,0}}_{initial\ forecast} + \sum_{h \geq 1} \mathcal{R}_h \underbrace{\left(d\boldsymbol{X}^{e,h} - d\boldsymbol{X}^{e,h-1}\right)}_{forecast\ revision} \tag{2.21}$$

*where the* ***forecast-revision Jacobian*** $\mathcal{R}_h$ *for any* $h > 1$ *is given by*

$$\mathcal{R}_h = \begin{bmatrix} 0 & \boldsymbol{0}_h' \\ \boldsymbol{0}_h & \mathcal{J} \end{bmatrix}$$

**Application to heterogeneous-agent models.** Propositions 7 and 8 apply to heterogeneous-agent models in which $Y_t = \int y_t dD_t$ is an aggregate of individual decisions $y_t$ for some non-trivial, time-varying distribution $D_t$. However, we need to impose restrictions on belief heterogeneity. In the exposition above, we assume that everyone has the same beliefs. More generally, this framework can be directly used even if expectations are heterogeneous as long as they are uncorrelated with other idiosyncratic characteristics in the cross-section. For this purpose, we redefine $X^{e,t}$ as the cross-sectional average expectation.

It is also possible to use this framework to allow for meaningful belief disagreement as long as beliefs are with permanent individual characteristics. In this case, one has to set up a heterogeneous-agent block for each permanent type, and apply the propositions type by type. Guerreiro (2022) follows this approach in his study of disagreements over the business cycle.

### 2.3.2 A flexible model of expectations

Propositions 7 and 8 enable us to compute linearized impulse responses to any shock $d\mathbf{X}$ given the path of expectations $\{d\mathbf{X}^{e,t}\}_t$ conditional on the same shock.[6] In some cases, the response of expectations may be estimated directly. We'll do so in section 2.4.6 with respect to unemployment. Another route is to impose a model of expectation formation. In this subsection, we present a tractable yet flexible specification that nests many popular models including: (1) sticky expectations (Mankiw and Reis, 2002; Carroll, Crawley, Slacalek, Tokuoka, and White, 2018), (2) noisy-information and rational expectations (Angeletos and Huo, 2021), (3) cognitive discounting (Gabaix, 2020), (4), sparsity (Gabaix, 2014, 2016; Guerreiro, 2022), (5) shallow reasoning (Angeletos and Sastry, 2021), (6) finite planning horizons (Woodford, 2018), (7) adaptive expectations (Cagan, 1956; Friedman, 1957), (8)

[6]Recall that deviations $d\mathbf{X}$ and $\{d\mathbf{X}^{e,t}\}_t$ are all relative to the paths around which one wishes to compute the Jacobian.

diagnostic expectations (Bordalo, Gennaioli, and Shleifer, 2018; Bianchi, Ilut, and Saijo, 2021), (9) noisy-information diagnostic expectations (Bordalo, Gennaioli, Ma, and Shleifer, 2020), among others. Appendix B.2.3 discusses how to map each of these models into our framework.

Propositions 7 and 8 deal with linear mappings in sequence space. So it's natural for us to model expectations in the same way. The most general linear sequence-space model of expectations we can write down—given a single time-variant input $d\mathbf{X}$—is a sequence of matrices $\Lambda_t \in \mathbb{R}^{T\times T}$ that map realized outcomes $d\mathbf{X} \in \mathbb{R}^T$ into time-$t$ expectations $d\mathbf{X}^{e,t} \in \mathbb{R}^T$ according to

$$d\mathbf{X}^{e,t} = \Lambda_t d\mathbf{X} \tag{2.22}$$

We maintain the assumption that expectations of current and past realizations of $d\mathbf{X}$ are correct. This implies that the upper-left block of $\Lambda_t$ is the identity matrix

$$\Lambda_t = \begin{bmatrix} \mathbf{I}_{t\times t} & \cdots \\ \vdots & \ddots \end{bmatrix} \tag{2.23}$$

Equation (2.22) looks simple but it can capture rich theories of expectation formation. It can account for an understanding of the data generating process as well as for updating of priors in light of new observations. In our expository environment, $d\mathbf{X}$ is the only input the agent has to form expectations about. So it's not restrictive to assume that $d\mathbf{X}^{e,t}$ depends only on the realized path of $d\mathbf{X}$ itself. In richer environments with multiple inputs, one may introduce additional linear terms, allowing the agent to think about cross-equation restrictions directly. For example, expectations of unemployment benefits and income taxes may be related via an understanding of the government budget constraint.

Corollaries 1 and 2 substitute (2.22) into propositions 7 and 8. The bracketed terms can be interpreted as the Jacobians of the non-FIRE decision problem represented by the function $g\left(\mathbf{X}, \{\mathbf{X}^{e,t}\}_t\right)$. These non-FIRE Jacobians account for the direct effect of $d\mathbf{X}$ on $d\mathbf{Y}$ as well its indirect effect through $\{\mathbf{X}^{e,t}\}_t$. Crucially, they're still $T \times T$ matrices just like the FIRE Jacobians of $f\left(\mathbf{X}\right)$. So, the rest of the SSJ machinery of Auclert, Bardóczy, Rognlie, and Straub (2021) applies without further modifications. In sum, we're now equipped to solve dynamic general equilibrium models with (or without) rich heterogeneity under general deviations from FIRE.

**Corollary 1.** *Consider the setup of proposition 7 with FIRE Jacobian $\mathcal{J}$, and forecast-error Jacobian $\mathcal{E}$. Let the constant expectations be $d\boldsymbol{X}^e = \Lambda d\boldsymbol{X}$, according to* (2.22)*. The linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is*

$$d\boldsymbol{Y} = \Big[\left(\mathcal{J} - \mathcal{E}\right)\Lambda + \mathcal{E}\Big] d\boldsymbol{X} \tag{2.24}$$

**Corollary 2.** *Consider the setup of proposition 8 with FIRE Jacobian $\mathcal{J}$, and forecast-revision Jacobians $\mathcal{R}^{\tau}$. Let expectations be $d\boldsymbol{X}^{e,t} = \Lambda_t d\boldsymbol{X}$, according to* (2.22)*. The linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is*

$$d\boldsymbol{Y} = \left[\mathcal{J}\Lambda_0 + \sum_{\tau \geq 1} \mathcal{R}_{\tau}\left(\Lambda_{\tau} - \Lambda_{\tau-1}\right)\right] d\boldsymbol{X} \tag{2.25}$$

Given a model for beliefs, the matrices given in equations (2.24) or (2.25) fully summarize the response of the heterogeneous agent block to the path $d\mathbf{X}$, taking into account forecast errors and revisions. These matrices are easy and fast to compute after obtaining the FIRE Jacobians $\mathcal{J}$.

## 2.4 HANK model with imperfect expectations

Next we present a full-fledged dynamic general equilibrium model that's suitable for a quantitative evaluation of unemployment benefit extensions with imperfect expectations. Proposition 6 highlights several features that we view as crucial for this exercise:

$$dY_0^{\text{rule}} - dY_0^* = - \underbrace{\mathcal{M}}_{\text{multiplier}} \cdot \underbrace{M_b}_{\text{aggregate demand response to UI}} \cdot \underbrace{\zeta_b(1-\lambda)dU_1}_{\text{forecast error of UI extension}} \tag{2.26}$$

First, we would like anticipated unemployment spells and unemployment benefits to have a reasonable impact on consumption. This requires that households have precautionary saving motive with respect to unemployment. As in our illustrative model, we assume that households are prudent and markets are incomplete. Quantitatively, it matters that households are heterogeneous in the degree of self-insurance against unemployment risk. Some households have ample savings to ride out a typical unemployment spell, while others are dependent on unemployment benefits. Second, we need a model with a reasonable feedback from aggregate demand to equilibrium output and employment. That is, it matters how aggregate demand translates into a distribution of income (from labor, capital, and transfers) over time, the expectations of these incomes, and the responses of consumption and investment.

In light of these considerations, we propose a New Keynesian model with heterogeneous households, search and matching unemployment, sticky prices and wages, investment adjustment costs, and smooth fiscal policy (gradual tax adjustments, long-term bonds). We build on models of automatic stabilizers (McKay and Reis, 2016; Kekre, 2021), and estimated medium-scale New Keynesian models (Christiano, Eichenbaum, and Trabandt, 2016; Auclert, Rognlie, and Straub, 2020; Bardóczy, Sim, and Tischbirek, 2022). Appendix B.3 contains detailed derivations of the equilibrium conditions.

### 2.4.1 Households

The household block is a standard incomplete markets model. There is a unit mass of ex-ante identical households. In any given period, households are heterogeneous with respect to employment status $e_{it} \in \{E, U, N\}$, productivity $z_{it} \in \mathcal{G}_z$, and liquid assets $a_{it-1} \geq \underline{a}$. The model frequency is quarterly and timing is as follows.

1. **Productivity shock.** Households draw a new productivity $z_{it}$ from a finite set $\mathcal{G}_z$. Productivity follows a discrete Markov process with fixed transition matrix $\Pi_z$.

2. **Labor market transitions.** First, employed workers lose their job with probability $s_t$. Second, unemployed workers (including those who separated in this quarter) find jobs with the endogenous probability $f_t$. Third, newly unemployed workers qualify for unemployment benefits with probability $\pi^{get}$, while other households on UI lose eligibility with probability $\pi_t^{lose}$. The probability of losing UI eligibility maps directly to the expected duration of benefits $1/\pi_t^{lose}$ and is the key policy variable. The combined transition matrix for labor market status $e_{it}$ is

$$\begin{array}{c} \\ E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{array} \begin{array}{c} \begin{array}{ccc} E_t & U_t & N_t \end{array} \\ \begin{pmatrix} 1 - s_t(1-f_t) & \pi^{get} s_t(1-f_t) & (1-\pi^{get}) s_t(1-f_t) \\ f_t & (1-\pi_t^{lose})(1-f_t) & \pi_t^{lose}(1-f_t) \\ f_t & 0 & 1-f_t \end{pmatrix} \end{array} \tag{2.27}$$

3. **Consumption-saving decision.** Households choose consumption $c_{it}$ and liquid assets $a_{it}$ to maximize their expected lifetime utility subject to a budget constraint and a borrowing constraint.

The Bellman equation at the consumption-saving stage is

$$
\begin{aligned}
V_t(e_{it}, z_{it}, a_{it-1}) &= \max_{c_{it}, a_{it}} u(c_{it}) + \beta_t E_t[V_{t+1}(e_{it+1}, z_{it+1}, a_{it})] \\
\text{s.t. } c_{it} + a_{it} &= (1 + r^a_{t-1})a_{it-1} + (1 - \tau_t)\Big[y_t(e_{it}, z_{it}) + d^{FI}_t(a_{it-1})\Big] + T_t \\
y_t(e_{it}, z_{it}) &= w_t z_{it} \mathbb{1}\{e_{it} = E\} + b_t z_{it} \mathbb{1}\{e_{it} = U\} \\
a_{it} &\geq \underline{a}
\end{aligned}
\tag{2.28}
$$

Households in state $E$ are employed and earn labor income $w_t z_{it}$. Households in state $U$ are unemployed and receive UI benefits $b_t z_{it}$. Households in state $N$ are unemployed and have exhausted their UI benefits. The lump-sum transfer $T_t$ ensures that every household can maintain positive consumption. Liquid assets are held as short-term deposits that earn riskless return $r^a_{t-1}$. Households also receive transfers $d^{FI}_t(a_{it-1})$ from a financial intermediary. These are indexed to liquid wealth, but are lump-sum in the sense that households don't internalize that accumulating more wealth will increase this transfer.[7]

## 2.4.2 Financial intermediary

All assets in the economy are held by a representative financial intermediary. The assets are three: shares in firm equity $v_t$, long-term nominal government bonds $B_t$, and short-term nominal reserves $M_t$. The liabilities of the financial intermediary are net worth $N^{FI}_t$ and short-term deposits $A_t$ from households. Thus the balance sheet, in date-$t$ real terms, is

$$
p_t v_t + q^B_t \frac{B^N_t}{P_t} + \frac{M_t}{P_t} = N^{FI}_t + A_t \tag{2.29}
$$

[7] This is a simple way to introduce illiquid assets that has three important benefits. First, the model is as easy to solve as any 1-asset model. Second, similarly to full-fledged two-asset models, it can reconcile high average MPC with a realistic amount of assets (including capital). Third, together with the setup of the financial intermediary, the model can match moderate MPC out of asset price fluctuations, which is a challenge for standard two-asset models even with large portfolio adjustment costs and imperfect expectations. See Bardóczy, Sim, and Tischbirek (2022).

where $P_t$ is the price level, $p_t$ is the equity price, $q_t^B$ is price of long nominal bonds, and the price of reserves is 1. Going forward, let $\pi_t = P_t/P_{t-1} - 1$ denote the inflation rate.

The nominal return on these assets are the following. One share of equity purchased in period $t-1$ yields dividend stream $\{P_{t+s}d_{t+s}\}$ for all $s \geq 0$. One government bond purchased in period $t-1$ pays a coupon $\delta_B^s$ in period $t+s$ for all $s \geq 0$. One unit of reserves purchased in period $t-1$ pays $(1+i_{t-1})$ in period $t$. Finally, the intermediary pays out $d_t^{FI}$ as dividend to households in period $t$. This implies that net worth is

$$N_t^{FI} = \underbrace{(d_t + p_t)\nu_{t-1}}_{\text{gross return on equity}} + \underbrace{\frac{1+\delta_B q_t^B}{1+\pi_t}\frac{B_{t-1}}{P_{t-1}} + \frac{1+i_{t-1}}{1+\pi_t}\frac{M_{t-1}}{P_{t-1}}}_{\text{gross return on nominal assets}} - \underbrace{(1+r_{t-1}^a)A_{t-1} - d_t^{FI}}_{\text{pay to households}} \tag{2.30}$$

Payouts to households follow an ad hoc rule

$$\log\left(\frac{d_t^{FI}}{d_{ss}^{FI}}\right) = \phi_N \log\left(\frac{N_{t-1}^{FI}}{N_{ss}^{FI}}\right) \tag{2.31}$$

By choosing a low $\phi_N$, we can smooth out the financial income of households relative to fluctuations in the underlying asset prices. This prevents counterfactually large consumption responses out of asset price fluctuations.

The financial intermediary chooses $v_t$, $B_t^N$, $A_t$, and $M_t$ to maximize its expected return on net worth, $\mathbb{E}_t[N_{t+1}^{FI}/N_t^{FI}]$, subject to the constraints (2.30) and (2.31). This yields the no arbitrage conditions

$$1 + r_t^a = \mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t}\right] = \mathbb{E}_t\left[\frac{1+\delta_B q_{t+1}^B}{q_t^B(1+\pi_{t+1})}\right] = \mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\right] \equiv 1 + r_t \tag{2.32}$$

where we defined $r_t$ as the economy-wide ex-ante real interest rate.

### 2.4.3 Firms

Our specification of firms is standard. We consider three sectors: retailers (nominal rigidities), capital producer (investment adjustment cost), and labor agency (search and matching frictions). These sectors are connected by competitive markets, so one could model them as one type of firm that makes the same decisions subject to the same constraints.

**Retailers.** There is unit mass of retailers indexed by $j$ who engage in monopolistic competition. They produce differentiated goods using a Cobb-Douglas production function with the same productivity $y_{jt} = \Theta_t k_{jt}^{\alpha} n_{jt}^{1-\alpha}$. Firms hire capital $k_{jt}$ and labor $n_{jt}$ on spot markets at prices $r_t^k$ and $h_t$ and pay a fixed cost $\Xi$. They also set the price of their product, $p_{jt}$, subject to a demand curve with constant elasticity $\epsilon$ and a quadratic price adjustment cost à la Rotemberg (1982). We allow for price indexation, so the adjustment cost is paid on price changes relative to a fraction $\iota_p$ of last period's price change. The firms' objective is to maximize the present value of their future profits. The Bellman equation is

$$J_t^R(p_{jt-1}, p_{jt-2}) = \max_{k_{jt}, n_{jt}, y_{jt}, p_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - h_t n_{jt} - r_t^K k_{jt} - \Psi_{jt}^p - \Xi + E_t \left[ \frac{J_{t+1}^R(p_{jt}, p_{jt-1})}{1 + r_t} \right] \right\}$$

$$\text{s.t. } y_{jt} = \Theta_t k_{jt}^{\alpha} n_{jt}^{1-\alpha}$$

$$y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$$

$$\Psi_{jt}^p = \frac{\psi_p}{2} \left[ \log \left( \frac{p_{jt}}{p_{jt-1}} \right) - \iota_p \log \left( \frac{p_{jt-1}}{p_{jt-2}} \right) \right]^2 Y_t$$

In a symmetric equilibrium, all firms choose the same level of output, capital, and labor. So, they have the same marginal cost:

$$mc_t = \frac{1}{\Theta_t} \left( \frac{r_t^K}{\alpha} \right)^{\alpha} \left( \frac{h_t}{1 - \alpha} \right)^{1-\alpha} \tag{2.33}$$

and set the same prices according to the Phillips curve

$$\pi_t - \iota_p \pi_{t-1} = \frac{\psi_p}{\epsilon}\left(mc_t - \frac{\epsilon - 1}{\epsilon}\right) + \frac{1}{1+r_t} E_t\left[\frac{Y_{t+1}}{Y_t}\left(\pi_{t+1} - \iota_p \pi_t\right)\right] \tag{2.34}$$

**Capital producer.** A representative firm owns the capital stock and rents it to retailers at rate $r_t^K$. It's Bellman equation is

$$\begin{aligned} J_t^K(K_{t-1}, I_{t-1}) &= \max_{K_t, I_t}\left\{r_t^K K_{t-1} - I_t + E_t\left[\frac{J_{t+1}^K(K_t, I_t)}{1+r_t}\right]\right\} \\ \text{s.t. } K_t &= (1-\delta)K_{t-1} + \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \end{aligned} \tag{2.35}$$

where $\delta \in (0,1)$ is the depreciation rate, $I_t$ is investment, $\mu_t$ is the marginal efficiency of investment as in Justiniano, Primiceri, and Tambalotti (2010), and $S(\bullet)$ is a convex function that satisfies $S(1) = S'(1) = 0$.

Defining Tobin's $Q$ as the marginal value of capital at the end of period $t$, investment dynamics is characterized by

$$Q_t = \frac{r_{t+1}^K + E_t\left[Q_{t+1}(1-\delta)\right]}{1+r_t} \tag{2.36}$$

$$1 = Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right)\right] + E_t\left[\frac{\mu_{t+1} Q_{t+1}}{1+r_t}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] \tag{2.37}$$

**Labor agency.** A representative firm hires workers on a frictional labor market and rents homogeneous labor services to retailers at rate $h_t$. The agency posts vacancies $v_t$, each of which is filled with probability $q_t$. Following Christiano, Eichenbaum, and Trabandt (2016), we assume a two-tiered cost of hiring. The firm pays $\kappa_v$ to create a vacancy and then $\kappa_h$

for each vacancy it fills.[8] Incumbent workers separate with probability $s_t$. The Bellman equation is

$$J_t^L(N_{t-1}) = \max_{N_t, v_t} \left\{ (h_t - w_t)N_t - (\kappa_v + \kappa_h q_t)v_t + E_t \left[ \frac{J_{t+1}^L(N_t)}{1+r_t} \right] \right\}$$
$$\text{s.t. } N_t = (1 - s_t)N_{t-1} + q_t v_t \tag{2.38}$$

Optimization yields a standard job creation curve, equating the cost and benefit of hiring the marginal worker

$$\frac{\kappa_v}{q_t} + \kappa_h = h_t - w_t + E_t \left[ \frac{1 - s_{t+1}}{1 + r_t} \left( \frac{\kappa}{q_{t+1}} + \kappa_h \right) \right] \tag{2.39}$$

### 2.4.4 Government policy

The fiscal authority issues long-term nominal bonds, collects income taxes, and provides unemployment benefits. Let $U_t$ denote the mass of workers eligible for unemployment benefits. The government budget constraint is

$$G_t + T_t + (1 - \tau_t)b_t U_t + \frac{(1 + \delta_B q_t^B)}{1 + \pi_t} \frac{B_{t-1}}{P_{t-1}} = \tau_t(w_t N_t + d_t^{FI}) + q_t^B \frac{B_t}{P_t} \tag{2.40}$$

Spending $G_t$ and lump-sum transfers $T_t$ are exogenous. The income tax rate $\tau_t$ is chosen according to a rule that can prevent large swings in the tax rate, while ensuring that real government debt is stationary

$$\tau_t - \tau_{ss} = \phi_B q_{ss}^B \left( \frac{B_{t-1}}{P_{t-1}} - \frac{B_{ss}}{P_{ss}} \right) \tag{2.41}$$

[8] The role of $\kappa_h$ is similar to wage stickiness in models without search and matching. It dampens the procyclicality of marginal costs and hence profits. This is especially important in HANK models where strongly countercyclical profits can have large—and unrealistic—redistributive effects (Broer, Harbo Hansen, Krusell, and Öberg, 2020).

In the announcement-based policy, UI duration $1/\pi_t^{lose}$ is exogenous. In the rule-based policy, it is indexed to the end-of-period unemployment rate

$$\frac{1}{\pi_t^{lose}} - \frac{1}{\pi_{ss}^{lose}} = -\zeta_b(N_t - N_{ss}) \tag{2.42}$$

The monetary authority sets the short-term nominal interest rate according to

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\left(i_{ss} + \phi_\pi \pi_t\right) + \epsilon_t^m \tag{2.43}$$

where $\epsilon_t^m$ is a monetary policy shock.

### 2.4.5 Equilibrium

**Wage setting.** A risk-neutral labor union bargains with the labor agency on behalf of employed workers. The surplus of the union is

$$H_t = w_t - b_t + E_t\left[\frac{(1-s_{t+1})(1-f_{t+1})}{1+r_t} H_{t+1}\right] \tag{2.44}$$

and the surplus of the labor agency is

$$J_t = h_t - w_t - \Psi^w(w_t, w_{t-1}) + E_t\left[\frac{1-s_{t+1}}{1+r_t} J_{t+1}\right] \tag{2.45}$$

where $\Psi_w(\bullet)$ is a convex function that captures real wage rigidities

$$\Psi^w(w_t, w_{t-1}) = \frac{\psi_w}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 \tag{2.46}$$

The real wage $w_t$ is then set to maximize $H_t^\eta J_t^{1-\eta}$.

**Matching.** New matches are formed on the labor market according to a Cobb-Douglas matching function

$$M(JS_t, v_t) = A_m (JS_t)^{\ell} v_t^{1-\ell} \tag{2.47}$$

where the mass of job seekers equals the mass of unemployed workers from last period plus the mass of newly separated workers

$$JS_t = 1 - N_{t-1} + s_t N_{t-1} \tag{2.48}$$

Let $\theta_t \equiv v_t / JS_t$ denote labor market tightness. Job finding and vacancy filling probabilities are

$$f_t = A_m \theta_t^{1-\ell} \qquad \text{and} \qquad q_t = \frac{f_t}{\theta_t}. \tag{2.49}$$

**Market clearing.** Factor market clearing requires

$$N_t = \int n_{jt} dj \tag{2.50}$$

$$K_{t-1} = \int k_{jt} dj \tag{2.51}$$

The notation reflects that capital is predetermined from the perspective of capital producers but not from the perspective of retailers. Aggregate dividends are given by

$$d_t = d_t^R + d_t^K + d_t^L = Y_t - w_t N_t - I_t - \Psi_t^p - (\kappa_v + \kappa_h q_t) v_t - \Xi \tag{2.52}$$

Firm equity is then priced according to (2.32). Asset market clearing corresponds to the balance sheet of the financial intermediary (2.29), imposing that the intermediary holds all shares $v_t = 1$, and nominal reserves are zero $M_t = 0$. Nominal reserves are in zero net supply, the purpose of including them is to deliver a Fisher equation in (2.32). Goods market clearing requires that the final good is used for household consumption, investment (including adjustment costs), government spending, price adjustment costs, hiring costs, and the fixed cost.

$$Y_t = C_t + I_t + G_t + \Psi_t^p + (\kappa_v + \kappa_h q_t)v_t + \Xi \tag{2.53}$$

**Definition.** Given initial conditions for the distribution of households $D_{-1}$, net worth $N_{-1}^{FI}$, government debt $B_{-1}$, price level $\{P_{-1}, P_{-2}\}$, investment $I_{-1}$, capital $K_{-1}$, real wage $w_{-1}$, and sequences of exogenous variables $\{\beta_t, \Theta_t, \mu_t, s_t, G_t, T_t, \epsilon_t^m\}$, competitive equilibrium is a sequence of prices $\{P_t, w_t, h_t, r_t^K, r_t, q_t^B, p_t, \pi_t, i_t, r_t^a\}$, aggregates $\{Y_t, N_t, K_t, I_t, Q_t, v_t, q_t, f_t, \theta_t, B_t, \tau_t, \pi_t^{lose}, d_t^{FI}, b_t, d_t, N_t^{FI}, mc_t, U_t, H_t, J_t\}$, policy functions $\{a_t, c_t\}$, and distributions $\{D_t\}$ such that households optimize, financial intermediary optimizes, firms optimize, monetary and fiscal authorities follow their rules, markets clear, job-finding and vacancy-filling probabilities are consistent with the matching function, and the employment and UI eligibility rates are consistent with the distribution's law of motion.

### 2.4.6 Estimation

We estimate the model in two steps, similarly to Christiano, Eichenbaum, and Evans (2005) and Auclert, Rognlie, and Straub (2020). In the first step, we pin down all the parameters that affect the steady state. We fix some parameters to conventional values from the literature, and calibrate others internally to hit steady-state moments. In the second step, we estimate the remaining parameters by impulse response matching.

**Calibration of steady state.** Households have CRRA utility over consumption $u(c) = c^{1-\sigma}/(1-\sigma)$ with an EIS of $\sigma = 0.5$. We set the borrowing limit to $\underline{a} = 0$. We assume that the annual economy-wide real interest rate is $r = 2\%$. The discount factor $\beta$ is calibrated internally to deliver 20% average quarterly MPC out of a one-time lump-sum transfer. Our

Markov process for labor productivity $(\mathcal{G}_z, \Pi_z)$ is the discrete-time equivalent of the process estimated by Kaplan, Moll, and Violante (2018). To account for progressive taxation, we scale down the cross-sectional variance of log productivity by $(1-0.181)^2$, where 0.181 is the degree of progressivity in the log-linear retention function of Heathcote, Storesletten, and Violante (2017). Mean productivity is normalized to 1.

For labor market transitions, we set the job-finding rate to $f = 0.6$ and calibrate the separation rate to deliver an unemployment rate of 4.5%. We assume that UI benefits replace 50% of the steady-state wage, and all unemployed workers qualify for benefits initially, $\pi^{get} = 1$. In steady state, unemployment benefits last on average for 2 quarters, $\pi^{lose} = 0.5$. We set the lump-sum transfer to $T = 0.01$, enough to ensure that borrowing-constrained households who have exhausted their UI benefits can consume a positive amount. We set the vacancy filling rate to $q = 0.7$ quarterly, and assume that $\kappa_h$ accounts for 94% of total search cost, leaving 6% for vacancy posting cost per hire $\kappa_v/q$. We calibrate the bargaining power of the union $\eta$ such that total search cost is 7% of the quarterly wage of an average worker.

We calibrate total factor productivity, $\Theta$, to normalize output to $Y = 1$. We set government debt, $B/P$, to 46% of annual output, and choose the coupon, $\delta_B$, to match the average duration of U.S. government debt of 5 years. Having realistic duration prevents counterfactually large exposure of government budget to fluctuations in short-term interest rates. This matters in non-Ricardian models. We set government spending, $G$, to 16% of output, which leads to a marginal tax rate of $\tau = 0.27$. We set depreciation rate to $\delta_K = 0.083/4$ quarterly and calibrate the capital share $\alpha$ to match a quarterly capital to output ratio of 8.92. This implies that the steady-state labor share is 62%. The fix cost $\Xi$ is calibrated to make total wealth $p + q_b B/P$ equal to 382% of annual output.

One of the most important transition-specific parameters is $\zeta_b$, the semi-elasticity of average unemployment duration with respect to the unemployment rate. We pin this down from a linear approximation of the Emergency Unemployment Compensation (EUC08) program. Our goal is to work with a policy rule that is in the right ballpark, not to provide a serious quantitative evaluation of EUC08 per se.[9]

The unemployment rate in 2007Q1 was 4.6%, close to the steady state value of 4.5% in our model. Unemployment rate peaked at 10.1% in 2009Q4. During the same time, unemployment benefit duration was raised from 26 weeks to 99 weeks (in states with unemployment rate above 8.5%). So, our back of the envelope calculation for the semi-elasticity is $\zeta_b = (99 - 26)/13/(0.101 - 0.046) \approx 102$. That is, a one percentage point increase in the unemployment rate triggers 1.02 quarter increase in average UI duration.

**Estimation: IRF Matching.** As in Christiano, Eichenbaum, and Evans (2005), we estimate the model by matching the impulse response functions obtained in our model to their empirical counterparts obtained with a standard business-cycle shock. We generate the empirical impulse responses by following the empirical strategy in Angeletos, Huo, and Sastry (2021). That is, we estimate the regression

$$z_t = \alpha + \sum_{p=1}^{P} \gamma_p z_{t-p}^{IV} + \sum_{k=0}^{K} \beta_k \varepsilon_{t-k} + u_t \tag{2.54}$$

where $z_t$ is an outcome of interest (e.g. unemployment rate), $\varepsilon_t$ is an identified shock, and $z_{t-p}^{IV}$ are lagged values of $z_t$ instrumented by lagged values of $\varepsilon_t$. Our identified shock is the main business cycle (MBC) shock from Angeletos, Collard, and Dellas (2020). This shock is constructed to account for most of the business cycle fluctuations in unemployment rate. We generate impulse response functions not only for outcomes, but also generate the impulse response functions of expectation for the relevant variables at several horizons.

To perturb the economy from its steady-state level, we consider a single shock to the

[9] EUC08 features nonlinearities and a staggered rollout which we ignore. Kekre (2021) for example takes these features into account but assumes perfect foresight with respect to the announced policy.

marginal efficiency of investment (MEI) which follows an AR(1) process

$$\mu_t = \rho_\mu \mu_{t-1}.$$

The process for this shock has two free parameters: the persistence $\rho_\mu$ and the initial level of the shock $\mu_0$. Angeletos, Collard, and Dellas (2020) show that the MBC and MEI shocks are closely related. Using our model, we solve the impulse responses of variables in our model to this shock for a given set of parameters. In doing so, we attribute the estimated impulse responses for expectations directly into our household block using equation (2.21). We focus on the implications of imperfect expectations for aggregate demand and so assume that all other economic agents have full information and rational expectations.[10]

We recover the implied impulse response functions IRF($\Omega$), where $\Omega$ denotes the set of that we estimate which can be seen in Table 2.2. We choose values for these parameters so as to minimize the distance between our model's implied impulse response and those estimated in the data:

$$\hat{\Omega} = \arg\min_{\Omega} \left(\text{IRF}(\Omega) - \widehat{\text{IRF}}\right)' \Sigma^{-1} \left(\text{IRF}(\Omega) - \widehat{\text{IRF}}\right),$$

where $\widehat{\text{IRF}}$ denotes the estimated impulse response function. In our estimation, we include the impulse response functions for the unemployment rate, consumption, inflation rate, and the nominal interest rate.

**Expectations: Data limitations and solution.** In practice, the exercise described in the previous section cannot be fully implemented due to the unavailability of all necessary expectations data. In this dimension, we confront two limitations: (1) we may not have survey data on expectations to all relevant variables and (2) even for the variables for which we do have survey data for, we only observe expectations for a finite number of future

[10] It is easy to extend the estimation to incorporate imperfect expectations into the decision problems of other economic agents.

horizons, and not the infinite number of horizons which would be required to solve the model.

We use data for the Survey of Professional Forecasters (SPF). From this dataset, we use data for one to four quarters-ahead unemployment rate forecasts.[11] However, people must still form expectations about the real-interest rate, tax rates, job-finding rates, among others. Furthermore, they must also form expectations about the unemployment rate at horizons beyond the fourth quarter ahead. We solve both of these issues by imposing a parametric model of beliefs to generate beliefs of variables for which expectations data are lacking and extrapolate unemployment forecasts beyond the fourth quarter horizon.

As Angeletos, Huo, and Sastry (2021) point out, most popular models of belief formation generate either underreaction or overreaction at all horizons. This pattern is very clearly seen in the impulse response of forecasts observed in Figure 2.1. To capture the estimated pattern of initial underreaction followed by delayed overreaction, we combine noisy information with diagnostic expectations and long memory. In doing so, we build on Bordalo et al. (2020) (who combined noisy information with standard diagnostic expectations) and on Bianchi et al. (2021) (who introduced diagnostic expectations with long memory). In Appendix B.4.1, we discuss the merits of this model of beliefs relative to other popular models in the literature. As Figure B.1 shows, having both features is essential to match the estimated pattern.

As we discuss in Appendix B.2.3, the noisy-information and long-memory diagnostic expectations model implies that the time $t$ average expectation to a deterministic shock takes the following form:

$$\overline{E}_t[dX_{t+h}] = \underbrace{\left[(1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} - \theta\sum_{j=1}^{t}\alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right)\right]}_{\equiv\lambda_t} dX_{t+h}, \tag{2.55}$$

[11] We are currently working on incorporating SPF data for other variables into our framework.

where $\overline{E}_t[X_{t+h}]$ denotes the average expectation and $\mathbb{E}_t[X_{t+h}]$ denotes the full-information and rational expectations in that same economy, and uses the following convention

$$\sum_{j=1}^{t} \alpha_j \left( \frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j} \right) = 0$$

if $t = 0$. This model features several parameters: $\theta$ denotes the degree of belief overreaction, $\alpha_j \geq 0$ for $j \geq 1$ denote the memory weights and satisfy $\sum_{j=1}^{\infty} \alpha_j = 1$, and $\tau \equiv \tau_\epsilon/\tau_\nu$ denotes the ration of the precision of priors to the precision of the noisy signals.

Table 2.1: Belief parameters

| Parameter | Description | Value |
|---|---|---|
| $\theta$ | Diagnostic expectation param | 4.332 |
| $\tau$ | Noisy information param | 10.304 |
| $\alpha$ | Long memory param 1 | 7.536 |
| $\beta$ | Long memory param 2 | 24.907 |

This model nests four known models as special cases. First, assume that $\tau = 0$ and $\alpha_1 = 1$. Then, this model collapses to the standard diagnostic expectations, as in Bordalo, Gennaioli, and Shleifer (2018). Second, maintaining the assumption that $\tau = 0$ but allowing for the memory weights to assign mass to further away expectations, our model also nests the long-memory diagnostic expectation model used in Bianchi, Ilut, and Saijo (2021). Third, assuming that $\theta = 0$ but $\tau > 0$, this model collapses to the standard noisy-information and rational expectations model as in Angeletos and Huo (2021). Finally, allowing $\theta > 0$ and $\tau > 0$ but assuming that $\alpha_1 = 1$, then this model collapses to the standard noisy-information and diagnostic expectations model used in Bordalo, Gennaioli, Ma, and Shleifer (2020). Our model is best understood as extending this final model to allow for long-memory, which turns out to be essential in capturing the pattern of initial underreaction followed by delayed overreaction which can be seen in Figure 2.1.[12]

As in Bianchi, Ilut, and Saijo (2021), we assume that the $\alpha_j$ are determined by a Beta-binomial distribution with parameters $\alpha$ and $\beta$. This assumption implies that we have four parameters to calibrate in this model $\theta$, $\tau$, $\alpha$, and $\beta$. We calibrate these parameters so that the beliefs that they would imply for the unemployment rate forecasts line up with those that we observe in the data. However, note that, in solving the model, we actually use directly the observed unemployment rate forecasts and not the ones implied by this model.

The calibrated parameters are found in Table 2.1 and the models empirical match can be seen in Figure 2.2. Overall, the fit to the data we actually observe is good. Note that, in this model, the ratio of under or overreaction,

$$\frac{\overline{E}_t[dX_{t+h}]}{dX_{t+h}} = \lambda_t,$$

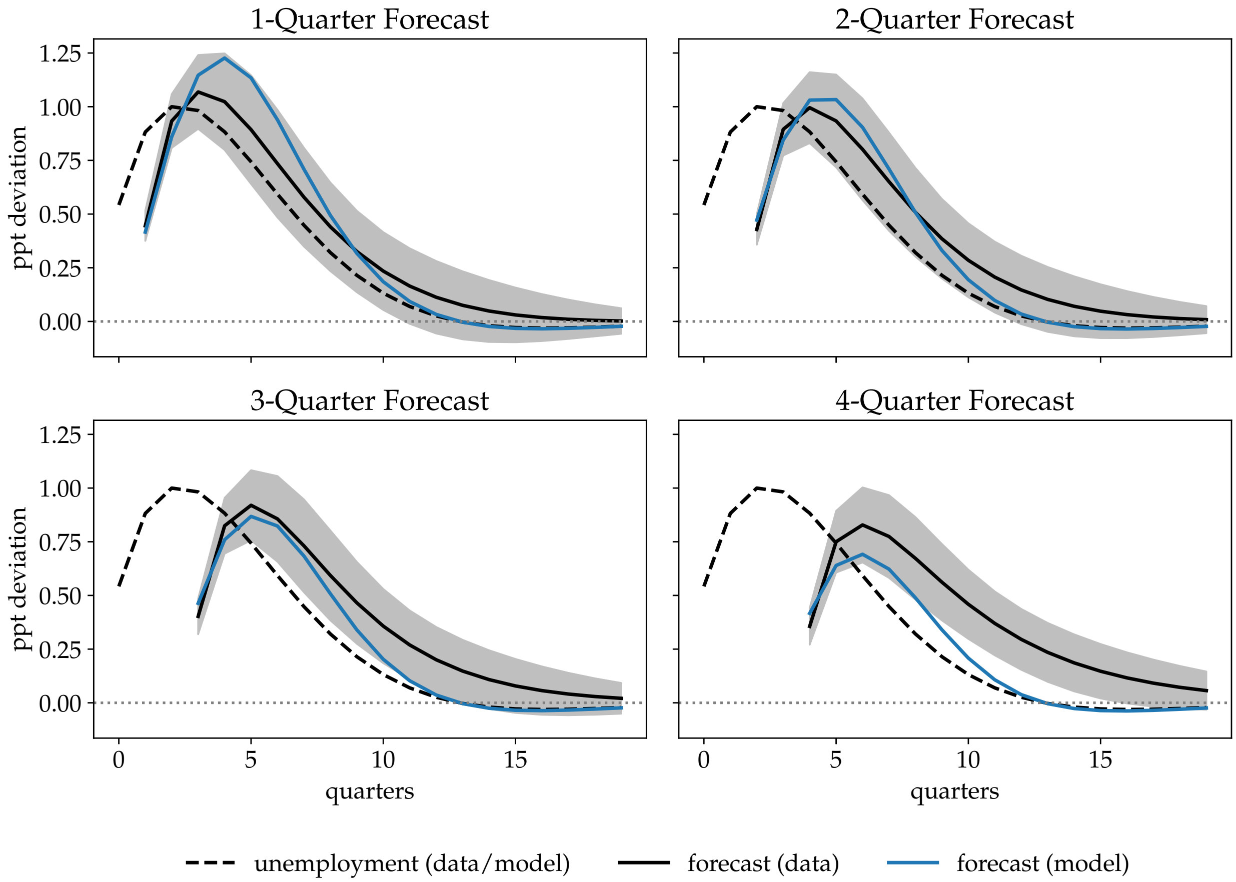
is constant across horizons. As it turns out, to match the data, the implied forecasts slightly exagerate the amount of overreaction at the shortest horizon while underestimating the amount of overreaction at longer horizon. We leave a more in depth analysis of this interesting fact for future work.

## 2.5 Results

In this section, we present the main results of our estimation exercise. Furthermore, we compare the implied impulse responses in our model to the benchmarks of perfect and no anticipation of future changes. This exercise allows us to understand the implications of the patterns of imperfect expectations observed in the data.

[12] See Appendix B.4.1 for a discussion.

Figure 2.2: Calibration of belief parameters




## 2.5.1 Estimation results

We estimate the remaining parameters which are relevant for the transition dynamics in our economy. These parameters are as follows. The monetary policy parameters: the Taylor-rule coefficient on inflation, $\phi_\pi$, the Taylor-rule coefficient on unemployment, $\phi_u$, and the Taylor-rule inertia, $\rho_m$. The investment adjustment cost, $\psi$, and the real-wage adjustment cost, $\psi_w$. The elasticity of the tax rate to debt, $\phi_B$. The nominal rigidity parameters: price indexation, $\iota_p$, and the slope of the Phillips curve, $\kappa_p$. The financial income payout rate, $\phi_N$. Finally, the parameters controlling the scale and the persistence of the MEI shock: $\mu_0$

and $\rho_\mu$, respectively. The estimated values for the parameters which affect the transition dynamics in our economy can be found in Table 2.2.

Table 2.2: Estimated parameters

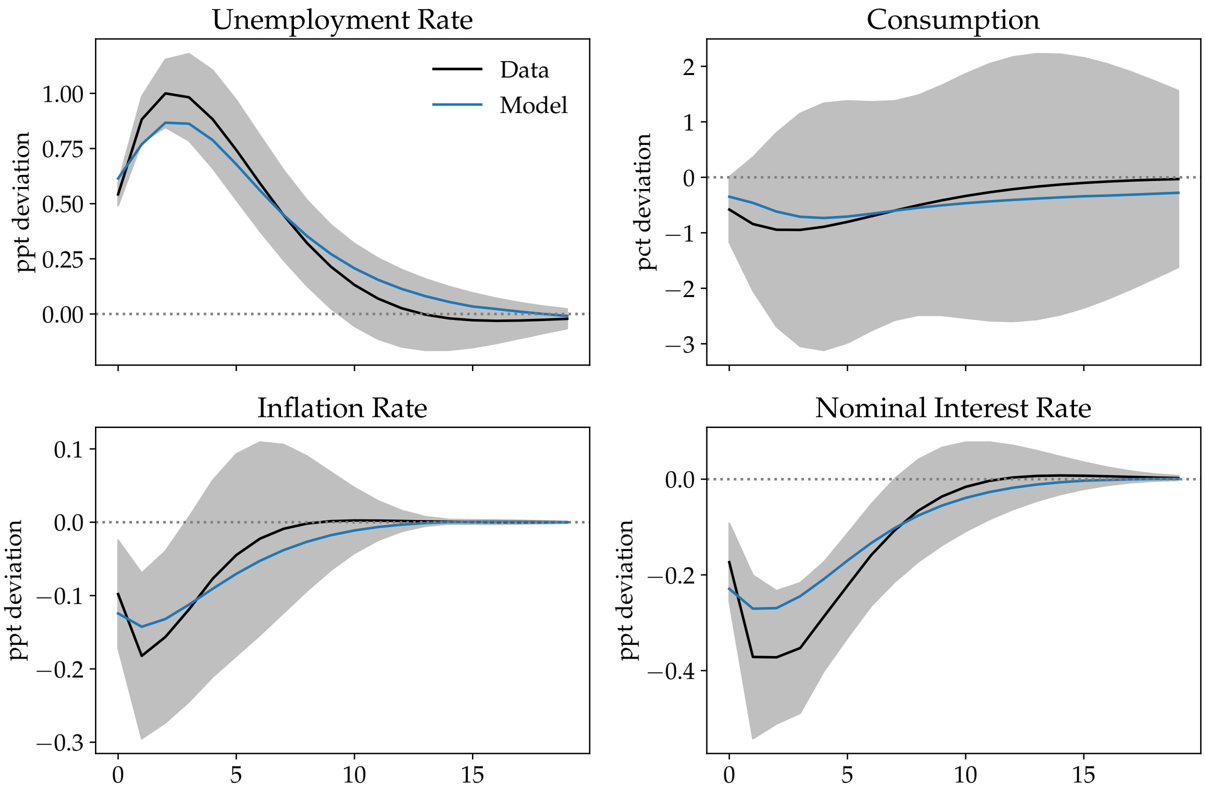
| Parameter | Description | Value |
|---|---|---|
| *Core model parameters* | | |
| $\phi_\pi$ | Taylor rule coef on inflation | 1.241 |
| $\phi_u$ | Taylor rule coef on unemployment | 0.122 |
| $\rho_m$ | Taylor rule inertia | 0.000 |
| $\psi$ | Investment adjustment cost | 1.788 |
| $\phi_B$ | Response of tax rate to debt | 0.054 |
| $\psi_w$ | Real wage adjustment cost | 1082.0 |
| $\iota_p$ | Price indexation | 0.249 |
| $\kappa_p$ | Phillips curve slope | 0.075 |
| $\phi_N$ | Financial income payout rate | 0.009 |
| *MEI shock process* | | |
| $\mu_0$ | Scale of MEI shock | 0.025 |
| $\rho_\mu$ | Persistence of MEI shock | 0.716 |

The model's impulse response functions are shown in the blue lines in Figure 2.3 for the unemployment rate, consumption, inflation rate, and the nominal interest rate. The black line shows the associated empirical impulse responses the the shaded region plots the 68% confidence interval around the empirical point estimates. The model provides a good fit to its targeted empirical counterparts.

### 2.5.2 Quantifying the consequences of imperfect expectations

In this section, we assess the efficacy of UI extensions on stimulating demand with imperfect expectations. The goal is to quantify the role of imperfect anticipation of endogenous UI extensions in affecting aggregate demand. As we have discussed before, we do so by working directly with the empirical response of expectations, avoiding the need to choose a particular model of belief formation. We show that the direct effect of UI extensions on the distribution

Figure 2.3: Targeted Impulse Responses: Outcomes

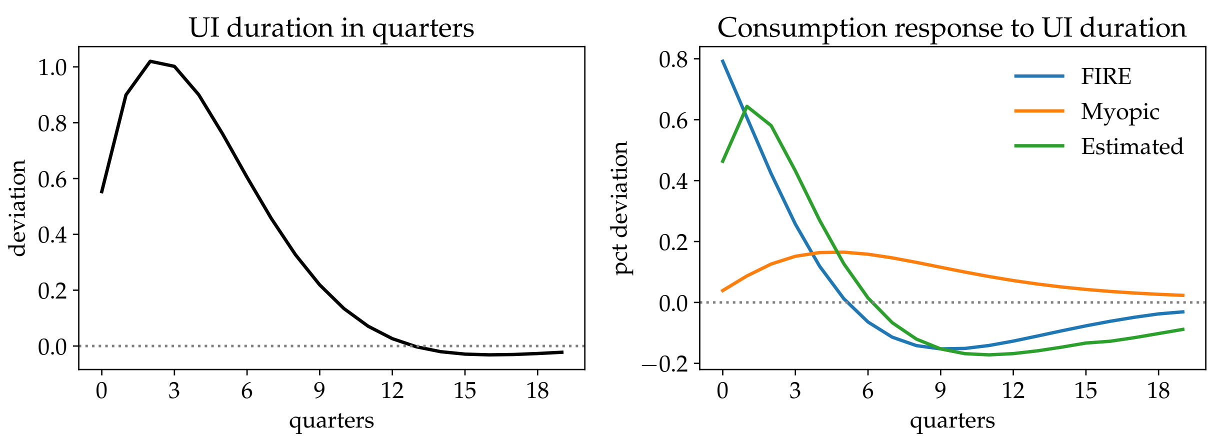



of income is less important than their indirect effect on precautionary saving. This implies that the power of UI extensions to boost aggregate demand is diminished if households do not anticipate them. We show this result in partial equilibrium (using only the calibrated household block) as well as in general equilibrium (using the full estimated HANK model).

**Partial-equilibrium analysis.** UI extensions can boost aggregate demand by two channels. First, directly, by raising the income of unemployed households who get to keep their benefits thanks to the extension. Second, indirectly, by reducing the precautionary savings of employed households facing the risk of job loss, and of unemployed households facing the risk of losing benefits. Our first goal is to establish that the precautionary saving channel is quantitatively relevant.

We consider a UI extension that would be triggered, according to policy rule (2.42), by the empirical impulse response of unemployment with respect to the main business cycle shock. The path of UI duration is plotted in the left panel of Figure 2.4. We feed this path of UI duration to the households of our HANK model and compute the response of aggregate consumption under different assumptions about expectations. For the purposes of this partial equilibrium exercise, we keep all other prices, income, and the job-finding rate constant at their steady-state level.[13] We contrast the response of the economy under the estimated beliefs with two extreme benchmarks: (1) full-information and rational expectations (FIRE) and (2) myopia. The first benchmark assumes that people have the correct expectations, which implies that they make no forecast errors. The second benchmark assumes that people never revise their beliefs about the future and so consistently make forecast errors. So the first benchmark features perfect anticipation of UI benefits, while the second benchmark features no anticipation of UI benefits.

Figure 2.4: Partial-equilibrium Consumption Responses to an UI Extension




[13] That is, our results depend only on the calibrated household block, and are independent from the supply side and policy blocks of the model.

The right panel of Figure 2.4 shows the impulse response of aggregate consumption. The blue line is computed assuming that households have FIRE or perfect foresight of the rise in UI duration from period 0 onwards. In this scenario, aggregate consumption rises sharply on impact due to an immediate reduction in precautionary savings, stays above steady state for five quarters, and then falls below steady state as households start to build back their normal buffer stock of savings. To understand why consumption falls below steady state before it recovers, note that a UI extension raises incomes only for those workers that lose their job and stay eligible longer. The majority of households remain continuously employed during the period of the UI extension. From their perspective, UI extension reduces risk, but provides no income. They optimally adjust their buffer stock in response to the change in unemployment risk.

The second scenario, myopia, isolates the role of actual transfers, as household (wrongly) forecast no change to their income prospects upon unemployment. The orange line shows that the resulting response of aggregate consumption is markedly different from the first scenario with FIRE. Conditional on their individual states, the households' consumption-saving decisions do not change at all. The hump-shaped aggregate consumption response is driven entirely by changes in the distribution. The mass of UI eligible households rises while the mass of ineligible unemployed households falls. Aggregate consumption rises moderately because the households who receive UI benefits consume more on average than those who exhausted their benefits. The comparison of this scenario with FIRE demonstrates the importance of anticipation of UI benefits in shaping the consumption response to the policy. In fact, the peak response of aggregate consumption to unemployment benefits is over four times as large with FIRE than with myopia, and it happens on impact as opposed to 5 quarters later.

In the third scenario, we give households the expectations estimated in the ARMA-IV regression (2.54). As figure 2.1 shows, estimated beliefs feature initial dampening followed by delayed overreaction relative to the actual path of the unemployment rate. Since we assume that households have first-order knowledge of the policy, the same pattern applies to beliefs about UI duration. It follows that the initial response of aggregate consumption is muted relative to FIRE, but much higher than the myopic scenario since people still anticipate some of the UI extension. However, because of the overreaction in beliefs, after a few periods the response of aggregate demand becomes even higher than under FIRE due to the effect of perceived UI duration on precautionary savings.

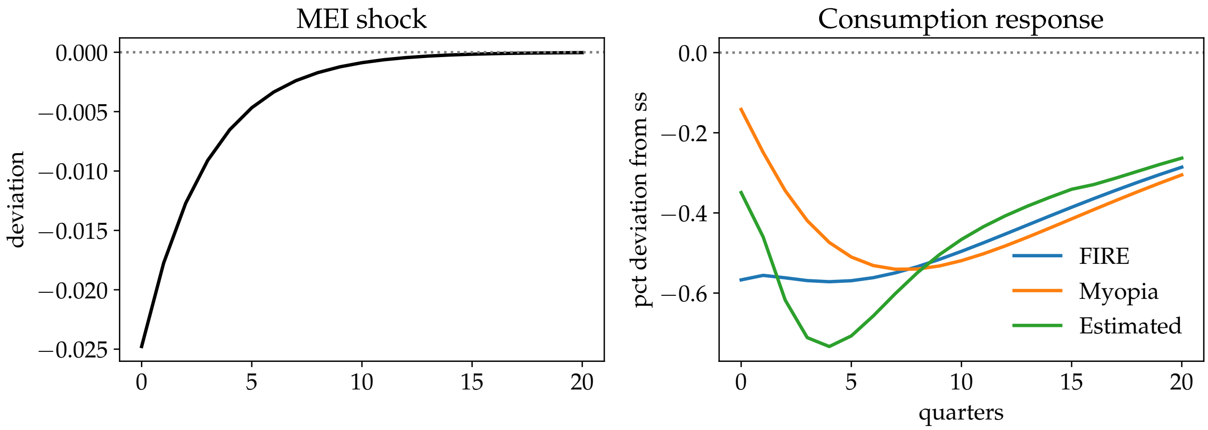
**General-equilibrium analysis.** We established that imperfect anticipation of UI extensions has a large impact on the partial-equilibrium response of aggregate demand to the policy. Next, we compute the consequences imperfect anticipation in the full dynamic general-equilibrium model.

Figure 2.5 displays the impulse response of aggregate consumption to the marginal efficiency of investment (MEI) shock. As in the previous section, we compare the response in our baseline economy with the estimated beliefs to the benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). In performing these comparisons, we fix all parameters (other than those relating to beliefs) to their estimated values (see section 2.4.6). We then compute the dynamic response of those benchmarks to the same MEI shock. The response of our baseline economy can be seen in green, while the response under FIRE and Myopia can be seen in blue and orange.

Aggregate consumption falls in response to this negative MEI shock for all models, mostly because firms invest less and hire less workers, leading to a rise in unemployment and a decline in incomes. As unemployment surges the government responds by increasing UI benefits, which helps stimulate the economy, but does not fully offset the shock.

The initial drop in consumption is less pronounced in the baseline model than with FIRE. This result is a consequence of the fact that individuals are more optimistic about

Figure 2.5: General Equilibrium Impulse Responses to MEI Shock



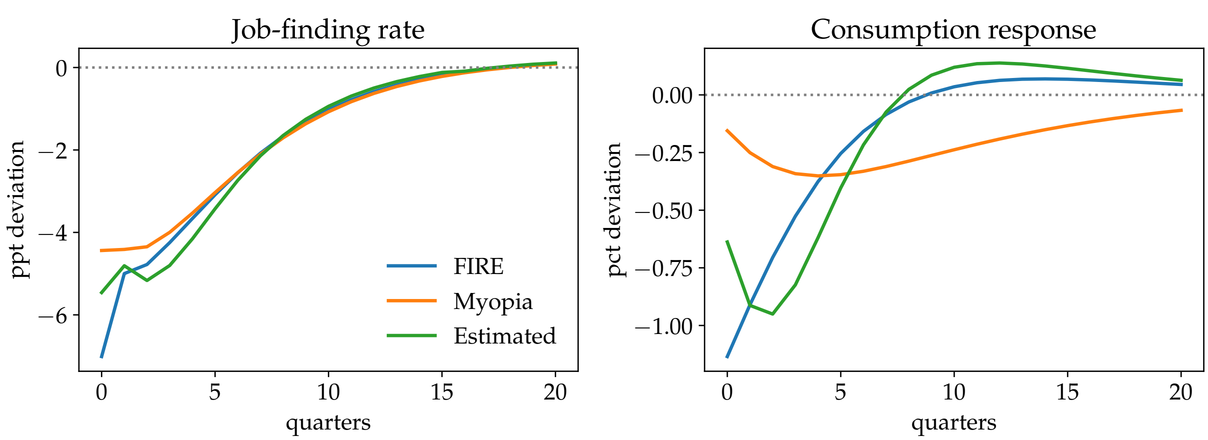

the depth of the recession due to the initial underreaction of beliefs, i.e., individuals think that unemployment will not rise as much. The same holds for the comparison of Myopia to the two other lines. The initial drop in consumption is -0.39, -0.58, and -0.16 percent for the baseline, FIRE, and Myopia economies.

However, after this initial period, individuals become more pessimistic about the future path of unemployment and job finding prospects than they would under FIRE or Myopia. It follows that individuals predict larger unemployment risk and so, despite also predicting higher UI benefits, they have a higher precautionary-savings motive and cut their consumption by more relative to FIRE and Myopia. These effects imply a hump-shaped response of aggregate consumption which would not be present with FIRE. The peak response of aggregate consumption with the estimated beliefs if -0.81, while for FIRE it is equal to the initial response -0.58. Over time, these very pessimistic expectations are not realized and individuals consume their excess savings, justifying the fact that consumption is higher after 10 quarters under the estimated beliefs than under both other benchmarks.

Figure 2.5 highlights the importance of expectations on the general-equilibrium response of aggregate consumption. However, it does not allow us to understand the independent effects of each general-equilibrium channel. To better understand the consequences of the fall in job finding rates and the endogenous rise in UI duration, we now decompose the overall GE effect. We isolate the effect of different channels on consumption, by taking the Jacobian with respect to that input and multiplying it by the impulse response of that input. We focus primarily on understanding the effects coming through these two channels due to their central importance in our analysis. In appendix B.4.2, we complement the analysis here by describing the effects of the remaining GE forces.

**Job-finding rate.** As a consequence of the MEI shock, firms post less vacancies which leads to a decline in the job-finding rate. In Figure 2.6, we evaluate the effect of this general-equilibrium channel on aggregate consumption for our baseline economy (Estimated) and the two benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). The left panel shows the IRF for the job-finding rate while the panel on the right computes the isolates the effect of the decline in the job-finding rate for aggregate consumption.

Figure 2.6: Partial effect of job-finding rate on consumption




The impulse responses for the job-finding rate are very similar across the three models.

The initial drop is larger under FIRE and smaller with myopic beliefs. The baseline economy sees an initial smaller drop in the job finding rate relative to FIRE, but the ranking is reversed after 3 periods.

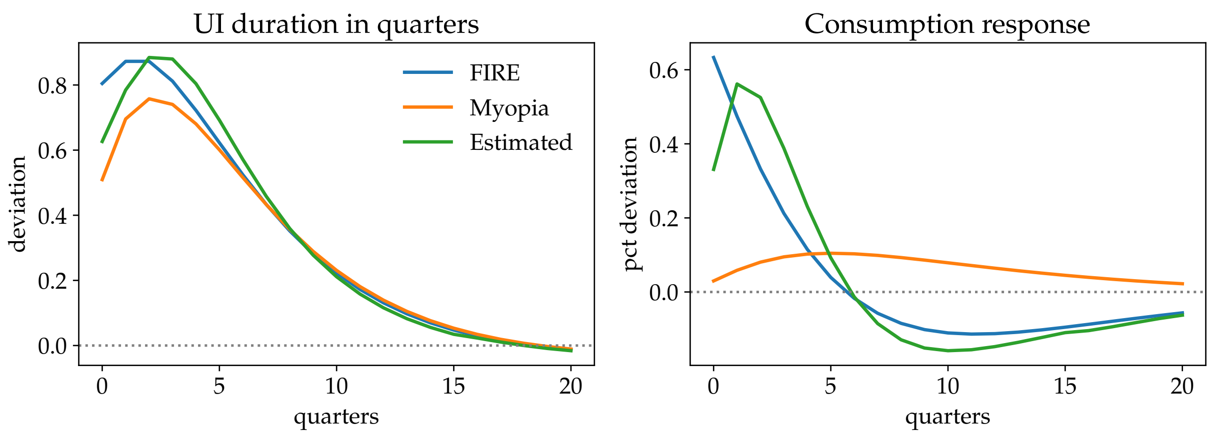
Despite featuring similar job-finding rates, there are large differences in the response of aggregate consumption. With myopic beliefs, since people do not anticipate the coming recession, the initial decline in consumption is small. However, as unemployment rises, workers are faced with unexpectedly longer unemployment spells which leads to a severe contraction of spending in later periods. Instead, with FIRE, there is a large initial drop in consumption which fully recovers by period 10.

In our baseline economy, we see that because beliefs initially underreact, the consumption response is muted relative to FIRE, but it is larger than under myopia. However, after this initial phase, individuals become more pessimistic and the average belief overreacts relative to FIRE. The consequence is a pronounced drop in aggregate consumption. It is noteworthy that this pattern of delayed overreaction in beliefs implies a hump-shaped response of consumption to the job-finding rate, which would not be possible under FIRE.

**UI duration extension.** As the job-finding rate drops, the unemployment rate rises which triggers an extension in the duration of unemployment insurance. In Figure 2.7, we evaluate the effect of this general-equilibrium channel on aggregate consumption for our baseline economy (Estimated) and the two benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). The left panel shows the IRF for the UI duration while the panel on the right computes the isolates the effect of the increase in UI duration for aggregate consumption.

The duration of UI tracks exactly the impulse response of the unemployment rate, which follows a hump shape. Unemployment rises initially the most for FIRE and the least for myopic beliefs. The baseline economy features an initial lower unemployment rate than

Figure 2.7: Partial effect of UI duration extension on consumption




FIRE, but a larger peak level of unemployment. These dynamics feed exactly to the UI duration.

As we have discussed before, extending UI stimulates consumption for two reason: the redistribution channel and the precautionary savings channel. The panel on the right shows that, under myopic beliefs, the impact of UI extensions on aggregate consumption is very small when compared to FIRE. This shows that the major source of stimulus comes from the precautionary savings channel rather than the redistribution channel. With FIRE, UI extensions are very powerful at stimulating consumption especially on impact. The strength of the initial impact is largely a consequence of the anticipation of higher future UI benefits, leading people to save less and thus consume more.

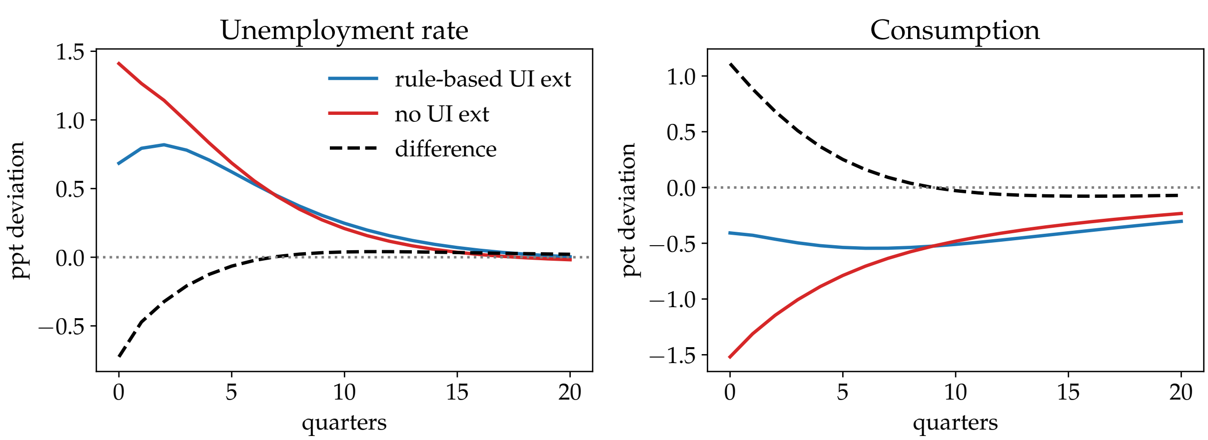
Our baseline economy features lower initial anticipation of UI extensions relative to the FIRE benchmark. Figure 2.7 shows that this initially mutes the consumption response to UI extensions. However, as noted before, the empirical pattern of beliefs implies that eventually beliefs overreact relative FIRE. It follows that they begin anticipating larger UI benefits and thus a stronger stimulative power for this policy.

## 2.6 Quantifying the stimulative power of UI extensions

The results in the previous section allow us to understand the importance of anticipating UI extensions in determining their efficacy in stimulating consumption. Furthermore, they allow us to compare the implications of the empirical patters of beliefs relative to two important benchmarks: FIRE and myopia. However, those results do not allow us to quantify the power of UI extensions. For that purpose, we need to compare the impulse responses obtained in the previous section with those obtained in a counterfactual economy assuming no extensions, i.e., $\zeta_b = 0$.

For the purposes of computing a counterfactual response, we must take into account how beliefs differ in this counterfactual economy relative to our baseline analysis. This means that we must specify a model for belief formation, and can no longer simply rely on the estimated beliefs. We assume that beliefs are determined by the noisy-information and long-memory diagnostic expectations model (baseline) that we estimate in Section 2.4.6. This choice is in line with our discussion in that section and further extended in Appendix B.4.1.

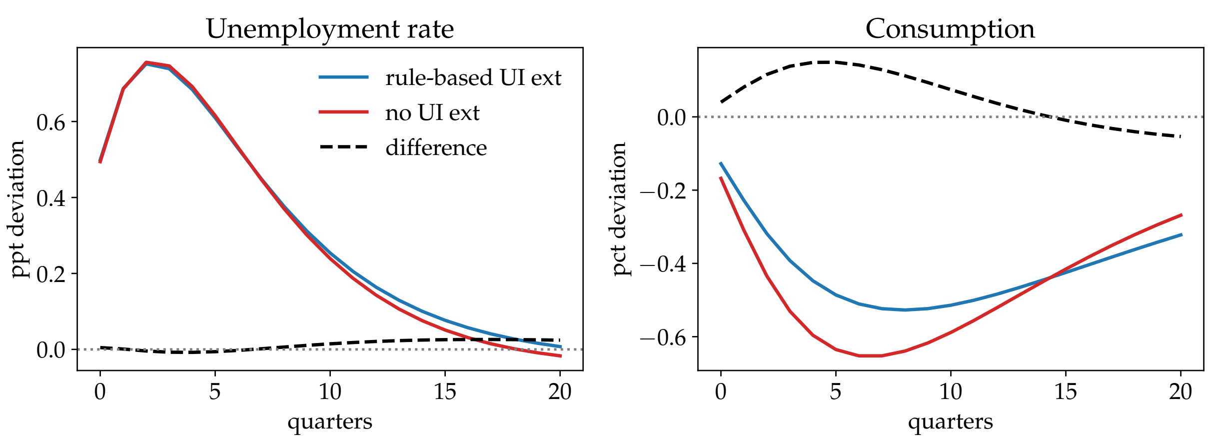
Figure 2.8: Impact of UI extension with FIRE



Figures 2.8, 2.9, and 2.10 display the impulse responses under FIRE, myopia, and baseline beliefs, respectively. For each figure, the left panel displays the impulse responses for the unemployment rate with the policy (blue) and without the policy (red). The black dashed line plots the difference between these two responses and is a measure of the stimulus. The right panel displays the analogous three responses for consumption.

Figure 2.8 shows that, with FIRE, the strongest effects of UI extensions happen immediately on impact of the shock. At this initial date, the policy leads to a decrease in the unemployment rate of 0.7 percentage points and in increase in consumption of 1 percentage point on impact. The stimulus effect then declines over time and dissipated by the eighth quarter.

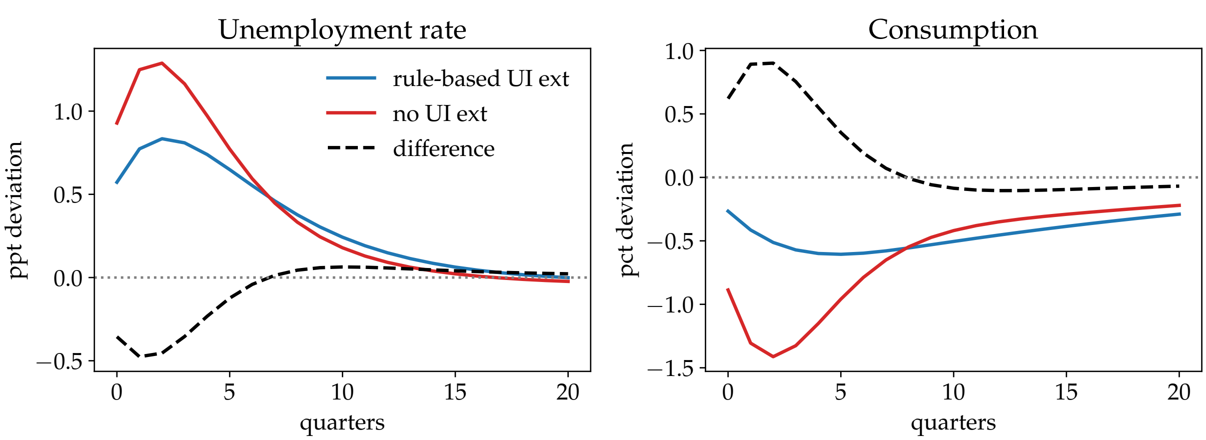
Figure 2.9: Impact of UI extension with Myopia




The positive impact of UI extensions are mostly a consequence of the effect that their anticipation has on precautionary savings. This fact is most clearly seen in the comparison of the efficacy of UI extensions with FIRE (Figure 2.8) and Myopia (Figure 2.9). In fact, with Myopia we see that the extensions have almost no impact on the unemployment rate and only a very moderate impact on consumption (echoing the partial equilibrium results in the previous section). We see that the peak impact of UI extensions is slightly delayed relative to FIRE, which is a consequence of the fact that without anticipation, only the redistribution channel is operative and so the impact of the policy tracks closely the UI duration at each point in time. Still, the peak impact of UI extensions is essentially zero for the unemployment rate and less than 0.1 percentage points for consumption.

Figure 2.10 shows the analogous results for the baseline economy. We can see several features. First, the impact of UI extensions are dampened relative to FIRE. This feature is especially true at the onset of the shock, where the promise of UI extensions are responsible for a decrease in the unemployment rate of close to 0.4 percentage points and an increase in consumption of 0.6 percentage points. Due to the delayed overreaction of beliefs, the impact of UI extensions also follows a hump shape. Intuitively, in the economy with delayed belief overreaction, individuals switch from their initial over-optimism to over-pessimism about the future unemployment rate. However, since people understand the policy rule, they believe that this increase in unemployment will trigger an expansion of UI generosity. The peak impact of UI extensions leads decreases the unemployment rate by over 0.5 percentage points and increases consumption by close to 1 percentage point.

Figure 2.10: Impact of UI extension in the baseline economy



## 2.7 Alternative policy implementation

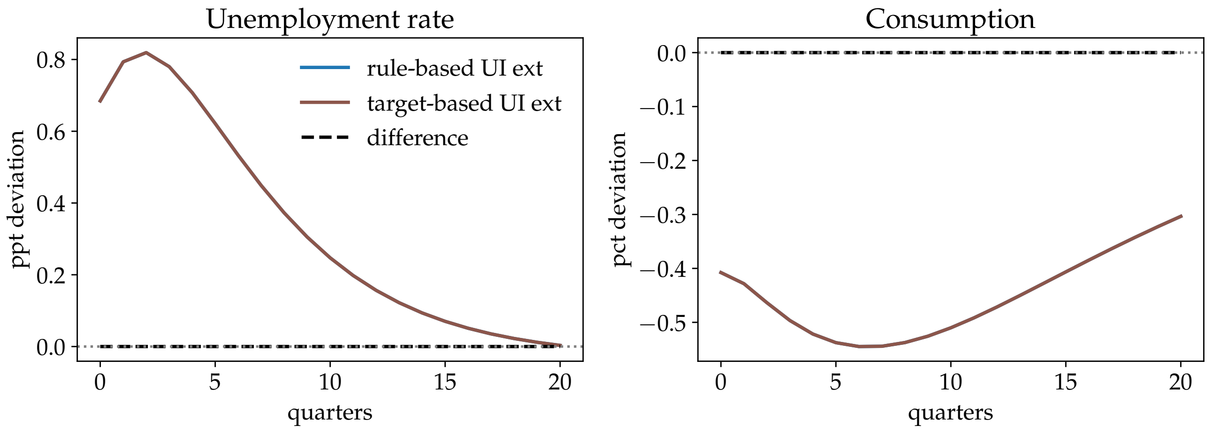
In this section, we are interested in understanding how the efficacy of UI duration extensions is affected by the way in which the policy is implemented. As in Bianchi-Vimercati, Eichenbaum, and Guerreiro (2021), we are interested in comparing the impact of the policy when it is implemented and announced as a rule versus when the path of UI duration is directly announced. In the latter case, we assume that the government directly announces a path for the policy variable $\pi_t^{lose}$ and that this announcement is immediately learned and understood by all market participants.

As in Section 2.6, making this comparison requires us to compute a counterfactual path for the economy under a different policy implementation. So in our baseline economy, we maintain the assumption that beliefs are given by the noisy-information and long-memory diagnostic expectations model (baseline) that we estimate in Section 2.4.6. We first compute the response of the economy under the assumption that the policy is implemented as a rule. We then recover the implied path for UI duration and compute the dynamic response in the counterfactual economy where the same policy is directly announced at the onset of the recession. As in the previous section, we do this analysis in our baseline economy and for comparison also perform the analysis under FIRE and myopia.

Figures 2.11, 2.12, and 2.13 display the impulse responses under FIRE, myopia, and baseline beliefs, respectively. For each figure, the left panel displays the impulse responses for the unemployment rate with the rules-based policy (blue) and with the instrument-announcement policy (brown). The black dashed line plots the difference between the equilibrium with announcement and that with rules. The right panel displays the analogous three responses for consumption.

Figure 2.11 shows that with perfect anticipation, i.e., with FIRE, the two forms of policy implementation lead to the same outcomes. With FIRE, it doesn't make any difference

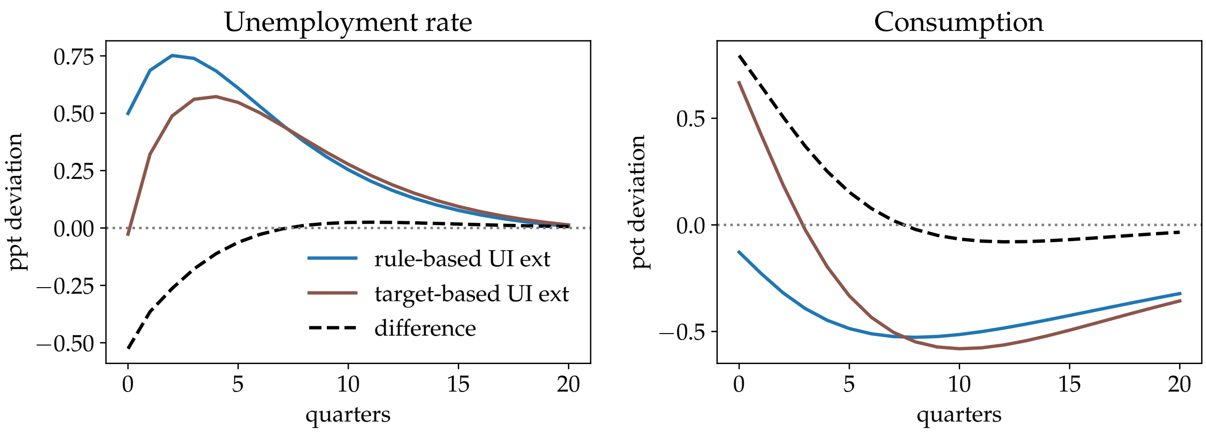
Figure 2.11: Instrument-rule vs announcement with FIRE




whether the instruments are directly announced or that they are announced as a rule, since people accurately forecast the behavior of the unemployment rate and can thus use it to perfectly predict the future path of unemployment duration (see also Angeletos and Sastry, 2021, and Bianchi-Vimercati, Eichenbaum, and Guerreiro, 2021). Instead, Figure 2.12 shows that with myopia there can be large differences between these two forms of implementation. When the instrument is directly announced to people, the model features large anticipation of UI benefits and so a strong stimulative power of the policy. Indeed, the unemployment rate falls by over 0.5 percentage points on impact and consumption is boosted by almost 0.7 percentage points on impact.

Finally, in our baseline economy, the results are mixed. The announcement-based policy limits the initial rise in the unemployment rate by over 0.25 percentage points, and the fall in consumption by 0.28 percentage points. After the initial period, the order is reversed and the unemployment rate becomes 0.15 percentage points higher in the economy with the announcement-based policy. Similarly, consumption falls 0.25 percentage points more. These results are a direct consequence of the pattern of delayed overreaction present in beliefs.

Figure 2.12: Instrument-rule vs announcement with myopia




Initially, beliefs underreact, so people under forecast the generosity of UI extensions in the economy with the policy rule. So it is more powerful to directly announce the extension in UI benefits.[14] However, after this initial period, beliefs overreact and people become overly pessimistic. So under the rule-based policy, people believe that UI benefits will be extended for longer. In other words, the rule-based policy exploits the belief overreaction and stimulates demand further without an actual increase in UI duration.

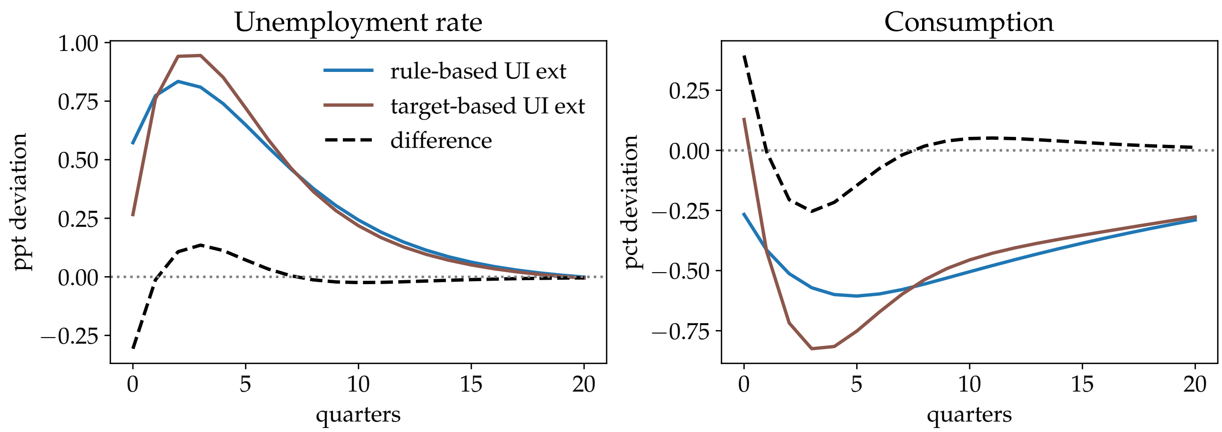
We conclude that in our baseline economy, changing the implementation of UI extensions to a direct announcement could help their stimulative power at the onset of the recession, but may lack efficacy past the peak of the recession when beliefs turn overly pessimistic.

## 2.8 Conclusion

Economists have long emphasized the benefits of linking UI benefits duration to aggregate economic conditions (see, e.g., Chodorow-Reich and Coglianese 2019; Eichenbaum 2019; Mitchell and Husak 2021). In this paper, we argue that expectations are critical in deter-

[14] This argument follows the same logic as in Bianchi-Vimercati et al. (2021).

Figure 2.13: Instrument-rule vs announcement in the baseline economy




mining the stabilization power of these policies.

We study the economic impact of UI extensions in a state-of-the-art Heterogeneous Agent New Keynesian model with search and matching frictions. We discuss a general framework to solve and analyze such models under arbitrary beliefs about macroeconomic outcomes. We leverage the framework to estimate the model to match the impulse responses of key aggregate variables and expectations to identified business-cycle shocks. By doing so, we demonstrate that expectations data can be used directly to solve the model, thus sidestepping the issue of choosing among the "wilderness" of alternative models for belief formation. Our results emphasize that the stimulative power of state-dependent UI extensions can be greatly affected by systematic forecast errors that people make in predicting the business cycle.

# Chapter 3

# Belief Disagreement and Business Cycles

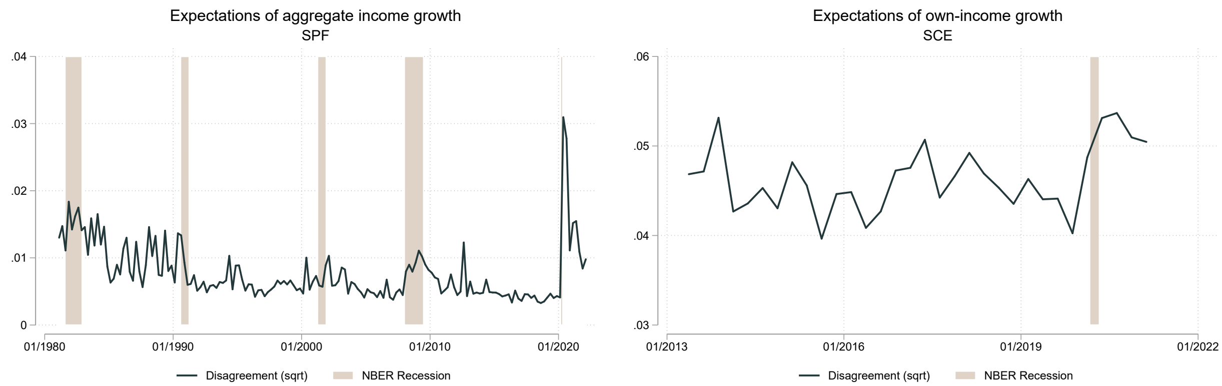
## 3.1 Introduction

Expectations about the future are central to macroeconomics. A variety of decisions made by households and firms fundamentally depend on what they expect for future income levels, interest rates, or inflation, e.g., consumption and savings, investment, or price setting. So, to understand how individuals make the decisions that shape aggregate outcomes, it is essential to understand how agents form beliefs. In this paper, I explore a particular facet of expectations: belief heterogeneity (or disagreement). Understanding the sources behind this form of heterogeneity is crucial to understand its aggregate implications.

Belief heterogeneity can be observed in surveys of expectations. In Figure 3.1, I plot the cross-sectional standard deviation of forecasts for one-year ahead income growth in two popular surveys of expectations: the *Survey of Professional Forecasters* (SPF) and the *Survey of Consumer Expectations* (SCE). Figure 3.1 shows two key findings: (1) the magnitude of disagreement is substantially high and (2) disagreement rises sharply during large events. The cross-sectional standard deviation is close to 1 percentage point in the SPF and 4.5 percentage points for the SCE.[1] Furthermore, there are also large fluctuations in disagreement

[1] Mankiw et al. (2003) emphasize that disagreement in inflation forecasts among households is much higher than for professional forecasters.

Figure 3.1: Belief disagreement in survey data




*Notes:* This figure displays the cross-sectional standard deviation of point forecasts for one-year ahead income growth both in the Survey of Professional Forecasters (SPF) in the left panel and the Survey of Consumer Expectations (SCE) in the right panel. See appendix A.3. for more details.

over time. For example, SPF disagreement rises to 3 percentage points during the Covid crisis.

How does belief heterogeneity affect aggregate outcomes? Despite having introduced multiple forms of heterogeneity into macroeconomics models, the literature has given relatively less attention to the impact of belief heterogeneity. In part, this is a consequence of the fact that the bedrock of modern macroeconomics is the full-information and rational expectations (FIRE) assumption, which implies that everyone shares the same beliefs and so eliminates any chance for disagreement. However, given the central importance of beliefs, understanding the sources and aggregate implications of this form of heterogeneity is crucial. In this paper, I study the impact of disagreement on aggregate demand, the transmission of macroeconomic shocks, and the efficacy of fiscal policy. I focus on aggregate demand because it has a central role in the macroeconomic transmission of shocks (see Remark 2 for a discussion).

The main findings in this paper are that (1) belief disagreement serves as a propagation mechanism which can substantially amplify business-cycle shocks and (2) the presence of belief disagreement implies that the sectoral composition of government spending affects the spending multiplier. I establish these results by considering a stylized New Keynesian model and empirical evidence. For quantification purposes, I then develop a Heterogeneous Agent New Keynesian (HANK) framework that embeds various forms of heterogeneity.

**Theory** I begin by considering a simple model with household heterogeneity and nominal rigidities. Households differ both in their beliefs and in their exposure to the business cycle due to heterogeneous income cyclicality. As is standard in the literature, I focus on the first-order response of this economy to shocks starting from a steady state. This model delivers two main insights.

First, I show that a single statistic summarizes the impact of belief heterogeneity on aggregate demand. This statistic, which I refer to as *correlated disagreement*, captures the extent to which belief heterogeneity is correlated with individual income cyclicality. Suppose that aggregate income ($y$) rises at time $t+h$. I show that the change in aggregate demand at time $t$, $c_t$, is given by:

$$\frac{\partial c_t}{\partial y_{t+h}} = \mathrm{MPC}_h \cdot (1+\mathrm{CD}) \cdot \overline{\lambda}. \tag{3.1}$$

$\mathrm{MPC}_h$ denotes the marginal propensity to consume out of this increase in future income, and $\overline{\lambda}$ captures the response of average beliefs of income to the actual change in income. This term is equal to one with full information and rational expectations, but generally differs from one away from that benchmark. CD is the correlated disagreement term, which is measured as the covariance between income cyclicality and the response of individual beliefs. Intuitively, this term arises from the fact that the beliefs of individuals more exposed to aggregate-income changes (higher cyclicality) receive a larger weight in determining the response of aggregate consumption. Belief disagreement is relevant to the extent that it correlates with other individual characteristics which determine individual consumption response to macroeconomic shocks.

Second, I show that the sign of correlated disagreement determines the extent to which shocks propagate through the economy. Equation (3.1) shows that correlated disagreement affects the strength of the impact of future income on current aggregate demand. When correlated disagreement is positive, this channel is stronger relative to an economy where all individuals have the same average belief (and so CD = 0). In this case, disagreement is a propagation mechanism that amplifies the initial shock. In other words, the aggregate demand response is higher than predicted by the simple average level of attention, so the shock's impact is larger. Conversely, when correlated disagreement is negative, the general-equilibrium (GE) channel is muted relative to the homogeneous-attention economy. In this case, disagreement dampens the consequences of the shock.

Whether business cycles are amplified or dampened crucially depends on the sign of correlated disagreement. It is ex ante unclear whether we should expect the responsiveness of beliefs to be positively or negatively related to income exposure. In this paper, I provide both theoretical and empirical evidence that the correlation is positive.

**Endogenous beliefs** First, I endogenize beliefs via a model of behavioral inattention in the tradition of Gabaix (2014, 2016). I show that this model unambiguously predicts positively correlated disagreement. Households with higher income cyclicality are more exposed to changes in aggregate conditions, i.e., a given change in aggregate income implies a larger individual-income response for high-cyclicality than for low-cyclicality workers. So, there is an incentive for workers with high income cyclicality to track shocks more closely, i.e., pay more attention. The result is a positive correlation between attention and income cyclicality.

**Evidence** Second, I provide empirical support for this positive correlation. I analyze the size of forecast errors as a function of income cyclicality. Forecast errors are the difference between realized income growth and expected income growth. Through the lens of the model, the magnitude of forecast errors is informative of how attentive people are. Using survey data from the *Survey of Consumer Expectations* (SCE) and data on actual outcomes from the *Current Population Survey* (CPS), I construct average forecast errors at the state-quarterly level from 2013 to 2021. I show that the magnitude of forecast errors is decreasing in the state's average income cyclicality. A 0.1 increase in average income elasticity at the state level is associated with a 16.3 percent decrease in the magnitude of forecast errors. This result supports the implication that attention is increasing in income cyclicality.

**Quantifying business-cycle amplification** Since correlated disagreement is positive; aggregate demand is more responsive to changes in macroeconomic conditions, and business cycles are amplified relative to a homogeneous-attention benchmark.

To assess the quantitative relevance of the mechanisms described in the simple model, I develop a Heterogeneous Agent New Keynesian model. This model aims to develop a more realistic description of aggregate demand, the crucial object of the analysis. The quantitative model extends the previous analysis along the following dimensions: (1) it introduces incomplete markets in the form of uninsurable idiosyncratic income risk and borrowing constraints; (2) income risk increases in recessions and decreases in expansions, as emphasized by the empirical literature,[2] (3) it allows for government spending, debt, and proportional taxation; (4) it assumes that monetary policy is conducted according to a standard Taylor rule; and (5) it allows for time-varying prices and nominal wages, subject to standard nominal frictions. I maintain the assumption that individuals are heterogeneously exposed to aggregate conditions and that they optimize their level of attention. In this more general model, individuals must form beliefs not only about their own income, but also about the real interest rate and the tax rate.

Models with this considerable degree heterogeneity are known for their computational

[2] For evidence on this fact, see Guvenen et al. (2017) and Coibion et al. (2017).

complexity. In this paper, non-rational heterogeneous beliefs create an additional dimension of complexity. I make the simplifying assumption that individuals choose their optimal attention in an ex-ante stage and not based on the current temporary state. Under this assumption, I can leverage recent advances in the literature to write the problem in a computationally-tractable way. The computational method is described in Section 3.4.6.

I calibrate the model to standard targets in the literature in addition to evidence on income cyclicality and forecast errors. I consider the response of aggregate output to a variety of standard business-cycle shocks: discount-factor, government-spending, monetary-policy, and productivity shocks. I show that amplification is more substantial when the shock is more persistent, or when the response of monetary policy to inflation is weaker. For example, calibrating productivity shocks to capture the effects of an oil shock implies that the response of aggregate income on impact is 16 percent larger because of correlated disagreement. The amount of amplification more than doubles, to 33 percent, if the response of monetary policy is weaker.

The intuition for these results is as follows. First, the more persistent the shock, the more relevant beliefs about future output are in shaping current actions. So, the correlated-disagreement mechanism becomes stronger if the shock is more long-lasting. Second, the weaker the monetary policy response, the higher the relative importance of general-equilibrium effects working from aggregate income to aggregate demand. If monetary policy is less responsive, the amplification mechanism becomes relatively more important, leading to further amplification.

**Government-spending multipliers** I then consider the impact of correlated disagreement on the effects of government-spending policy. In particular, I ask how the composition of government spending across worker cyclicality groups affects the government-spending multiplier, given the heterogeneity in attention.

When all individuals have the same attention or under full-information and rational expectations, the government-spending multiplier is independent of the group-composition of spending in this model.[3] In contrast, when attention is heterogeneous, the multiplier is higher if government spending targets the most cyclical groups of workers. The reason for this result is as follows. The first-round effect of spending increases workers' incomes. In response, workers that see their incomes increasing also choose to increase their consumption. The relevant statistics to determine the aggregate-demand response to this change in income are the *effective marginal propensities to consume*, i.e., the MPC weighted by the attention to income changes. It follows that, when spending targets the most cyclical groups, the government increases incomes for people with higher effective MPCs, because they are also the most attentive. The result is a larger first-round effect of government spending, which increases the government-spending multiplier.

Using the simple model, I show that if the government targets the most-cyclical workers, the multiplier can be larger than the FIRE multiplier, even if people are fully attentive to taxes.[4] Using the calibrated model, I quantify the consequences of targeting for the size of the government-spending multiplier. I show that the multiplier can depend substantially on the composition of spending. The multiplier is less than one when the government targets the least cyclical workers but rises above 1.2 if the government targets the most cyclical workers.

**Literature.** This paper belongs to a large literature analyzing the transmission of shocks and policies without the FIRE assumption. A large section of the literature focuses on

[3]To focus on the implications of heterogeneous attention, I assume that the distribution of MPCs is orthogonal to the workers' income cyclicality. So, under FIRE, targeting becomes irrelevant. Baqaee and Farhi (2018) and Flynn et al. (2021) study the implications of MPC heterogeneity for the design of fiscal policy.

[4]The literature has generally found that government-spending multipliers decrease when deviating from FIRE, see Angeletos and Lian (2018), Woodford and Xie (2019), Farhi et al. (2020), and Bianchi-Vimercati et al. (2021).

informational frictions and shows how this deviation from FIRE affects the response of the economy to shocks, see, e.g, Woodford (2001), Mankiw and Reis (2002), Lorenzoni (2009), Angeletos and La'O (2010, 2013), Nimark (2014), or Angeletos and Lian (2018). Another strand of the literature focuses instead on bounded rationality. Ilut and Schneider (2014) analyzes the implications of ambiguity aversion, and Bianchi et al. (2021) analyzes the impact of diagnostic expectations for the transmission of business cycles. Woodford (2018) and Woodford and Xie (2019, 2022) analyze fiscal and monetary policy when people have finite planning horizons. García-Schmidt and Woodford (2019), Iovino and Sergeyev (2018) and Bianchi-Vimercati et al. (2021) evaluate the effectiveness of monetary and fiscal policies in models in which people have level-$k$ thinking. Gabaix (2020) shows how to modify the standard New Keynesian model to account for expectational frictions in the form of cognitive discounting, while Angeletos and Sastry (2021) develop a model of shallow reasoning to analyze the relative performance of different forms of policy communication. However, these papers considered models where agents are ex-ante identical, implying that belief dispersion does not have first-order consequences to macroeconomic aggregates.

A more recent literature analyzes that question in models which allow for agent heterogeneity. Angeletos and Huo (2021) and Auclert et al. (2020) study environments with heterogeneous agents and incomplete and dispersed information, while Farhi and Werning (2019) and Farhi et al. (2020) analyze monetary and fiscal policies, respectively, with household heterogeneity and level-$k$ thinking. Pappa et al. (2023) study a HANK model with search and match frictions and incomplete information. Bardóczy and Guerreiro (2023) studies the stabilization effects of unemployment insurance in a general model with non-FIRE beliefs. However, all of these papers assume that the belief response is orthogonal to other sources of heterogeneity, which implies that correlated disagreement is always zero.

This paper shares the focus on the correlation between expectations and other individual characteristics with Broer et al. (2021). They document systematic heterogeneity in expectations across the income distribution about the macroeconomy and rationalize these findings with a theory of information in which individuals choose their optimal level of attention at every point in time. They argue that this leads to higher macroeconomic volatility. Instead, I focus on systematic heterogeneity in expectations across individuals with different income cyclicalities and assume that attention is a permanent characteristic of households. I derive in closed form the implications of heterogeneous beliefs in a simple model and calibrate a HANK model to study the impact of this form of heterogeneity for business cycles and fiscal stabilization policy.

There is a large literature on the empirical determinants of expectations using survey data, see, e.g., Coibion and Gorodnichenko (2012, 2015), Bordalo et al. (2020), or Angeletos et al. (2021). Relative to this literature, this paper provides empirical evidence on a determinant of heterogeneous attention to the macroeconomy and its implications for correlated disagreement. For further evidence on belief disagreement, see, e.g., Zarnowitz and Lambros (1987) and Mankiw et al. (2003), or more recently, Bordalo et al. (2020), Angeletos et al. (2021), and D'Acunto et al. (2019).

This paper shares the interest in targeted spending multipliers and the effects of the composition of government spending with a growing literature. Ramey and Shapiro (1998) focus on the implications of costly capital reallocation across sectors. Cox et al. (2020) analyze how the composition of spending affects the spending multiplier due to heterogeneous degrees of nominal rigidities in a multi-sector model. Baqaee (2015) and Bouakez et al. (2020) study how the production network affects the impact of government spending. Baqaee and Farhi (2018) and Flynn et al. (2021) focus instead on the role of MPC heterogeneity. In this paper, I contribute another perspective regarding heterogeneous attention and show how this implies that the composition of spending affects the associated multiplier.

Finally, there has been a long tradition of studying the implications of belief heterogeneity about asset returns in asset pricing and macrofinance following the seminal contributions of Harrison and Kreps (1978) and Scheinkman and Xiong (2003). Most closely related, Caballero and Simsek (2020) analyze the implications of disagreement about financial market returns for aggregate demand. Hassan and Mertens (2017) study how small correlated forecast errors across traders can be amplified by financial trading and distort real investment. They also show that their mechanism can deliver time-varying belief disagreement in a way that is consistent with empirical evidence. Instead, I focus on beliefs about future income and on how heterogeneity in those beliefs affects aggregate demand. In this paper, I abstract from the interaction between belief heterogeneity and the valuation of financial assets.

**Outline.** The paper is organized as follows. In Section 3.2, I develop the simple model. In Section 3.3, I present the empirical evidence. In Section 3.4, I introduce the quantitative model. In Section 3.5, I develop the implications for the spending multiplier. Finally, Section 3.6 concludes. The proofs for all propositions are contained in the appendix.

## 3.2 Simple model

In this section, I describe the simple model. This model allows me to characterize the main mechanisms transparently and in closed form. In Section 3.4, I generalize this framework to a more complete HANK framework and use that model for quantification purposes. Throughout the paper, I restrict attention to the first-order response of the economy to shocks, starting from the flexible price steady state.

Consider a simple New Keynesian economy in discrete time $t = 0, 1, ...$ For tractability, I assume that nominal wages are fully rigid.

This economy is populated by a continuum of households $i \in [0, 1]$. Households have preferences over consumption, $C_{i,t}$, and labor $N_{i,t}$, are given by

$$\mathcal{U}_i = \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t-1} \beta_{i,s} \right) \left[ u(C_{i,t}) - v(N_{i,t}) \right], \tag{3.2}$$

where $u(C) = C^{1-\sigma^{-1}}/(1-\sigma^{-1})$ and $v(N) = N^{1+\psi^{-1}}/(1+\psi^{-1})$, $C_{i,t}$ and $N_{i,t}$ denote individual consumption and labor hours, respectively, and $\sigma$ and $\psi$ are the elasticity of intertemporal substitution and the Frisch elasticity, respectively. I assume that the steady-state discount factor $\beta \in (0,1)$ is perturbed by discount-factor shocks and $\beta_{i,t}$ captures the effective subjective discount factor between periods $t$ and $t+1$. These demand shocks are the only disturbance present in this simple model. Furthermore, for simplicity, I also assume that the path of $\{\beta_{i,t}\}$ is realized at time zero, so there is no aggregate uncertainty in this economy from $t = 0$ on.

For simplicity, I assume that the production function is linear in labor $Y_t = N_t$, where $Y_t$ and $N_t$ denote aggregate income and labor, respectively. The goods market clearing condition is given by:

$$Y_t = \int_0^1 C_{i,t} di, \tag{3.3}$$

and labor market clearing requires $N_t = \int_0^1 N_{i,t} di$.

I assume that the economy is initially at a steady state and normalize the steady state output to one, $Y = N = 1$.

### 3.2.1 Firms

Firms are perfectly competitive and maximize profits, given by: $P_t Y_t - W_t N_t$. An interior solution to a firm's problem requires $W_t = P_t$, where $P_t$ denotes the final good's price. Note that these assumptions also imply that, in equilibrium, there are no profits and that the real wage is given by:

$$w_t = \frac{W_t}{P_t} = 1. \tag{3.4}$$

Because wages are fully rigid, there is no price inflation:

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} = 1. \tag{3.5}$$

### 3.2.2 Monetary policy

I assume that monetary policy directly controls the nominal interest rate $i_t$. Furthermore, to keep the analysis simple, I assume that the monetary authority keeps the nominal interest rate constant and equal to the steady-state interest rate:

$$i_t = r = \beta^{-1} - 1. \tag{3.6}$$

**Remark 1.** *It is well known that there may be multiple equilibria when monetary policy pegs the nominal interest rate. In this simple model, I maintain this interest-rate peg assumption and sidestep discussions regarding indeterminacy by assuming that the economy converges back to its flexible price steady state. In the model of section 3.4, I assume that monetary policy is given by a standard Taylor rule.*

### 3.2.3 Households

**Labor income** Each household belongs to a group $g = 1, ..., n$. The total mass of group $g$ is given by $\pi_g$, where $\sum_g \pi_g = 1$. If household $i$ belongs to group $g$, then their labor income is given by:

$$Y_{g,t} = w_t N_{g,t}, \tag{3.7}$$

where group labor supply, $N_{g,t}$, satisfies $\sum_g \pi_g N_{g,t} = N_t$.

Because nominal wages are fully rigid, labor supply is determined by firm demand, i.e., the amount of labor supplied needs to meet firms' labor demand at this nominal wage. To model heterogeneous income cyclicality, I assume that changes in aggregate demand have a different incidence across groups. Formally, I assume that:

$$N_{g,t} = \Gamma_g(N_t). \tag{3.8}$$

I assume that in a steady state, all people work the same number of hours $\Gamma_g(N) = N$. Furthermore, the income elasticity of group $g$ is defined as:

$$\gamma_g \equiv \Gamma_g'(N) \geq 0, \tag{3.9}$$

which must satisfy $\sum_g \pi_g \gamma_g = 1$. If we think of $g$ as different sectors, then this assumption generates sectoral heterogeneity in the incidence of business cycles, sidestepping the exact microfoundations necessary to achieve this result.[5]

The household's time-$t$ flow of funds constraint is given by

$$C_{i,t} + A_{i,t+1} = Y_{g,t} + (1+r)A_{i,t}, \tag{3.10}$$

where $A_{i,t}$ denotes savings from period $t-1$ brought to period $t$. They are also subject to a standard no-Ponzi games condition $\lim_{T\to\infty} \frac{A_{i,T}}{(1+r)^T} \geq 0$.

**Beliefs** At every point in time, the household must anticipate the behavior of future variables which are relevant to their problem. In particular, the household must anticipate the discount-factor shock, labor income, and the real interest rate. I maintain the assumption that people have the correct expectations regarding their preference shock $\beta_{i,t}$. Furthermore,

[5] A way of achieving this result would require developing a multi-sector model in which workers had sector-specific skills and non-homothetic preferences.

since real interest rates remain constant at their steady-state value, I assume that people correctly anticipate that $r_t = r$. So, the household's forecasting problem is simply one of forecasting their own future income.

For now, I let beliefs about future income be arbitrary and write $E_{i,t}[\cdot]$ as the expectation operator given household $i$'s beliefs. In line with the literature, I assume that people know their current level of income. This assumption implies that markets clear in the current period given households' present consumption and savings decisions, and that basic macroeconomic identities hold. In what follows, I begin by describing household behavior and equilibrium properties for these beliefs. In subsection 3.2.5, I endogenize beliefs and show what it implies for disagreement and aggregate dynamics.

**Consumption and savings decisions** The household's utility maximization is standard; they choose consumption and savings to maximize expected utility $E_{i,t}[\mathcal{U}_i]$ subject to (3.10).

In Appendix C.1.1, I show that to first order the individual demand can be written as:

$$c_{i,t} = (1-\beta)\left\{\sum_{h=0}^{\infty}\beta^h E_{i,t}[y_{g,t+h}] + \beta^{-1}a_{i,t}\right\} + \sigma\sum_{h=0}^{\infty}\beta^{h+1}r^n_{i,t+h}, \tag{3.11}$$

where lower-case letters $c_t$, $y_{g,t}$, and $a_{i,t}$ denote the deviation of consumption, income, and assets from their steady-state values, respectively, and $r^n_{i,t} \equiv -d\log(\beta_{i,t})$ is the discount-factor shock. The interpretation of this equation follows from standard Permanent Income Hypothesis logic: $(1-\beta)$ denotes the household's marginal propensity to consume and $\sum_{h=0}^{\infty}\beta^h E_{i,t}[y_{g,t+h}]$ denotes the household's expected permanent income. The term $\beta^{-1}a_{i,t}$ denotes the household's financial wealth, while the final term $\sum_{h=0}^{\infty}\beta^{h+1}r^n_{i,t+h}$ captures the demand-shift induced by the discount-factor shock, which is multiplied by the intertemporal elasticity of substitution $\sigma$. Finally, note that $y_{g,t} = \gamma_g y_t$ where $y_t$ denotes the deviation of aggregate output from steady state. I assume that workers understand that $y_{g,t} = \gamma_g y_t$.

### 3.2.4 Aggregation and equilibrium

Because this economy features constant wages and prices, equilibrium output is fully demand determined, i.e., output and employment adjust so as to clear the goods market $n_t = y_t = c_t$ where $c_t$ denotes aggregate demand:

$$c_t \equiv \int_0^1 c_{i,t} di,$$

and where $c_{i,t}$ is given by (3.11). So, the crucial object is aggregate demand.

In characterizing aggregate demand, it also becomes evident how belief heterogeneity matters for aggregates. In what follows, I emphasize two results: (1) within-group belief heterogeneity is irrelevant, and (2) correlated disagreement determines the strength of general equilibrium forces working through aggregate income. To show this, I proceed in steps: first, I compute average consumption in group $g$:

$$\bar{c}_{g,t} \equiv \int_{i \in g} \frac{1}{\pi_g} c_{i,t} di,$$

and then aggregate across groups $c_t = \sum_g \pi_g \bar{c}_{g,t}$.

Aggregating individual demand (3.11) for all members of group $g$, implies that:

$$\bar{c}_{g,t} = (1-\beta) \left\{ \sum_{h=0}^{\infty} \beta^h \overline{E}_{g,t}[y_{g,t+h}] + \beta^{-1} \bar{a}_{g,t} \right\} + \sigma \sum_{h=0}^{\infty} \beta^{h+1} \bar{r}^n_{g,t+h}. \tag{3.12}$$

This is exactly the same expression as (3.11), except that individual beliefs, assets, and discount-factor shock have been replaced by group average belief, $\overline{E}_{g,t}[y_{g,t+h}]$, assets, $\bar{a}_{g,t}$, and demand shock, $\bar{r}^n_{g,t+h}$, respectively. Only the average belief of group $g$ affects the group's average consumption. In other words, belief heterogeneity within the groups is irrelevant in the sense that the dispersion of beliefs around the average belief does not affect aggregate demand. The intuition for this result is as follows. More optimistic people perceive a higher permanent income than pessimistic people do and so choose to consume more today. However, because all households have the same MPC, the higher demand of relatively optimistic people exactly offsets the lower demand of relatively pessimistic ones. So, this type of belief dispersion does not affect average group consumption.

Next, we want to use average group demand to find aggregate demand. First, note that by asset market clearing:

$$\int_0^1 a_{i,t} di = \sum_g \pi_g \overline{a}_{g,t} = 0.$$

Furthermore, aggregating expectations of future income implies that:

$$\sum_g \pi_g \cdot \overline{E}_{g,t}[\underbrace{y_{g,t+h}}_{=\gamma_g y_{t+h}}] = \sum_g \pi_g \cdot \gamma_g \cdot \overline{E}_{g,t}[y_{t+h}] = (1 + \mathrm{CD}_{t,h}) \cdot \overline{E}_t[y_{t+h}], \tag{3.13}$$

where $\overline{E}_t[y_{t+h}]$ denotes the economy-wide average belief, and

$$\mathrm{CD}_{t,h} \equiv \mathbb{C}\mathrm{ov}\left(\gamma_g, \overline{E}_{g,t}[y_{t+h}]/\overline{E}_t[y_{t+h}]\right)$$

denotes the covariance between income cyclicality $\gamma_g$ and "normalized" average beliefs. I call this term *correlated disagreement*. It captures the extent to which belief disagreement is correlated with individual income cyclicality. It follows that aggregate demand is given by:

$$c_t = (1-\beta) \sum_{h=0}^{\infty} \beta^h \cdot (1 + \mathrm{CD}_{t,h}) \cdot \overline{E}_t[y_{t+h}] + \sigma \sum_{h=0}^{\infty} \beta^{h+1} r^n_{t+h}, \tag{3.14}$$

and $r^n_t \equiv \int_0^1 r^n_{i,t} di$ denotes the average discount-factor shock. Aggregate demand is not solely a function of average beliefs and the demand shock. Correlated disagreement determines the aggregate response of consumption to changes in income, i.e., it affects the strength of the general-equilibrium channel. The reason for this result is intuitive. Because agents are heterogeneously exposed to the cycle, not all beliefs matter equally. The beliefs of more cyclical individuals are more relevant for aggregate consumption than those of less cyclical individuals, which follows from the fact that, given a change in aggregate income, more cyclical workers have higher changes to individual income and adjust their individual consumption by more than less cyclical workers. Correlated disagreement captures exactly the term that corrects for the fact that beliefs of more cyclical workers receive a higher weight in determining aggregate demand. For instance, the weight on the beliefs of a worker with zero income cyclicality would be zero because this worker does not adjust their individual consumption in response to changes in their beliefs about aggregate income.

Equating aggregate demand to aggregate output, we find

$$y_t = (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\cdot(1+\mathrm{CD}_{t,h})\cdot\overline{E}_t[y_{t+h}] + \sigma\sum_{h=0}^{\infty}\beta^h r^n_{t+h}, \tag{3.15}$$

which solves for output at time $t$, given beliefs future output and the discount-factor shock.

Note that under full information and rational expectations, then $\overline{E}_{i,t}[y_{t+h}] = \mathbb{E}_t[y_{t+h}] = y_{t+h}$. This fact also implies that $\mathrm{CD}_{t,h} = 0$. So, under FIRE, this model collapses to an as-if representative agent model where heterogeneous income cyclicality is irrelevant. More generally, as long as beliefs are homogeneous, we find that $\mathrm{CD}_{t,h} = 0$. So the economy behaves as an as-if representative agent model with average beliefs, and heterogeneous income cyclicality would be irrelevant. Instead, suppose that beliefs are heterogeneous, but all workers have the same income cyclicality. It follows that $\mathrm{CD}_{t,h} = 0$. So, the model collapses to an as-if representative agent model with non-FIRE beliefs, where belief heterogeneity is irrelevant conditional on average beliefs. This logic shows that only the combination (and correlation) of heterogeneous income cyclicality and heterogeneous beliefs affects aggregate output. One form of heterogeneity without the other does not affect aggregate quantities.

To further analyze the effects of correlated disagreement, it is useful to make a further structural assumption on beliefs. I assume that average beliefs of group $g$ move proportionally with the rational expectations belief:

$$\overline{E}_{g,t}[y_{t+h}] = \lambda_{g,t}\mathbb{E}_t[y_{t+h}]. \tag{3.16}$$

This assumption can be satisfied by several widely used models expectations which deviate from FIRE. For example, this assumption nests: (1) incomplete and dispersed information following the tradition of Lucas (1972), (2) rational inattention in the tradition of Sims (2003) in which individuals obtain signals about their permanent income, (3) sticky information as in Mankiw and Reis (2002), (4) behavioral inattention or sparsity as in Gabaix (2014, 2016), or (5) shallow reasoning as in Angeletos and Sastry (2021).[6] I refer to $\lambda_{g,t}$ as the "attention" of individuals in group $g$, with the understanding that in different models of beliefs it may be a consequence of different microfoundations.

For simplicity, I further assume that attention is constant over time $\lambda_{g,t} = \lambda_g$. The average level of attention is given by $\overline{\lambda} = \sum_g \pi_g \lambda_g$. This object has been the focus of study in the empirical and theoretical literature, where the consensus is that $\overline{\lambda} < 1$. In this paper, I focus instead on the correlated disagreement term. Under these assumptions, the correlated disagreement term is also constant over time:

$$\text{CD}_{t,h} = \mathbb{C}\text{ov}\left(\gamma_g, \lambda_g/\overline{\lambda}\right) \equiv \text{CD}. \tag{3.17}$$

[6]The assumption outlined in equation 3.16 implicitly assumes that attention to output changes at different dates does not depend on the forecasting horizon, $h$. This assumption is always satisfied if people obtain information that helps them determine their permanent income. As it turns out, this assumption is not consequential for the analysis, and it is possible to allow the attention parameter to also vary with the horizon. I elaborate on this more complex model in Appendix C.1.5.

**Proposition 9.** *Suppose that beliefs satisfy* (3.16) *with* $\lambda_{g,t} = \lambda_g$*, then*

$$y_t = (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\cdot(1+CD)\cdot\overline{\lambda}y_{t+h} + \sigma\sum_{h=0}^{\infty}\beta^h r^n_{t+h}. \tag{3.18}$$

1. *If* $CD = 0$*, then the economy behaves as if it was populated by a representative agent with the average level of attention* $\overline{\lambda}$*. Correlated disagreement is zero if one of the following conditions hold: (1) attention is constant* $\lambda_g = \overline{\lambda}$ *for all* $g$*, (2) income cyclicality is constant* $\gamma_g = 1$ *for all* $g$*, or (3) attention and cyclicality are orthogonal.*

2. *If* $CD > 0$*, then the effects of changes in future output are amplified with respect to an economy with homogeneous attention, i.e., it is as if the* MPC *was higher. Correlated disagreement is positive if* $\lambda_g$ *is increasing in* $\gamma_g$*.*

3. *If* $CD < 0$*, then the effects of changes in future output are dampened with respect to an economy with homogeneous attention, i.e., it is as if the* MPC *was lower. Correlated disagreement is negative if* $\lambda_g$ *is decreasing in* $\gamma_g$*.*

Proposition 1 summarizes the effects of correlated disagreement. If correlated disagreement is zero, the economy is equivalent to a representative-agent (RA) economy with the average level of attention $\overline{\lambda}$.[7] Interestingly, this situation occurs whenever there is no heterogeneity in income cyclicality or there is no heterogeneity in attention. This fact also demonstrates that each type of heterogeneity in isolation would be irrelevant to first-order output dynamics.

Instead, when these two forms of heterogeneity are present, there can be departures from the RA benchmark. If attention increases with income cyclicality, then the individuals most affected by business cycles have higher levels of attention, which implies that their beliefs

[7] I call this case the RA benchmark, with the understanding that under some models of beliefs, it would require belief heterogeneity (e.g., incomplete and dispersed information or sticky expectations). However, this type of belief heterogeneity would not be consequential for aggregate outcomes.

move by more. This correlation then results in an amplification of general-equilibrium forces when compared to the RA benchmark. If attention decreases with income cyclicality, then the individuals most affected by business cycles have lower levels of attention, implying that their beliefs move by less. Compared to the RA benchmark, this correlation results in a dampening of general-equilibrium forces.

We can also think about correlated disagreement as affecting the effective marginal propensity to consume out of income changes at the aggregate level. In the RA economy, the MPC out of changes to aggregate income would be $(1-\beta)\overline{\lambda}_t$, which is the micro-level MPC multiplied by the average attention to aggregate income changes. Instead, when correlated disagreement is present, it is as if the MPC was $(1-\beta)(1+\text{CD})\overline{\lambda}$. This term is higher if more cyclical households are more attentive and it is lower if more cyclical households are less attentive.[8]

**Remark 2.** *In this paper, I focus on the implications of correlated disagreement for aggregate demand. However, belief disagreement naturally affects multiple dimensions of individual decision-making and market interaction, shaping aggregate outcomes. I focus solely on the consequences for aggregate demand to cleanly characterize this particular channel and because aggregate demand is known to play a crucial role in the macroeconomic transmission of business-cycle shocks. Furthermore, stimulating aggregate demand is also the central focus of various monetary and fiscal policies.*

**Propagation mechanism** We can use the previous results to think about how correlated disagreement affects the propagation of shocks. The main finding is that correlated disagreement can amplify shocks if it is positive or dampen shocks if it is negative.

[8]The formal logic behind these results is similar to the relationship between static MPC heterogeneity and the incidence of a change in output developed in Patterson (2019). However, note that the mechanisms presented in this paper result from heterogeneous attention rather than MPC heterogeneity. Furthermore, these mechanisms are related to attention to the economy's future performance, i.e., it is an inherently dynamic problem.

Suppose that the aggregate shock is such that $r_t^n = \rho^t r_0^n$, where $\rho \in [0, 1]$ denotes the persistence of the shock and $r_0^n < 0$ denotes the initial impulse. I am assuming the shock is negative, which implies that the economy is entering a recession. The logic for an expansion would be symmetrical. Using equation (3.18), we can show that the equilibrium is

$$y_t = \frac{\rho^t \sigma r_0^n}{1 - \rho \left\{ \beta + (1 - \beta) \cdot (1 + \text{CD}) \cdot \overline{\lambda} \right\}}, \tag{3.19}$$

Note that replacing $\overline{\lambda} = 1$ and $\text{CD} = 0$ this expression obtains $y_t^{\text{FIRE}} = \rho^t \sigma r_0^n / (1 - \rho)$, i.e., the full-information and rational expectations equilibrium. If, instead, we replace $\text{CD} = 0$ but maintain $\overline{\lambda} < 1$ we obtain the homogeneous-attention equilibrium:

$$y_t^{\text{RA}} = \frac{\rho^t \sigma r_0^n}{1 - \rho \left\{ \beta + (1 - \beta) \cdot \overline{\lambda} \right\}}.$$

As it is well known, inattention $\left( \overline{\lambda} < 1 \right)$ dampens general-equilibrium forces compared to the FIRE benchmark. This fact also implies that the response of the economy is muted versus that benchmark, i.e., the recession is less severe $y_t^{\text{RA}} > y_t^{\text{FIRE}}$.

However, in this paper, I am interested in comparing the response of the economy with disagreement versus the economy without disagreement keeping the average level of attention constant, i.e., I am interested in comparing $y_t$ and $y_t^{\text{RA}}$. If $\text{CD} > 0$, the economy's response is larger than in the homogeneous attention economy. Instead, if $\text{CD} < 0$, the response is smaller than in the homogeneous attention economy. The intuition for this result is as follows. When correlated disagreement is positive, then attention and incidence are positively correlated, i.e., the individuals who are more cyclical and thus more responsive are also more attentive. In this case, as shown in the previous section, the general-equilibrium effects are amplified, leading to larger cuts in aggregate consumption and, thus, a larger recession. When correlated disagreement is negative, attention and incidence are negatively correlated, and the logic is exactly reversed.

To evaluate the amount of amplification generated by correlated disagreement, I define amplification as the proportional response relative to the RA benchmark:

$$\mathcal{A}_t \equiv \frac{y_t - y_t^{\mathrm{RA}}}{y_t^{\mathrm{RA}}}. \tag{3.20}$$

Proposition 10 summarizes the main results regarding the amount of amplification generated by correlated disagreement.

**Proposition 10.** *Suppose that $r_t^n = \rho^t r_0^n$, then amplification, as defined in equation (3.20), is constant over time and given by*

$$\mathcal{A}_t = \frac{(1-\beta)\,\rho \cdot CD \cdot \overline{\lambda}}{1-\rho\left\{\beta + (1-\beta)\,(1+CD)\cdot \overline{\lambda}\right\}}. \tag{3.21}$$

*Furthermore,*

1. *Amplification is increasing in correlated disagreement, $d\mathcal{A}_t/dCD > 0$.*
2. *Amplification is increasing with persistence, $\rho$, if and only if correlated disagreement is positive, $sign\,(d\mathcal{A}_t/d\rho) = sign\,(CD)$.*
3. *Amplification is decreasing with the discount factor if and only if correlated disagreement is positive, $sign\,(d\mathcal{A}_t/d\beta) = -sign\,(CD)$.*

Proposition 10 shows that when correlated disagreement is positive, the response of output in the economy is amplified by the presence of disagreement. Instead, when correlated disagreement is negative, the response of output in the economy is dampened by correlated disagreement. Furthermore, I also show that the higher is correlated disagreement, the more amplification is generated, which follows from the logic described above. Finally, I provide two additional results.

First, I show that the effect of shock persistence, $\rho$, depends on the sign of correlated disagreement. If correlated disagreement is positive, higher shock persistence leads to further amplification. Instead, if correlated disagreement is negative, higher persistence leads to less amplification. This result follows from the fact that the higher the shock's persistence, the stronger the effect of expectations about the future in determining present consumption. As a result, the correlated disagreement propagation mechanism is stronger the more persistent shocks are.

Second, I show that the effect of a higher marginal propensity to consume, i.e., a lower discount factor, also depends on the sign of correlated disagreement. A higher marginal propensity to consume, if correlated disagreement is positive, leads to larger amplification. Instead, if correlated disagreement is negative, higher MPCs lead to less amplification. A higher marginal propensity to consume makes the general-equilibrium effects through which correlated disagreement operates more relevant. It follows that, with a higher MPC, the impact of correlated disagreement is more substantial. Thus, if CD is positive, it leads to more significant amplification; if CD is negative, it leads to less amplification.

This discussion emphasizes the central role of the sign of correlated disagreement in determining the amount of amplification in this economy. If correlated disagreement is positive, then it amplifies business cycles. Instead, if correlated disagreement is negative, it dampens business cycles. But what sign should we expect for correlated disagreement? It is ex ante unclear whether the correlation between beliefs and business-cycle exposure should be positive or negative. In what follows, I provide both theoretical and empirical evidence on the sign of this correlation. First, I show that, with endogenous attention, correlated disagreement is unambiguously positive. Second, I provide empirical evidence in favor of this theoretical prediction in Section 3.3.

### 3.2.5 Endogenous attention

I endogeneize beliefs by modelling attention following the sparsity-based bounded rationality model introduced by Gabaix (2014) and further extended to dynamic programming problems by Gabaix (2016).[9] I assume that

$$E_{i,t}\big[y_{g,t+h}\big] = \lambda_i \mathbb{E}_t[y_{g,t+h}], \tag{3.22}$$

where $\lambda_i \in [0,1]$ is the attention to permanent income. This equation is similar to equation (3.16) but cast in terms of individual beliefs. When $\lambda_i = 0$, this person "does not pay attention", and so their belief is just equal to their default belief. When $\lambda_i = 1$, this person "pays full attention" and has FIRE beliefs. When $\lambda_i \in (0,1)$, they have a partial perception of the true value of income.

Attention and consumption-savings decisions are made as follows. In the first ex-ante stage, individuals choose their optimal level of attention $\lambda_i$ to minimize expected inattention costs due to misoptimized consumption-savings choices, plus a cognitive cost of attention $\kappa\lambda_i$, where $\kappa > 0$. Assuming that the cost of attention is linear simplifies the exposition but is not central and can be easily generalized. In the second stage, shocks realize, and beliefs are determined by equation (3.22). Given these beliefs, individuals make their consumption and savings choices at each date, which implies that consumption is given by equation (3.11) replacing beliefs.

**Remark 3.** *A strict interpretation of this behavioral inattention model requires the assumption that individuals are boundedly rational in the second stage. Otherwise, they would be able to infer the correct beliefs using their knowledge of $\lambda_i$ and $E_{i,t}[y_{t+h}]$. In the language of Gabaix (2019), this model features deterministic attention and action. Instead, a large litera-*

[9] A useful review of models of behavioral inattention can be found in Gabaix (2019).

*ture considers models where agents optimally choose to receive noisy signals, as in models of rational inattention in the tradition of Sims (2003). In this tradition, individuals need not be boundedly rational in the second stage. However, this type of orthogonal noise in forecasts is not consequential for the aggregate implications of the model in this paper. So, for simplicity of exposition, I eliminate the noise. A further consequence of this assumption is that agents do not have any uncertainty regarding their point forecast. Because I restrict attention to the first-order effects of shocks, this implication is also not consequential for the aggregate dynamics in this economy.*

As in Gabaix (2016), I assume that people choose their optimal level of attention to minimize the second-order losses from inattention relative to the full-information level of utility. In Appendix C.1.4, I show that, to second order, the costs of inattention are given by:

$$C_g(\lambda_i) \equiv -\frac{1}{2}\frac{\partial^2 v}{\partial c^2} \cdot \sum_{h=1}^{\infty}\sum_{\tilde{h}=1}^{\infty} \frac{\partial c}{\partial Y_h}\frac{\partial c}{\partial Y_{\tilde{h}}} \cdot (1-\lambda_i)^2 \cdot \gamma_g^2 \sigma_{h,\tilde{h}}, \tag{3.23}$$

where $\partial^2 v/\partial c^2 = \beta^{-1}u''(1)$ is the utility cost of consumption misoptimization, $\partial c/\partial Y_h = (1-\beta)\beta^h$ denotes the response of consumption today to an increase in income in $h$ periods, i.e., the individual intertemporal marginal propensity to consume, and $\sigma_{h,\tilde{h}}$ denotes the perceived covariance between aggregate output at horizons $h$ and $\tilde{h}$.

Intuitively, this formula can be read as follows: $\gamma_k^2\sigma_{h,\tilde{h}}$ denotes the expected magnitude of the changes to income, $(1-\lambda_i)$ captures the extent of inattention and the MPC captures how these expectational mistakes translate into consumption decisions. Finally, $\frac{1}{2}\frac{\partial^2 \tilde{v}}{\partial c^2}$ captures how these forecast errors matter for individual utility losses.

I also assume that attention creates a psychological cost. For simplicity, I assume that this cost is linear $\kappa\lambda_i$, for $\kappa > 0$. The optimal attention then solves

$$\min_{\lambda_i \in [0,1]} C_g(\lambda_i) + \kappa\lambda_i. \tag{3.24}$$

**Proposition 11.** *Optimal attention is given by*

$$\lambda_i = \lambda_g \equiv \max\left\{0, 1 - \frac{\kappa}{\Lambda\gamma_g^2}\right\}, \tag{3.25}$$

*where* $\Lambda \equiv -\frac{1}{2}\frac{\partial^2 v}{\partial c^2}\sum_{h=1}^{\infty}\sum_{\tilde{h}=1}^{\infty}\frac{\partial c}{\partial Y_h}\frac{\partial c}{\partial Y_{\tilde{h}}}\sigma_{h,\tilde{h}}$. *This expression shows that* $\lambda_g$ *is increasing in* $\gamma_g$.

Proposition 11 shows that optimal attention is increasing in income cyclicality. This result is intuitive: since more cyclical people have more volatile incomes, they would suffer more from not paying as much attention to changes in economic conditions. As a result, they optimally choose to pay a higher psychological cost to pay more attention. With linear costs, it may be true that individuals with very low income cyclicality decide not to pay any attention.

Note that, under the assumptions in Proposition 11, we find that the correlated disagreement term is constant over time and always positive:

$$\text{CD} \equiv \mathbb{C}\text{ov}\left(\gamma_g, \lambda_g/\overline{\lambda}\right) > 0.$$

**Corollary 3.** *If attention is endogenous and given by* (3.25)*, then correlated disagreement is positive, CD* $> 0$. *This fact implies that the effects of changes in future output are amplified with respect to an economy with homogeneous attention.*

Endogeneizing attention implies that general-equilibrium effects are amplified relative to the RA benchmark with homogeneous attention. As a result, considering the determinants of heterogeneous attention implies that discount-factor shocks are amplified. In other words, disagreement behaves as a propagation mechanism amplifying the effects of shocks on the economy.

The lesson behind this model is more general than the simple framework considered here. First, the lesson that attention increases in individual exposure to shocks is general and also holds if one considers different models for endogenous attention, such as rational inattention. Second, the implication that this type of correlation should also predict a larger response than would be obtained in a simple homogeneous attention model also has broader implications than just for the framework considered here.

## 3.3 Forecast errors and income cyclicality

In this section, I present empirical evidence in favor of the main implication that correlated disagreement is positive. For this purpose, I use microdata on household expectations from the Survey of Consumer Expectations (SCE),[10] and to measure realized outcomes, I use the Current Population Survey (CPS).[11] The SCE is a household panel surveying expectations that has been running since 2013 and covers around 1,300 households each month. For comparability with the model, I use mean forecasts for one-year ahead own income growth, as measured by the New York Fed using the response to question 24.[12] I consider the magnitude of forecast errors as a function of income cyclicality. To show why forecast errors are informative of the level of attention of these individuals, I need to introduce some more notation.

Let $\Delta y_{i,t+h} \equiv y_{i,t+h} - y_{i,t}$ denote individual income growth, then the individual forecast error is defined as follows:

$$\text{FE}_{i,t} \equiv \Delta y_{i,t+h} - E_{i,t}[\Delta y_{i,t+h}]. \tag{3.26}$$

[10] This data is publicly available from the New York Federal Reserve Bank (NYFed) website: https://www.newyorkfed.org/microeconomics/sce#/.

[11] This data can be obtained from IPUMS, see Flood et al. (2021).

[12] This question asks individuals to suppose that, in 12 months, they are still working the same job and then report the probability that their income growth falls within various ranges. The NYFed estimates a probability density function using the method described in Engelberg et al. (2009).

Note that, under the assumptions on beliefs in Section 3.2.5, we can write

$$\mathrm{FE}_{i,t} = (1-\lambda_i)\mathbb{E}_t[\Delta y_{i,t+h}] + \varepsilon_{i,t+h} = (1-\lambda_i)\gamma_i\mathbb{E}_t[\Delta y_{t+h}] + \varepsilon_{i,t+h}, \tag{3.27}$$

where $\varepsilon_{i,t+h} \equiv \Delta y_{i,t+h} - \mathbb{E}_t[\Delta y_{i,t+h}]$ denotes the unforecastable component of income growth.

I derive the relationship between the magnitude of forecast errors, $\mathrm{FE}^2_{i,t}$, and income cycliality as follows. Fixing an individual, the expected magnitude of forecast error is given by:

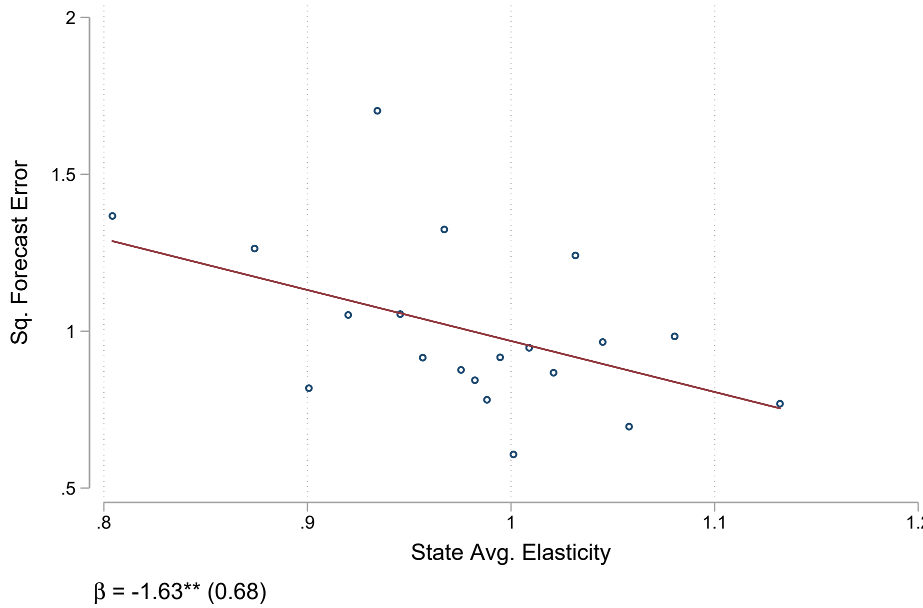
$$\mathbb{E}\left[\mathrm{FE}^2_{i,t}\right] = (1-\lambda_i)^2\gamma_i^2 C + \sigma^2_{i,\varepsilon},$$

where $\sigma^2_{i,\varepsilon}$ denotes the variance of $\varepsilon_{i,t+h}$ and $C$ is a strictly positive constant. So, the expected magnitude of forecast errors can decrease in cyclicality only if $\lambda_i$ increases in $\gamma_i$. The intuition for this result is as follows. First, note that keeping attention constant, higher cyclicality means that the magnitude of forecast errors rises via a mechanical effect on the variance of income growth. If attention is not increasing in $\gamma_i$, then the magnitude of forecast errors unambiguously increases. If attention is increasing in income cyclicality, then it provides a force in the opposite direction, pushing the magnitude of forecast errors closer to zero. Only if this opposing force is sufficiently strong is it possible to observe the magnitude of forecast errors declining with $\gamma_i$.

In evaluating this implication in the data, the ideal experiment would be to be able to know individual income cyclicality and match that to belief data. However, due to data constraints, this is not possible. Instead, I exploit the state-level dimension to merge the two separate surveys.

First, I construct the state's average income elasticity $\overline{\gamma}_{S,t}$ using individual-level regressions to back out elasticities at the industry level using detailed micro-data on individual income growth from the Current Population Survey (CPS). I then aggregate these elasticities using state industry shares also measured in CPS data. More details about this procedure

Figure 3.2: Income cyclicality and the magnitude of corrected-forecast errors




*Notes:* This figure shows the relationship between states' average income cyclicality and the magnitude of forecast errors. See text for more details.

can be found in appendix C.2.2. Then, using the SCE, I construct the average income growth forecast at the state level by aggregating all forecasts within a quarter, $\overline{E}_{S,t}[\Delta y_{i,t+h}]$ where $S$ stands for state and $t$ denotes the respective quarter. In the SCE, we observe one-year ahead forecasts, which implies that the horizon is four quarters, $h = 4$. Then, I use direct observation of individuals within a state to construct the average yearly income growth using $\Delta y_{S,t+h}$ using CPS data. I define the state-level corrected forecast error analogously to the individual one

$$\overline{\text{FE}}_{S,t} \equiv \Delta y_{S,t+h} - \overline{E}_{S,t}[\Delta y_{i,t+h}]. \tag{3.28}$$

The main result can be found in Figure 3.2 and is presented as a binscatter plot. This figure's y-axis is normalized so that 1 represents the average squared corrected-forecast-error.

Figure 3.2 shows that the magnitude of forecast errors falls with the state's average income cyclicality. This means that the average forecast error is lower in states with higher income cyclicality.The coefficient is statistically significant and economically significant: a 0.1 increase in the state's cyclicality is associated with a decrease in the magnitude of forecast errors by 16.3 percent on average. This result suggests that the average level of attention is higher in states with higher levels of income cyclicality. In Appendix C.2.3, I show that this result is robust to including a variety of controls including quarter fixed effects, the share of high-skill workers, among others.

## 3.4 Quantitative model

In Section 3.2, I emphasize the role of correlated disagreement in determining the strength of GE effects and the amplification of business-cycle shocks. To deliver clean results, the model in that section is intentionally stylized. In this section, I generalize the simple model in various dimensions to evaluate the quantitative relevance of the main mechanisms described in this paper. The extensions included in this section are done to achieve a more realistic description of aggregate demand, the crucial object in the analysis.

The modeling approach is based on the rapidly expanding literature on Heterogeneous-Agent New-Keynesian (HANK) models.[13] I augment a standard HANK model with the two ingredients present in the simple model. First, I assume that each household belongs to a group $g$ which determines their income cyclicality. Second, I assume that households may be inattentive to future economic variables relevant to their decision-making. As in the simple model, I choose parameters to normalize the steady-state level of output to one, $Y = 1$.

[13]See, e.g., Gornemann, Kuester, and Nakajima (2016), McKay and Reis (2016), Guerrieri and Lorenzoni (2017), Auclert (2019), McKay, Nakamura, and Steinsson (2016), Kaplan, Moll, and Violante (2018), Werning (2015) or Auclert, Rognlie, and Straub (2018, 2020).

### 3.4.1 Households

I extend the household description in Section 3.2.3 to allow for incomplete markets and countercyclical uninsurable idiosyncratic income risk. The economy is inhabited by a continuum of infinitely-lived households indexed by $i \in [0,1]$.

**Labor income** Each household belongs to a group $g = 1, ..., n$. The average labor income of group $g$ is given by:

$$Y_{g,t} = w_t N_{g,t}, \tag{3.29}$$

where $N_{g,t} = \Gamma_g(N_t)$ and $\gamma_g \equiv \Gamma'_g(1)$. To model income risk, I assume that individual households draw idiosyncratic productivity states $z_{i,t}$ from a finite support and with Markov transition matrix $\Pi^z$. Note that this transition matrix is constant across groups. I let $\pi^z(z)$ denote the steady-state mass of households in state $z$ and normalize productivity levels so that $\sum_z \pi^z(z)z = 1$. If a household in group $g$ has productivity $z_{i,t}$, then their pre-tax labor income is given by

$$y(z_{i,t}, Y_{g,t}) = \chi(z_{i,t}, Y_{g,t}) \cdot z_{i,t} \cdot Y_{g,t}, \tag{3.30}$$

where the function $\chi(z, Y)$ satisfies:

$$\chi(z, 1) = 1. \tag{3.31}$$

This function allows us to parameterize the cyclicality of income risk conveniently. The standard assumption in models of idiosyncratic income risk is $\chi(z, Y) = 1$. In this case, the cross-sectional variance of log-income is constant $\mathbb{V}\text{ar}(\log(y)) = \mathbb{V}\text{ar}(\log(z))$. To allow the variance of income to vary over the business cycle, I use the simple parameterization in Auclert and Rognlie (2018):

$$\chi(z, Y) = \frac{z^{\zeta \log Y}}{\sum_z \pi_z z^{1+\zeta \log Y}}. \tag{3.32}$$

In this case, the cross-sectional variance of the log income of individuals in group $g$ is given by

$$\mathbb{V}\text{ar}_g\left(\log(y)\right) = \mathbb{V}\text{ar}\left(\log(z)\right) \cdot \left[1 + \zeta \log(Y_g)\right].$$

If $\zeta$ is negative, recessions lead to an endogenous widening of the distribution of income, i.e., income risk rises during a recession and falls during an expansion. Instead, if $\zeta$ is positive, income risk rises during an expansion and falls during a recession. Recent evidence suggests that $\zeta < 0$ is the empirically relevant case, see, e.g., Coibion et al. (2017) and Guvenen et al. (2017).

**Assets and budget and borrowing constraints** I assume that the household can trade one-period non-contingent risk-free government bonds. The household enters period $t$ with $a_{i,t}$ assets, on which they earn the real interest rate $r_t$. The household's time-$t$ budget constraint is given by

$$c_{i,t} + a_{i,t+1} = (1 - \tau_t) y(z_{i,t}, Y_{g,t}) + (1 + r_t) a_{i,t}, \tag{3.33}$$

where $c_{i,t}$ and $a_{i,t+1}$ denote the choices of consumption and savings, respectively, $\tau_t$ is the labor-income tax rate, and $r_t$ is the real interest rate on savings.[14] I further assume that the household is subject to a standard no-borrowing constraint

$$a_{i,t+1} \geq 0. \tag{3.34}$$

**Beliefs** The household must forecast the discount-factor shock, labor income, tax rates, and the real interest rate. I maintain the assumption that people have the correct expectations regarding their preference shock $\beta_{i,t}$. Furthermore, the household's labor income

[14]Note that lower-case letters now represent the level of consumption of individual $i$ at time $t$ and not deviations from steady state.

depends on individual productivity $z_{i,t}$ and group-average income $Y_{g,t}$. I assume that people know their current productivity state and the distribution from which these states are drawn. This assumption also means that individuals are fully attentive to idiosyncratic shocks but may be inattentive to the aggregate shocks, which determine the aggregate component of their labor income.[15]

As in the simple model, I begin by allowing for general beliefs and write $E_{i,t}[\cdot]$ as the expectation operator given household $i$'s beliefs. As before, I also assume that people know their current level of income, taxes, and real interest rates. In this model with borrowing constraints, this assumption is also useful in guaranteeing that these constraints are not violated because of misperception of current income. I discuss how to endogenize attention in Section 3.4.7.

**Consumption and savings decisions** The household's consumption and savings decisions are characterized by the problem:

$$V_{i,t}(a,z) = \max_{c,a'} u(c) - v(n) + \beta_t E_{i,t}[V_{i,t+1}(a',z')] \tag{3.35}$$

$$c + a' = (1-\tau_t)y(z, Y_{g,t}) + (1+r_t)a_{i,t} \tag{3.36}$$

$$a' \geq 0. \tag{3.37}$$

This problem defines a consumption and assets policy function: $c_{i,t}(a,z)$ and $a_{i,t}(a,z)$, respectively. The term $\beta_t$ captures discount-factor shocks. This dynamic problem implicitly assumes that the law of iterated expectations holds at the individual level, i.e., $E_{i,t}[E_{i,t+1}[\cdot]] = E_{i,t}[\cdot]$. This implies that individuals expect not to make forecast revisions in

[15] The assumption of full attention to idiosyncratic income shocks is extreme. I maintain this assumption to focus on how inattention to aggregates affects the transmission of business-cycle shocks. This assumption also makes the computational task easier because it implies that bounded rationality does not affect the economy's steady state. See the discussion in Section 3.4.6.

the future.[16]

**Remark 4.** *This model assumes that households in different groups are symmetric across all dimensions except for their income cyclicality and expectations. In particular, I have assumed that all households draw from the same idiosyncratic productivity distribution and have the same discount factors. Furthermore, I assume that, in a steady state, all groups have the same average income. It follows that groups will be perfectly symmetric in a steady state. These assumptions also imply that the marginal propensities to consume out of own income are the same across groups. Patterson (2019) provides evidence of a positive correlation between income cyclicality and the marginal propensity to consume out of contemporaneous income. In this model, I have decided to abstract from this correlation for two reasons: (1) to emphasize the role of heterogeneous attention, and (2) because the mechanism at play in this paper relates more to forward-looking* MPC *(the* MPC *out of income in the future) for which we have fewer data. However, note that if these forward-looking* MPC *are also positively related to income cyclicality, this will work to reinforce the mechanism which is the focus of this paper.*

### 3.4.2 Firms

Firms are perfectly competitive and maximize profits. They operate a linear technology:

$$Y_t = \Theta_t N_t, \tag{3.38}$$

[16]This assumption is also present in the anticipated-utility approach pioneered by Kreps (1998), in which agents behave as if they believe their expectations wouldn't change. See also Cogley and Sargent (2008) for a further discussion.

where $Y_t$ denotes aggregate output, $N_t$ is aggregate labor, and $\Theta_t$ denotes productivity. I maintain the assumption that firms have flexible prices, which implies that the real wage is:

$$w_t = \frac{W_t}{P_t} = \Theta_t. \tag{3.39}$$

This equilibrium condition implies that inflation $\pi_t = \log(P_t/P_{t-1})$ is given by:

$$\pi_t = \pi_t^w - \log(\Theta_t/\Theta_{t-1}), \tag{3.40}$$

where $\pi_t^w = \log(W_t/W_{t-1})$ denotes wage inflation.

### 3.4.3 Unions and sticky wages

I follow the New Keynesian sticky-wage literature and model sticky wages via a model of labor unions, as in Erceg et al. (2000); Schmitt-Grohé and Uribe (2005) and Auclert et al. (2018).

There is a continuum of labor unions that determine wages and labor supply. At each date, person $i$ supplies $n_{u,i,t}$ hours of work to union $u$, where $u \in [0,1]$ and $n_{i,t} = \int n_{u,i,t} du$ denotes total hours of work by person $i$. Each union aggregates the efficiency hours of work into total hours for their specific task $N_{u,t} = \int \chi(z_{i,t}, Y_{g(i),t}) \cdot z_{i,t} \cdot n_{u,i,t} di$. The union-specific labor supply is aggregated with those of other unions by a competitive labor-market packer via a CES technology

$$N_t = \left( \int_0^1 N_{u,t}^{\frac{\mu_w - 1}{\mu_w}} du \right)^{\frac{\mu_w}{\mu_w - 1}}, \tag{3.41}$$

which then sells these labor services to the final goods producer at $W_t$.

In order to generate nominal rigidities, I assume that unions face a quadratic cost of wage adjustment $\frac{1}{2\tilde{\kappa}_w} \left( \frac{W_{u,t}}{W_{u,t-1}} - 1 \right)^2$, which is measured in terms of household utility. At every date, the union chooses a new wage $W_{u,t}$ and is required to elicit labor by each of its members according to a uniform rule $n_{u,i,t} = N_{u,t}$. I assume that the union sets the wage $W_{u,t}$ and labor supply $N_{u,t}$ to maximize the aggregate welfare valuation of its income and labor supply. Furthermore, I assume that all unions are symmetric. So, in equilibrium, all unions set the same wage. In Appendix C.3.1, I outline these details and show that the linearized NK wage Phillips curve is given by:

$$\pi_t^w = \kappa_w \left[\sigma^{-1}\hat{c}_t + \psi^{-1}\hat{n}_t - (\hat{y}_t - \hat{\tau}_t - \hat{n}_t)\right] + \beta\mathbb{E}_t[\pi_{t+1}^w], \tag{3.42}$$

where $\kappa_w$ denotes the wage-stickiness parameter, and $\sigma$ and $\psi$ denote the intertemporal elasticity of substitution and the Frisch elasticity, respectively. Also, $\hat{c}_t$, $\hat{n}_t$ and $\hat{y}_t$ denote the log-deviation of aggregate consumption, labor, and output from steady state, respectively, and $\hat{\tau}_t \equiv d\tau_t/(1-\tau)$ denotes the deviation of taxes from steady state.

### 3.4.4 Fiscal and monetary policy

I assume that the government spends $\{G_t\}$, issues debt $\{B_t\}$, and taxes labor income at the rate $\{\tau_t\}$. The government budget constraint is given by:

$$G_t + (1 + r_t)B_t = \tau_t Y_t + B_{t+1}. \tag{3.43}$$

I assume that the government sets a path for spending exogenously and that government debt is constant at its steady-state level $B_t = B$. So, the tax $\tau_t$ adjusts every period to make this budget constraint hold. In Appendix C.5.3, I consider a more general case in which government debt is allowed to vary over time.

I assume that the monetary authority controls the nominal interest rate. Monetary policy is given by a Taylor interest-rate rule:

$$1 + i_t = (1 + r_t^*) \cdot e^{\phi_\pi \pi_t}, \tag{3.44}$$

where $\{r_t^*\}$ denotes a monetary policy shock and $\phi_\pi > 1$ is the Taylor coefficient. The real interest rate is given by:

$$(1 + r_t) = (1 + i_t)/e^{\pi_t}. \tag{3.45}$$

### 3.4.5 Aggregation and equilibrium

As before, aggregate demand is given by:

$$C_t = \int_0^1 c_{i,t} di,$$

and aggregate asset demand is given by:

$$A_{t+1} = \int_0^a a_{i,t+1} di.$$

Given beliefs, initial conditions on wages, government debt, and the distribution of individuals over assets and productivity states, and the path of government spending, an equilibrium is a sequence for prices $\{W_t, P_t, w_t, r_t, \pi_t\}$, aggregate quantities $\{C_t, N_t, Y_{g,t}, N_{g,t}\}$, policies $\{\tau_t, i_t\}$, and individual allocations $\{c_{i,t}, n_{i,t}, a_{i,t+1}\}$, such that households optimize, firms optimality conditions are satisfied, unions optimize, the government budget constraint is satisfied, interest rates satisfy the Taylor rule, and markets clear:

$$C_t + G_t = Y_t = \Theta_t N_t,$$

$$A_t = B.$$

### 3.4.6 Computational method

The goal is to use a calibrated version of this model to quantify the response of aggregate variables to four different shocks: discount-factor, $\{\beta_t\}$, government spending, $\{G_t\}$, monetary policy, $\{r_t^*\}$, and productivity, $\{\Theta_t\}$, shocks. There are two features of this model that make the computation difficult. The first is the presence of household heterogeneity with incomplete markets and uninsurable idiosyncratic income risk. The second is the presence of heterogeneous non-FIRE beliefs. I leverage recent contributions in the Sequence-Space representation HANK literature by Auclert et al. (2021) and Auclert et al. (2020).

The computational strategy is to first solve for the stationary equilibrium of this economy and then solve for the first-order impulse-response functions for a given shock in sequence space.

**Steady state** In a steady state, aggregate quantities and prices are constant over time. I assume that monetary policy sets the nominal interest rate to support the zero inflation equilibrium, $\pi = \pi^w = 0$. In steady state, there are no shocks so $\beta_t = \beta$, $G_t = G$, $r_t^* = r$, and $\Theta_t = 1$.

I assume that the steady state is common knowledge. So, if the economy stays in a steady state, everyone understands that the relevant variables will remain at their steady-state levels. In other words, in a steady-state allocation, people make no errors in forecasting aggregate income, interest rates, or taxes.[17] This implies that the problem of a consumer is

[17] This assumption can be justified by the fact that the households have spent a long time in a steady state and have thus learned the stationary equilibrium.

given by:

$$V(a,z) = \max_{c,a'} u(c) - v(N) + \beta\mathbb{E}[V(a',z')|z],$$
$$c + a' = (1-\tau)\cdot z\cdot Y + (1+r)a$$
$$a' \geq 0,$$

where the expectation $\mathbb{E}[\cdot|z]$ is taken with respect to the distribution of $z'$ given $z$. Note that group heterogeneity is irrelevant in a steady state because all groups are symmetric. The solution to this problem determines policy rules $c(a,z)$ and $a'(a,z)$, which determine optimal consumption and asset holdings given the individual state variables.

Aggregate consumption and asset demand are given by:

$$C = \sum_z \int c(a,z)D(da,z), \quad \text{and} \quad A = \sum_z \int a'(a,z)D(da,z), \tag{3.46}$$

where $D(\cdot,\cdot)$ denotes the endogenous distribution of asset holdings and productivities. Note also that because all groups are symmetrical, the distribution $D$ is constant across groups in a steady state. The market clearing conditions are:

$$C + G = Y \quad \text{and} \quad A = B.$$

Computing the equilibrium requires solving for quantities and prices that satisfy all agents' private optimality, the above steady state restrictions, and market clearing.

**Transition dynamics with FIRE** First, I discuss how to compute the full-information rational expectations (FIRE) equilibrium in this economy. Then, I discuss how this approach can be generalized to models with more general beliefs.

With FIRE computing the transition dynamics would require solving for the path of each variable satisfying all the conditions. I use the method described in Auclert et al. (2021) to solve the equilibrium. This requires splitting the model into "blocks" that take in specific inputs and produce other aggregate sequences as outputs. For instance, a group-$g$ household block can be constructed taking in the sequences of discount factor shocks, group average income, taxes, and real interest rates while outputting sequences for average consumption and savings for this group.

Following Auclert et al. (2021), I construct the Jacobians, $\mathcal{J}$, of each block. These Jacobians summarize the partial derivative of a given output of the block with respect to that block's inputs. For example, one Jacobian of the household block is $\mathcal{J}_g^{C,r} = [\partial \overline{C}_{g,t}/\partial r_h]_{t=0,1,\ldots;h=0,1,\ldots}$, which summarizes how average demand of group $g$ at each date $t$ responds to an increase in the real interest rate at time $h$. These Jacobians summarize the relevant responses of the different blocks of the economy to every variable. They can thus be used to compute the first-order response in this economy to various exogenous impulses/shocks. In this model, the computationally complex components are the household blocks. I elaborate on these blocks below.

We are interested in the response of group $g$'s average consumption and asset demand to changes in the objects that are relevant to their decisions: group-average income, $Y_{g,t}$, tax rates, $\tau_t$, real interest rates, $r_t$, and discount factors, $\beta_t$. I describe how the approach solves for changes in average consumption demand, with the understanding that similar expressions can be written for asset demand. The response of average consumption is given by:

$$d\overline{\mathbf{C}}_g = \mathcal{J}_g^{C,Y} \cdot d\mathbf{Y}_g + \mathcal{J}_g^{C,\tau} \cdot d\boldsymbol{\tau} + \mathcal{J}_g^{C,r} \cdot d\mathbf{r} + \mathcal{J}_g^{C,\beta} \cdot d\boldsymbol{\beta}, \tag{3.47}$$

where bold letters denote the column-vector of realizations of that variable for each date, e.g., $\mathbf{Y}_g = \begin{bmatrix} Y_{g,0} & Y_{g,1} & \ldots \end{bmatrix}'$. Auclert et al. (2021) provide efficient ways of computing the relevant Jacobians that matter to solve for the response of this average consumption. We can then aggregate these blocks to find aggregate demand:

$$d\mathbf{C} = \sum_g \pi_g d\overline{\mathbf{C}}_g. \tag{3.48}$$

It follows from the fact that all groups are symmetrical in a steady state that the partial equilibrium responses summarized by the Jacobians are the same across all groups, i.e., $\mathcal{J}_g^{C,X} = \mathcal{J}^{C,X}$ for each variable $X$. Furthermore, as before, we know that $d\mathbf{Y}_g = \gamma_g \cdot d\mathbf{Y}$. As a result, the change in aggregate consumption is given by:

$$\begin{aligned} d\mathbf{C} &= \sum_g \pi_g \left[\mathcal{J}^{C,Y} \cdot \gamma_g \cdot d\mathbf{Y} + \mathcal{J}^{C,\tau} \cdot d\boldsymbol{\tau} + \mathcal{J}^{C,r} \cdot d\mathbf{r} + \mathcal{J}^{C,\beta} \cdot d\boldsymbol{\beta}\right] \\ &= \mathcal{J}^{C,Y} \cdot d\mathbf{Y} + \mathcal{J}^{C,\tau} \cdot d\boldsymbol{\tau} + \mathcal{J}^{C,r} \cdot d\mathbf{r} + \mathcal{J}^{C,\beta} \cdot d\boldsymbol{\beta}, \end{aligned}$$

where the equality follows from the fact that $\sum_g \pi_g \gamma_g = 1$. This result shows that, with FIRE, the aggregate demand response in this economy is exactly the same as in an economy without group income cyclicality heterogeneity. Intuitively, as in the simple model, this is a consequence of the fact that the average marginal propensities to consume are the same across groups.

**Transition dynamics without FIRE** However, the goal of this paper is to be able to compute the equilibrium in this economy for more general beliefs, which need not coincide with the realization (people may make forecast errors), which may be heterogeneous (to capture disagreement), and may change over time (to capture learning). In this section, I discuss how, under some restrictions, the Jacobians of the household block can be computed at almost no additional computational cost from the FIRE Jacobians. The central insight and computational method used here were originally developed in Auclert et al. (2020).

The computational complexity arises from the fact that, without FIRE, the average consumption function of group $g$ is a function not only of the realized path for each input but also of the entire distribution of beliefs that individuals hold about this path at every point in time. Auclert et al. (2020) show that, assuming that the distribution of beliefs is orthogonal to the individual states $(a, z)$, the Jacobians without FIRE can be computed directly from the FIRE Jacobians at almost no extra computational cost.

In this paper, I use a different representation of the Jacobians without FIRE from the one in Auclert et al. (2020). This representation is also used in Bardóczy and Guerreiro (2023), where the equivalence between the alternative forms is discussed in detail.[18] The response of group $g$'s consumption can be written as:

$$d\mathbf{C}_g = \sum_{\mathbf{X}\in\{Y,\tau,r,\beta\}} \left[ \mathcal{J}^{C,X} \cdot \underbrace{\overline{E}_{g,0}[d\mathbf{X}]}_{\text{Initial belief}} + \sum_{t\geq 1} \mathcal{R}_t^{C,X} \cdot \underbrace{\left(\overline{E}_{g,t}[d\mathbf{X}] - \overline{E}_{g,t-1}[d\mathbf{X}]\right)}_{\text{Forecast Revision at time } t} \right], \tag{3.49}$$

where $\overline{E}_{g,t}[\cdot]$ denotes the average expectation of group $g$ at time $t$ and

$$\mathcal{R}_t^{C,X} \equiv \begin{bmatrix} 0 & \mathbf{0}_t' \\ \mathbf{0}_t' & \mathcal{J}^{C,X} \end{bmatrix}.$$

Note that if the cross-sectional distribution of beliefs is orthogonal to the distribution of idiosyncratic states $(a, z)$, then all that matters to first order is the response of average beliefs in group $g$. This assumption is essential in allowing us to compute the relevant Jacobians, $\mathcal{R}$, from their FIRE counterparts, $\mathcal{J}$. The FIRE Jacobian multiplies the initial beliefs $\overline{E}_{g,0}[\cdot]$ and transformations of this Jacobian then multiply the successive forecast revisions that people make at each date $t$, $\overline{E}_{g,t}[d\mathbf{X}] - \overline{E}_{g,t-1}[d\mathbf{X}]$. Note that, by construction, the $t$ element of the forecast revision is the forecast error, i.e., the $t$-th element of the vector $\overline{E}_{g,t}[d\mathbf{X}] - \overline{E}_{g,t-1}[d\mathbf{X}]$ is $dX_t - \overline{E}_{g,t-1}[dX_t]$. So, the forecast revision term also captures the impact of forecast errors.

[18] In Appendix C.3.2, I briefly present the details behind these results.

This expression has a natural interpretation. Note that the FIRE Jacobian $\mathcal{J}^{C,X}$ multiplies the initial beliefs. Because the shocks are unanticipated, the initial response in beliefs is unanticipated both under FIRE and for any other model of beliefs. It follows that the slopes that determine the response to the initial change in beliefs are always the same. However, as time advances, people can learn more about the shocks and revise their beliefs. Forecast revisions would never happen under FIRE because people have perfect foresight. Without FIRE, people change their views over time as they suffer forecast errors and learn more. These successive forecast revisions lead people to adjust their consumption behavior relative to their original plan. The slopes which determine the revision in consumption decisions are captured by the matrix $\mathcal{R}_t^{C,X}$, where $t$ denotes the time in which the forecast revision occurs. This matrix implies that there is no consumption response prior to date $t$. This result follows from the fact that people could not have anticipated the forecast revision before date $t$ when those decisions were taken. Furthermore, the way in which consumption at current and subsequent dates is revised is captured exactly by the FIRE Jacobian $\mathcal{J}^{C,X}$. Intuitively, this result is also a consequence of the fact that the forecast revision was not anticipated. The response in current and future consumptions to the forecast revision is the same as if these had been time-0 belief updates, approppriately shifted. To further develop these intuitions, I now discuss two particular cases.

First, suppose that beliefs are never updated. Then, $\overline{E}_{g,t}[d\mathbf{X}] = \overline{E}_{g,0}[d\mathbf{X}]$. Since $\overline{E}_{g,t}[dX_t] = dX_t$, it follows that $\overline{E}_{g,t}[d\mathbf{X}] = d\mathbf{X}$, i.e., the initial beliefs were correct. It follows that in this case households consume exactly the same at every date as if they had full information and rational expectations. More generally, households may make forecast revisions and errors. However, at time 0 they do not anticipate any forecast error and so they make their consumption decisions as if the initial beliefs were fully accurate. This logic shows why the FIRE Jacobian multiplies the initial beliefs.

Now, suppose that beliefs are revised at some date $t$:

$$\overline{E}_{g,t}[d\mathbf{X}] - \overline{E}_{g,t-1}[d\mathbf{X}] = \begin{bmatrix} 0 & \dots & X_t - \overline{E}_{g,t-1}[X_t] & \overline{E}_{g,t}[X_{t+1}] - \overline{E}_{g,t-1}[X_{t+1}] & \cdots \end{bmatrix}'.$$

The consumption response to this forecast revision is given by $\mathcal{R}_t^{C,X}$. For instance, its implications for consumption at time $t$ are given by

$$\mathcal{R}_{t,(t,:)}^{C,X} \cdot (\overline{E}_{g,t}[d\mathbf{X}] - \overline{E}_{g,t-1}[d\mathbf{X}]) = \sum_{h\geq 0} \frac{\partial C_0}{\partial X_h} \cdot \left(\overline{E}_{g,t}[X_{t+h}] - \overline{E}_{g,t-1}[X_{t+h}]\right).$$

Note that the way in which time-$t$ consumption responds to a forecast revision is exactly the same as the way in which time-0 consumption would react to an unanticipated perfect-foresight shock under FIRE. Intuitively, this result follows from the fact that the forecast error could not have been anticipated, and so, to first order, it leads to the same consumption response as if it was a time-0 unanticipated shock.

To impose more structure, suppose that beliefs respond proportionally to the full-information and rational expectation: $\overline{E}_{g,t}[dX_{t+h}] = \lambda_{g,t,h} dX_{t+h}$. Under this assumption, we can write:

$$\overline{E}_{g,t}[d\mathbf{X}] = \Lambda_{g,t} d\mathbf{X}, \tag{3.50}$$

where $\Lambda_{g,t} = \text{diag}\left(\{1, ..., 1, \lambda_{t,1}, \lambda_{t,2}, ...\}\right)$ is a diagonal matrix. It follows that:

$$d\mathbf{C}_g = \sum_{\mathbf{X}\in\{Y,\tau,r,\beta\}} \tilde{\mathcal{J}}_g^{C,X} d\mathbf{X}, \tag{3.51}$$

where $\tilde{\mathcal{J}}_g^{C,X} \equiv \mathcal{J}^{C,X} \cdot \Lambda_{g,t} + \sum_{t\geq 1} \mathcal{R}_t^{C,X} \cdot (\Lambda_{g,t} - \Lambda_{g,t-1})$ are simple manipulations of the FIRE Jacobians.

### 3.4.7 Optimal attention

As in the simple model, I endogenize beliefs following Gabaix (2016). I extend that model by assuming that:

$$E_{i,t}[dX_{t+h}] = \lambda_{i,h}^{X} \cdot \mathbb{E}_t[dX_{t+h}] + (1 - \lambda_{i,h}^{X}) \cdot E_{i,t-1}[dX_{t+h}], \tag{3.52}$$

with initial condition $E_{i,-1}[dX_{t+h}] = 0$. The additional term implies that individuals learn over time, so new information accumulates to past knowledge. I also allow the attention variables to depend on the forecast horizon, allowing individuals to have more accurate forecasts regarding variables that are closer in time than those that are farther away. However, I maintain the assumption that attention for all states and periods is chosen once and for all in the pre-period, and households cannot re-optimize this plan in future times or states.

In Appendix C.3.3, I show that the utility cost of inattention takes a similar form to that found in the simple model:

$$\mathcal{C}_g(\boldsymbol{\lambda}_i, a, z) = -\frac{1}{2}\frac{\partial^2 v(c(a,z); a, z)}{\partial c^2} \sum_{X,\tilde{X},h,\tilde{h}} \frac{\partial c(a,z)}{\partial X_h}\frac{\partial c(a,z)}{\partial \tilde{X}_{\tilde{h}}} \left(1 - \lambda_{i,h}^{X}\right)\left(1 - \lambda_{i,\tilde{h}}^{\tilde{X}}\right)\sigma_{X_h,\tilde{X}_{\tilde{h}}}, \tag{3.53}$$

where $c(a,z)$ denotes the steady-state policy function.

The utility costs of inattention are now a function of the idiosyncratic asset and productivity states. I assume that the attention costs are linear $\kappa^X \lambda_{i,h}^X$. The cost of attention does not depend on the horizon but may depend on the variable being forecasted. This assumption will allow us to calibrate the model to match survey-data facts, see Section 3.4.8.

I assume that attention is chosen once and for all, to minimize ex-ante expected costs of inattention weighted by the ergodic distribution, i.e.,

$$\boldsymbol{\lambda}_i = \arg\min_{\boldsymbol{\lambda}} \sum_z \int \mathcal{C}_g(\boldsymbol{\lambda}, a, z) D(da, z) + \sum_{X,h} \kappa^X \lambda_{i,h}^X. \tag{3.54}$$

This assumption implies that attention is constant for all members of group $g$, $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_g$ and so beliefs are orthogonal to asset and productivity states. Under this assumption, we can use the computational method discussed in the previous section.

Following Gabaix (2014), I make the simplifying assumption that people believe the correlation across variables to be zero.[19] This assumption implies that the optimal attention can be easily solved and it is given by:

$$\lambda_{g,h}^Y = \max\left\{0, 1 - \frac{\kappa^Y}{\sum_z \int \frac{\partial^2 v(a,z)}{\partial c^2} \left(\frac{\partial c(a,z)}{\partial Y_{g,h}}\right)^2 D(da, z) \cdot \gamma_g^2 \sigma_Y^2}\right\} \tag{3.55}$$

and

$$\lambda_{g,h}^X = \max\left\{0, 1 - \frac{\kappa^X}{\sum_z \int \frac{\partial^2 v(a,z)}{\partial c^2} \left(\frac{\partial c(a,z)}{\partial X_h}\right)^2 D(da, z) \cdot \sigma_X^2}\right\} \tag{3.56}$$

for $X = \tau, r$.

**Forecast errors** Note that an individual's forecast error in predicting a variable $h$ periods ahead is given by:

$$\text{FE}_{i,t,t+h}^X = X_{t+h} - E_{i,t}[X_{t+h}] = \frac{1 - \lambda_{i,h}^X}{\lambda_{i,h}^X} \cdot \text{FR}_{i,t,t+h}^X + \varepsilon_{t,t+h}^X,$$

[19]In the appendix, I generalize these results to allow people to perceive correlations across variables. The results are consistent with the ones in the baseline model.

where $\varepsilon^X_{t,t+h} \equiv X_{t+h} - \mathbb{E}_t[X_{t+h}]$ denotes the unpredictable component of forecast errors and

$$\text{FR}^X_{i,t,t+h} \equiv E_{i,t}[X_{t+h}] - E_{i,t-1}[X_{t+h}]$$

denotes the individuals forecast revision at time $t$. This result means that it would be possible to obtain individual attention, $\lambda^X_{i,h}$, from the regressions of forecast errors on forecast revisions in Coibion and Gorodnichenko (2015), Bordalo et al. (2020), and Angeletos et al. (2021).

**Remark 5.** *In endogenizing attention, I assume that beliefs are chosen in an ex-ante stage, so they are not conditional on individual productivity and asset states. As discussed in the previous section, this assumption greatly facilitates the computational task. It is unclear how allowing for a correlation between* $(a, z)$ *would affect the results in this paper. In this remark, I briefly comment on the consequences of allowing for this correlation.*

*On the one hand, individuals with fewer assets or lower productivity are more likely to be borrowing constrained. It follows that they have lower* MPC *out of future income. In the limit, a borrowing-constrained individual has a zero* MPC *out of future income. All else equal, this fact would imply that individuals with fewer assets or lower productivity have a lower incentive to pay attention to changes in future income than individuals with more assets or higher productivity. Similar logic to heterogeneous income cyclicality would generate an even stronger correlation between attention and responsiveness, reinforcing the results in this paper.*

*On the other hand, individuals with lower assets and productivity also have higher marginal utility of consumption. It follows that consumption misoptimization is more costly for these individuals than for individuals with more assets or high productivity. This force would then work in the opposite direction mitigating the correlation.*

*In general, it is not clear which force dominates. Therefore, I think of the assumptions here as a useful and conservative benchmark to study the implications of heterogeneous in-*

*come cyclicality.*

Finally, note that we can write beliefs of an individual in group $g$ at time $t$ as $E_{i,t}[dX_{t+h}] = dX_{t+h}$ if $h \leq 0$ and

$$E_{i,t}[dX_{t+h}] = \sum_{s=0}^{t} \lambda_{g,h+s}^{X} \prod_{m=0}^{s-1} \left(1 - \lambda_{g,h+m}^{X}\right) \cdot dX_{t+h}, \tag{3.57}$$

if $h \geq 1$. It follows that this framework fits into the framework in equation (3.50).

### 3.4.8 Calibration

I first discuss the calibration of the economy's steady state and then elaborate on the calibration of the remaining parameters. The model is calibrated to a quarterly frequency. In steady state, I shut down shocks. So productivity is normalized to one $\Theta = 1$, the discount factor is equal to its steady-state value $\beta$, government spending is constant $G$, nominal interest rates are equal to real interest rates $i = r$, and inflation is zero $\pi = \pi^{w} = 0$.

Table 3.1 shows the calibrated parameters relevant to compute the steady state. I assume the household's utility function has constant elasticity over consumption and labor. This means that $u(c) = c^{1-\sigma^{-1}}/(1-\sigma^{-1})$ where $\sigma$ is the intertemporal elasticity of substitution. Following Auclert et al. (2018), I set this elasticity to 0.5. The disutility of labor is given by $v(n) = \xi n^{1+\psi^{-1}}/(1+\psi^{-1})$, where $\psi$ is the Frisch elasticity. Following Chetty et al. (2011), I set the Frisch elasticity to 0.5 and calibrate the disutility parameter $\xi$ so that the steady state features zero inflation with $Y = N = 1$. This calibration yields $\xi = 0.64$.

The productivity shocks are drawn from a discretized AR(1) process with persistence $\rho_z = 0.95$ and standard deviation $\sigma_z = 0.5$, which is in line with the parameters traditionally used in the literature. I set the interest rate to an annual rate of 2 percent or 0.5 percent quarterly. The government spending-to-GDP ratio is calibrated to 16 percent. I choose the level of assets-to-GDP and the discount factor to match an average marginal propensity to consume of 0.25. This yields $B/Y = 1.92$ and $\beta = 0.97$, which is in line with the values found in the literature.

Table 3.1: Calibration

| Parameter | Description | Value | Param. | Description | Value |
|---|---|---|---|---|---|
| $\sigma$ | IES | 0.5 | $r$ | Real int. rate | 0.5% |
| $\psi$ | Frisch | 0.5 | $G/Y$ | Spending-to-GDP | 16% |
| $\rho_z$ | Persistence $z$ | 0.95 | $B/Y$ | Assets-to-GDP | 1.92 |
| $\sigma_z$ | St. Dev. $z$ | 0.5 | $\beta$ | Discount factor | 0.97 |
| $\zeta$ | Cyclicality of income risk | $-0.5$ | $\phi_\pi$ | Taylor Coefficient | 1.5 |
| $\kappa_w$ | Wage rigidity | 0.0062 | $\rho_\beta$ | $\beta$ shock – Persistence | 0.9 |
| $\rho_G$ | Spend. shock – Persistence | 0.91 | $\rho_r$ | $r^*$ shock – Persistence | 0.89 |
| $\rho_\Theta$ | Product. shock – Persistence | 0.98 | $\xi$ | Labor disutility | 0.64 |

I assume there are 14 household groups, $n = 14$, one for each census industry group. The estimation of the elasticities $\gamma_g$ follows the procedure described in Appendix C.2.2. I assume that the group shares $\pi_g$ are equal to the shares of each industry in the US economy in 2018. I also use CPS data to estimate these shares. The results can be found in Table 3.2.

Table 3.2: Group shares and income cyclicality

| | Industry | $\pi_g$ | $\gamma_g$ | | Industry | $\pi_g$ | $\gamma_g$ |
|---|---|---|---|---|---|---|---|
| 1 | Agriculture, Forestry, Fishery | 1.76% | 0.05 | 8 | Non-durable Man. | 4.35% | 0.80 |
| 2 | Public Administration | 5.84% | 0.12 | 9 | Durable Man. | 7.60% | 1.44 |
| 3 | Bus. and Repair Services | 7.10% | 0.14 | 10 | Retail Trade | 14.69% | 1.77 |
| 4 | Prof. and Related Serv. | 30.90% | 0.43 | 11 | Wholesale Trade | 2.63% | 2.26 |
| 5 | Mining | 0.58% | 0.48 | 12 | Personal Services | 2.39% | 2.41 |
| 6 | Transp., Commun., Public Util. | 7.34% | 0.59 | 13 | Finance, Insur., Real Est. | 7.02% | 2.45 |
| 7 | Construction | 6.09% | 0.62 | 14 | Ent. and Recr. Serv. | 1.70% | 4.24 |

Following Auclert and Rognlie (2018), I assume that $\zeta = -0.5$, which implies that income risk is countercyclical and provides a good fit to the empirical findings with a single parameter. Furthermore, I assume that the Taylor coefficient is $\phi_\pi = 1.5$, which is standard in the literature. Consistent with the findings in Hazell et al. (2022), I set the wage flexibility parameter to $\kappa_w = 0.0062$.

I calibrate the attention cost parameters so that the average level of attention matches the regression results in Bordalo et al. (2020). To match these results, I calibrate the cost parameters $\kappa^X$, so that $\overline{\lambda}_3^Y = 0.69$ and $\overline{\lambda}_3^r = 0.45$. Because there are no forecasts for the tax rate, I cannot obtain $\overline{\lambda}_3^\tau$ in this way. Instead, I assume that attention to taxes is the same as attention to the aggregate component of income.

I consider four different shocks: to discount factors, $\beta$, to government spending, $G$, to the monetary policy rate, $r^*$, and productivity, $\Theta$. For each shock, I assume that the initial impulse evolves geometrically over time $X_{t+1} = \rho_X X_t$, where $\rho_X$ captures the persistence of the shock. I set the persistence for each shock in line with standard parameters in the literature. The persistence of discount factor shocks is set to 0.9, see Justiniano et al. (2010). The persistence of government-spending shocks is set to 0.91, see Auclert et al. (2018). The persistence of monetary policy shocks is set to 0.89, as estimated by Auclert et al. (2020). Finally, I set the persistence of TFP shocks to 0.98, which captures an oil shock in reduced form, see Blanchard and Gali (2007).

**Remark 6.** *Note that the model calibration does not directly use the empirical findings in section 3.3. Instead, I calibrate the model to a standard target in the literature and let the forces at play in the model determine attention heterogeneity. I do not pursue a more data-driven approach to recovering heterogeneous attention for three reasons. First, due to data constraints, I cannot conduct the analysis in Section 3.3 directly at the sector level, which could allow us to recover attention at the sector level. It is not easy to back out* *these attentions from state-aggregated data due to time-varying state characteristics, such as industrial composition. Second, because we do not have multiple forecast horizons in the SCE, we cannot compute the necessary forecast revision statistics which allow us to run the regressions in Coibion and Gorodnichenko (2015), Bordalo et al. (2020), and Angeletos et al. (2021). Finally, calibrating to the SPF's empirical findings has become standard in the literature. So, this choice also maximizes the comparability of my results to those in the literature.*

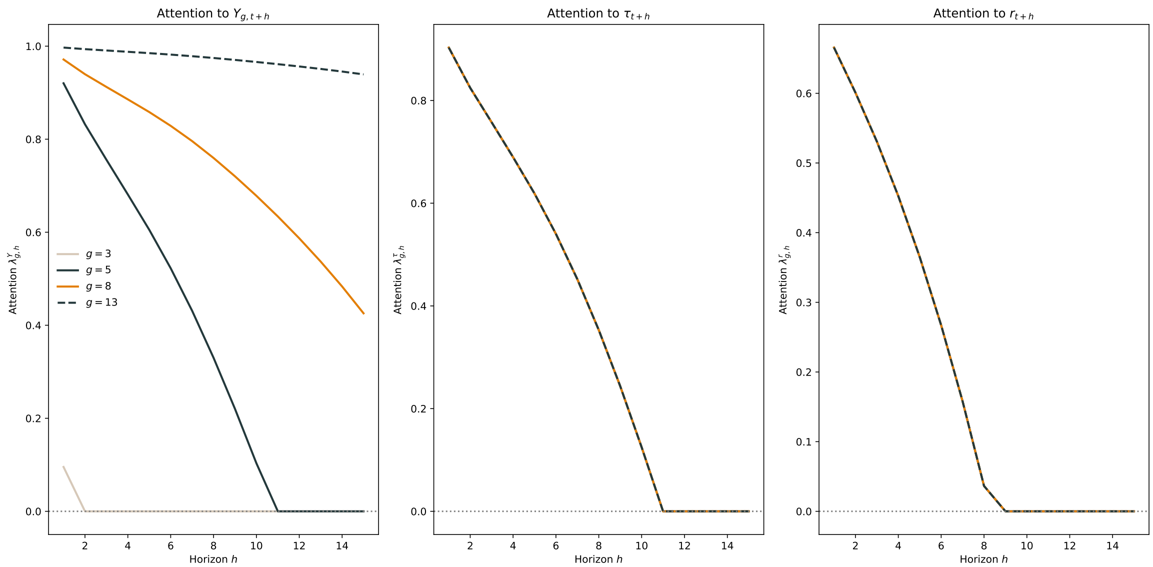
### 3.4.9 Quantitative results

In this section, I present the main quantitative findings. I begin by discussing the optimal level of attention generated by the model. I then discuss how heterogeneous attention impacts the response of group demand to changes in aggregate income. Finally, I present the main quantitative results on business cycle amplification.

**Optimal attention** Figure 3.3 displays the optimal level of attention to income, taxes, and interest rates on the left, middle, and right panels, respectively, for four different groups $g = 3, 5, 8$, and 13.

As expected from equation (3.56), the optimal attention to tax and interest rates is not affected by income cyclicality. So, all groups have the same levels of attention for every horizon. Instead, attention to changes in income depends on income cyclicality. Workers in more cyclical occupations choose a higher level of attention than workers who are less exposed to changes in aggregate conditions. We see that people in $g = 13$ are very close to full attention. Instead, people in $g = 3$ have such a low cyclicality that they optimally devote no attention to the aggregate component of their income from $h = 2$ on. In this model, people only disagree about income changes, not tax or interest rates. It would be easy to modify the assumptions to allow for disagreement about these other variables. However, since all

Figure 3.3: Optimal attention



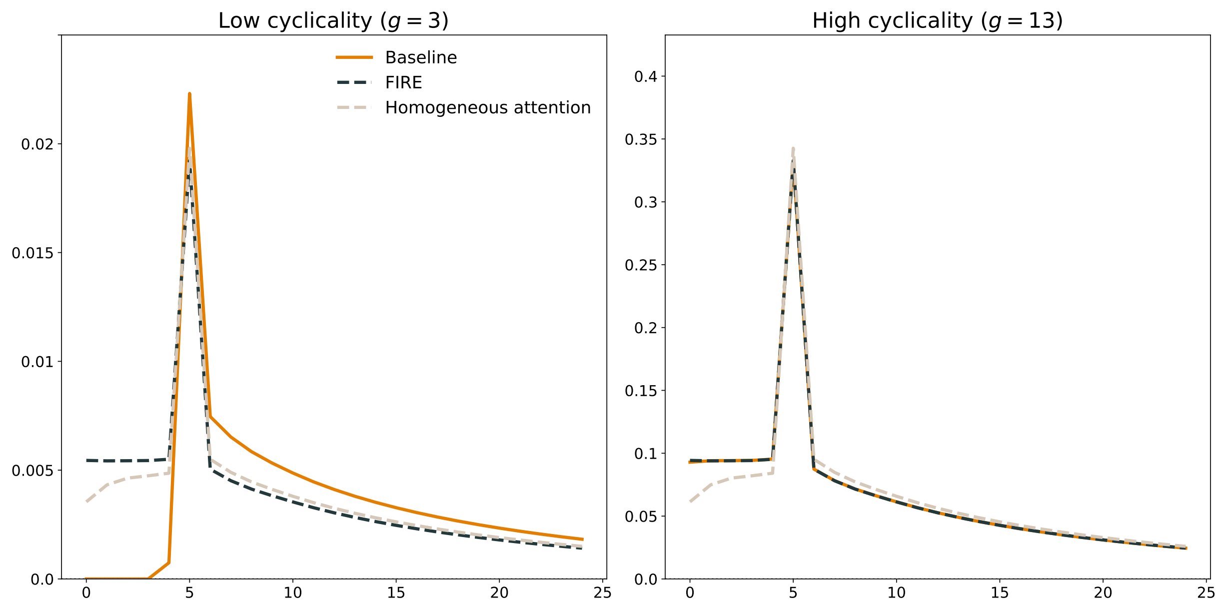

*Notes:* This figure displays optimal attention in the quantitative model for four different household groups $g = 3, 5, 8$ and 13, where $\gamma_3 = 0.14$, $\gamma_5 = 0.48$, $\gamma_8 = 0.8$, and $\gamma_{13} = 2.45$. The left panel displays attention to the aggregate component of income at various horizons, the middle panel displays attention to tax rates, and the right panel displays attention to real interest rates. See text for more details.

agents respond equally to tax and interest rate changes, this form of disagreement would not affect aggregate outcomes in this economy.

Overall, we see that attention decreases with the horizon $h$ of the forecast. The reason for this result is that shocks to variables that are far in the future have a lower impact on present decisions than shocks to variables that are closer in time, e.g., the marginal propensity to consume out of income two quarters ahead is higher than the marginal propensity to consume of income ten quarters ahead. As a result, people devote less cognitive effort to forecasting far-off variables. At far enough horizons, the value of predicting a variable is so tiny that individuals choose not to pay any attention, so $\lambda^X_{g,h} \to 0$ and $h \to \infty$. It is interesting to note that an additional contribution of this framework is to provide a microfoundation by which people behave as if they had finite planning horizons as in Woodford (2018) and Woodford and Xie (2019, 2022).

**Response to aggregate income changes** How does inattention affect the response of consumption to increases in aggregate income? To shed light on this question, in Figure 3.4, I compare the full-information and rational expectations response to an increase in income at time 5, with the response obtained in the economy with heterogeneous attention, for two groups $g = 3$ and $g = 13$. I also compute these responses in the counterfactual economy in which all individuals have the same level of attention (homogeneous attention), which coincides with the average level of attention in the baseline economy.

Figure 3.4: Consumption response

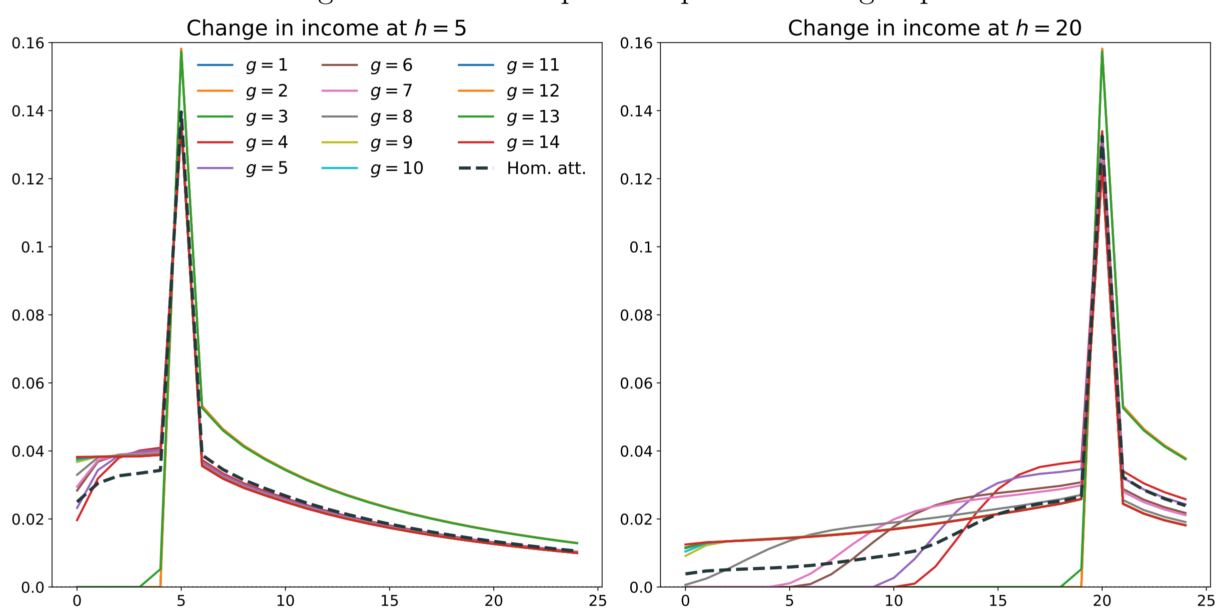



*Notes:* This figure displays the fifth column of the Jacobian multiplied by $\gamma_g$, or the partial-equilibrium response of consumption to an increase in aggregate income at time 5, of two groups: $g = 3$ in the left panel and $g = 13$ in the right panel, to an increase in aggregate income at time 5. For each group, the figure plots the response of consumption under full information and rational expectations (FIRE), heterogeneous attention (Baseline), and the counterfactual homogeneous attention (Homogeneous attention), which assumes that all agents have the same level of attentiveness.

Note that the shape of the FIRE responses in the two panels is exactly the same but they have different magnitudes. This result follows from the fact that, under FIRE, the Jacobians are the same for all groups. The different magnitudes result from the fact that an increase in aggregate income has a higher impact on the incomes of households with higher income cyclicality than for households with lower income cyclicality.

The same facts regarding the shape and size of the response are true for the economy with homogeneous attention. However, relative to the FIRE response, we see that the initial impact is dampened, i.e., agents consume less in anticipation of higher income in the future. Since individuals are inattentive, they do not fully incorporate how much their future income is rising into their present decicions. So, they do not consume as much early on. Since they have dissaved less relative FIRE, the increase in consumption at time 5 and subsequent dates is higher.

Figure 3.5: Consumption response for all groups




*Notes:* This figure displays the partial-equilibrium response of consumption for all groups to an increase in aggregate income at horizons 5 and 20 in the left and right panels, respectively. The figure also displays the response that would be obtained under homogeneous attention. The responses are divided by $\gamma_g$ for comparability.

Instead, the consumption responses to an increase in aggregate income in the baseline economy are quite different. The differences are not just in magnitude but also in their shape.

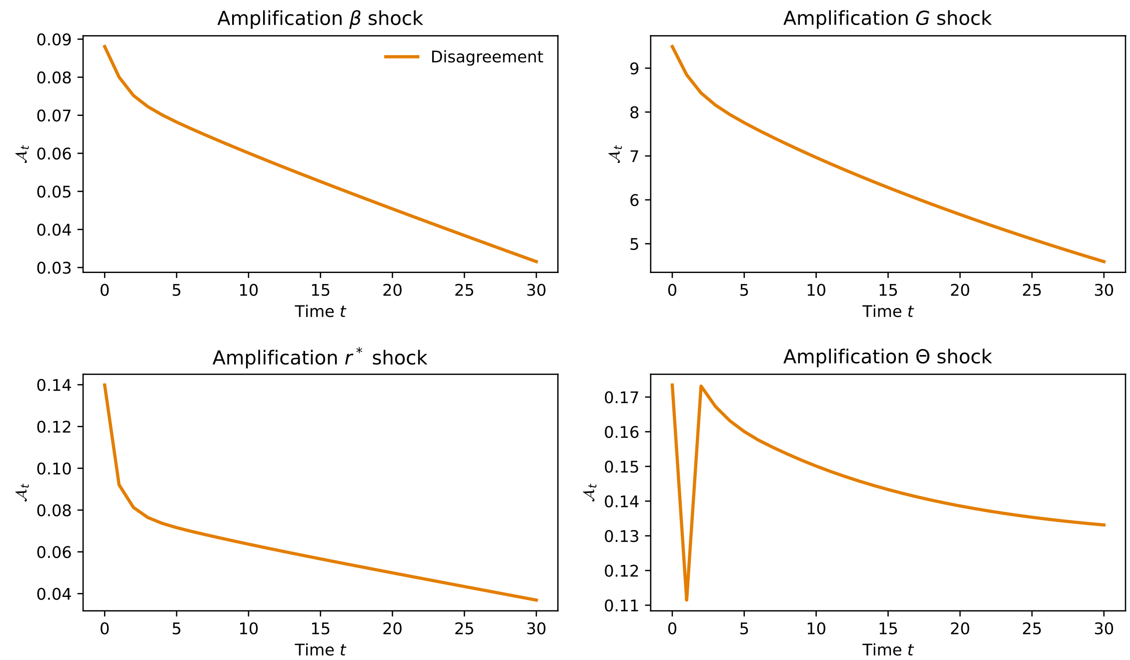
The low cyclicality group chooses to pay almost no attention to this income component. So, they do not increase consumption before time 4. At time 4, they become aware that there will be some increase in their future income, and so we see a mild response in average consumption. At the moment of the income increase, they finally become fully aware and increase their consumption. Because they have not consumed as much in earlier periods, they have not dissaved, so their consumption increase and time 5 and subsequent dates is higher relative to FIRE or homogeneous beliefs. Instead, the high cyclicality group displayed on the right panel has a very high level of attention to $Y_5$. As a result, their consumption in the baseline economy is essentially the same as under FIRE.

Figure 3.5 displays the response of consumption for all groups to an increase in aggregate income at two horizons $h = 5$ and $h = 20$. For comparison, I also plot the same response in the counterfactual economy with homogeneous attention. I divide the responses by $\gamma_g$ to fit the same scale. We can see that the message from the analysis above extends to all groups. Generally, the higher the cyclicality of a group, the higher their level of attention. This fact implies a higher consumption response before the income realization.

**Amplification of Business Cycles** How does disagreement affect the transmission and propagation of business cycles? In Section 3.2, I argue that heterogeneous attention can amplify discount factor shocks in a simple model with rigid wages. I now discuss how disagreement affects the amplification of business cycles in the quantitative model. For example, I show that the impact of an oil shock interpreted in the model as a productivity shock with high persistence can be amplified on impact by over 17 percent.

I compute the response of the economy in response to four different shocks: discount factor, $\beta$, government spending, $G$, interest rate, $r^*$, and productivity, $\Theta$, shocks. For each of these, I compute the impulse response function to an innovation at time 0 dissipating with persistence $\rho_x$, where $\rho_\beta = 0.9$, $\rho_G = 0.91$, $\rho_r = 0.89$, and $\rho_\Theta = 0.98$. I compute the

Figure 3.6: Business-cycle amplification




*Notes:* This figure displays amplification in the response of output, as defined in equation (3.20). I consider the response of the economy to four different shocks: a discount-factor shock in the top left panel, a government spending shock in the top right panel, an interest rate shock in the bottom left panel, and a productivity shock in the bottom right panel. See text for more details.

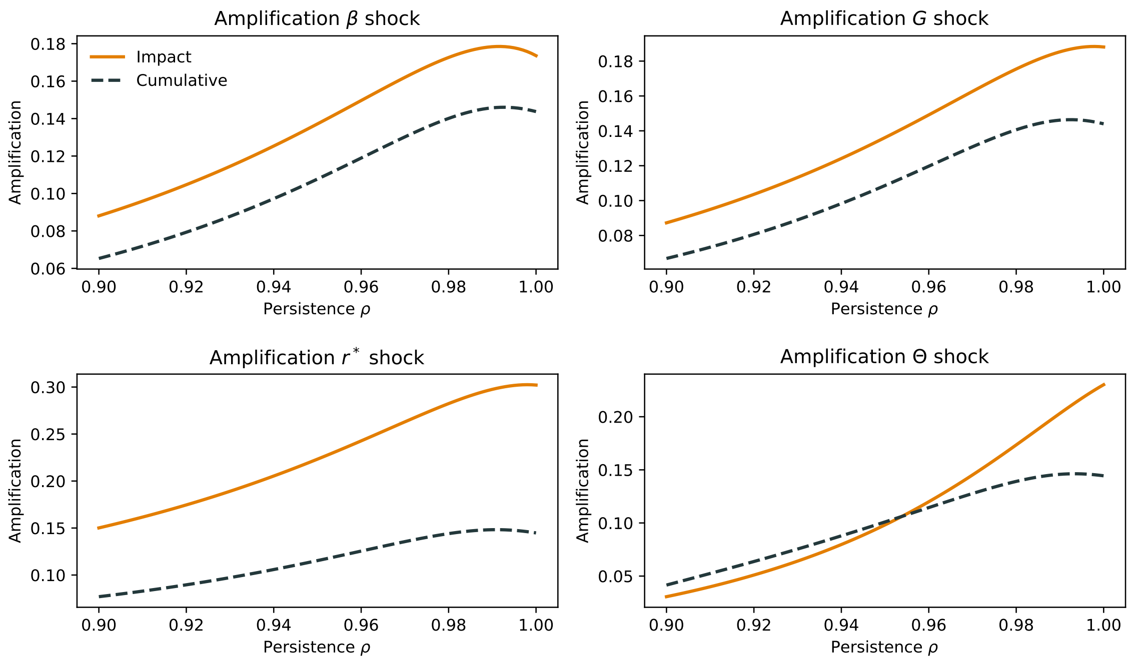
impulse response function under heterogeneous attention and in the counterfactual economy with homogeneous attention and use them to compute amplification as in equation (3.20).

Figure 3.6 displays the amount of amplification for each date $t$. We find that the correlated disagreement mechanism can significantly amplify the output response on impact. Discount-factor and government spending shocks can be amplified almost 10 percent, while interest rate shocks are amplified by over 14 percent, and productivity shocks are amplified by over 17 percent.

As highlighted in the simple model, the amount of amplification generated by correlated disagreement increases in the shock's persistence. To see this, Figure 3.7 displays impact and cumulative amplification for each of these shocks. Impact amplification is defined as $\mathcal{A}_0$ as in equation (3.20). Cumulative amplification summarizes the extent of amplification for the whole impulse response function, and I define it as:

$$\mathcal{CA} \equiv \frac{\sum_{t\geq 0}(1+r)^{-t}(y_t - y_t^{RA})}{\sum_{t\geq 0}(1+r)^{-t}y_t^{RA}}. \tag{3.58}$$

Figure 3.7: Business-cycle amplification: The role of persistence

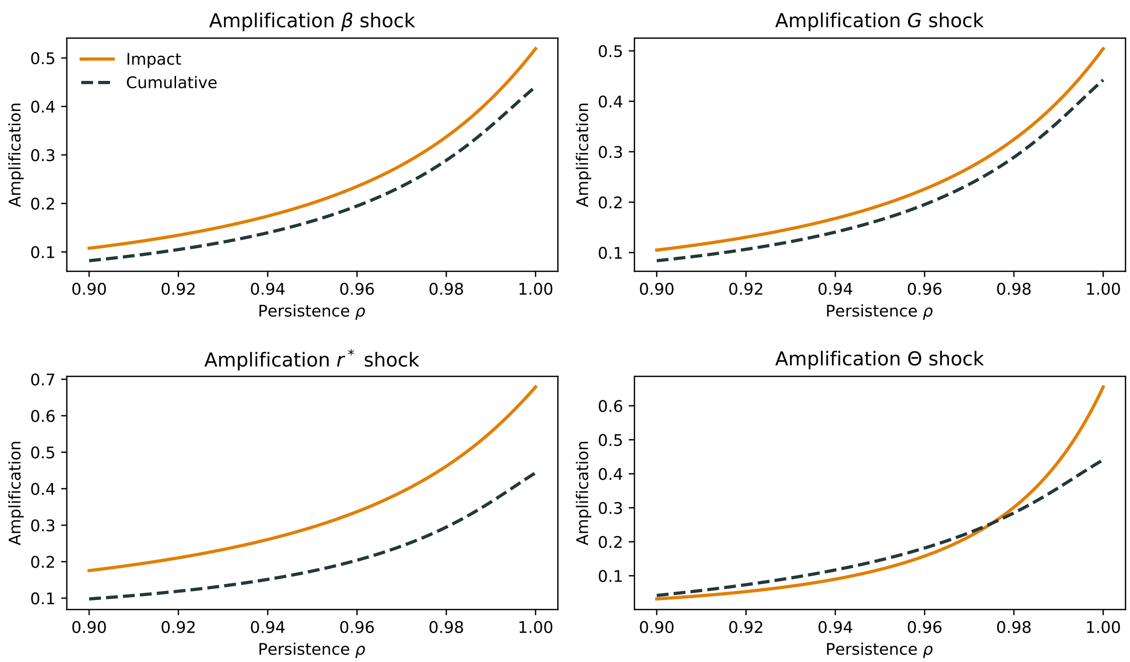



*Notes:* This figure displays impact and cumulative amplification in the response of output as a function of the persistence of the shock. Impact amplification is defined by $\mathcal{A}_0$, and cumulative amplification is defined as (3.58). I consider the response of the economy to four different shocks: a discount-factor shock in the top left panel, a government spending shock in the top right panel, an interest rate shock in the bottom left panel, and a productivity shock in the bottom right panel. See text for more details.

Figure 3.7 shows that impact amplification in the quantitative model can increase substantially with shock persistence. The amplification of discount-factor and spending shocks almost increases to over 18 percent, while it is as high as 25 percent for productivity shocks. Interest rate shocks are amplified by over 30 percent.

**The role of monetary policy** In this model, disagreement amplifies the response of output because it affects the response of aggregate demand to the general-equilibrium income channel. The strength of this channel crucially depends on how strongly monetary policy reacts to inflation. So, a natural question is how monetary policy affects the amount of amplification resulting from correlated disagreement. To answer this question, Figure 3.8 reconsiders the exercise of Figure 3.7 but with a lower Taylor coefficient of $\phi_\pi = 1.1$ instead of 1.5.

Figure 3.8: Business-cycle amplification: The role of monetary policy




*Notes:* This figure displays impact and cumulative amplification in the response of output as a function of the persistence of the shock when the Taylor coefficient is reduced from $\phi_\pi = 1.5$ to $\phi_\pi = 1.1$. Impact amplification is defined by $\mathcal{A}_0$, and cumulative amplification is defined as (3.58). I consider the response of the economy to four different shocks: a discount-factor shock in the top left panel, a government spending shock in the top right panel, an interest rate shock in the bottom left panel, and a productivity shock in the bottom right panel. See text for more details.

Comparing Figure 3.8 to 3.7 shows that there is much stronger amplification if the monetary policy response to inflation is weaker. The response to an oil shock is now amplified by almost 30 percent on impact and similarly if evaluated in terms of cumulative effects.

When monetary policy response is weaker, a more significant share of general-equilibrium forces operates via the income channel. This fact means that the correlated disagreement mechanism has a larger role when monetary policy is relatively unresponsive, leading to more considerable amplification.

## 3.5 Fiscal policy

In this section, I analyze the impact of correlated disagreement on the transmission of fiscal policy. In particular, I analyze how the composition of government spending can affect the size of the spending multiplier. I conduct this analysis from a purely positive perspective and do not consider the welfare and distributional consequences of this policy. It is well known that the desirability of stabilizing spending policy in stimulating aggregate demand depends crucially on which other constraints are imposed on monetary policy and other fiscal instruments. In this section, I do not try to assess the desirability of government spending policy.

To highlight the central intuition, I first analyze fiscal policy in the simple model of Section 3.2. I then evaluate these results quantitatively in Section 3.5.2.

### 3.5.1 Fiscal policy in the simple model

I extend the model in Section 3.2 to include government spending $\{G_t\}$ and proportional labor taxation $\{\tau_t\}$. For simplicity, I assume that there is no government debt, so the government runs a balanced budget. The government budget constraint is given by:

$$G_t = \tau_t Y_t, \tag{3.59}$$

and the modified household budget constraint is given by:

$$C_{i,t} + A_{i,t+1} = (1 - \tau_t) \cdot Y_{g,t} + (1 + r)A_{i,t}. \tag{3.60}$$

The market clearing condition is now given by:

$$C_t + G_t = Y_t. \tag{3.61}$$

For simplicity, I assume that steady-state spending and taxes equal zero.

**Untargeted spending** I first assume that the government buys units of the final good. In Appendix C.4.1, I show that to first order the individual demand can be written as:

$$c_{i,t} = (1-\beta)\sum_{h=0}^{\infty}\beta^h E_{i,t}[y_{g,t+h} - \tau_{t+h}] + (1-\beta)\beta^{-1}a_{i,t} + \sigma\sum_{h=0}^{\infty}\beta^{h+1}r^n_{i,t+h}. \tag{3.62}$$

Suppose furthermore that beliefs for income and taxes are proportional to their realized counterpart, $E_{i,t}[y_{g,t+h}] = \lambda_g^Y y_{g,t+h}$ and $E_{i,t}[\tau_{t+h}] = \lambda_g^\tau \tau_{t+h}$. Aggregate demand is given by:

$$c_t = (1-\beta)\sum_{h=0}^{\infty}\beta^h[(1+\text{CD})\cdot\overline{\lambda}^Y y_{t+h} - \overline{\lambda}^\tau \tau_{t+h}] + \sigma\sum_{h=0}^{\infty}\beta^{h+1}r^n_{t+h}, \tag{3.63}$$

where $\text{CD} \equiv \mathbb{C}\text{ov}\left(\gamma_g, \lambda_g^Y/\overline{\lambda}^Y\right)$ denotes correlated disagreement and $\overline{\lambda}^Y$ and $\overline{\lambda}^\tau$ denote the average attention to income and tax rates, respectively.

Equation (3.63) is the modified aggregate demand taking government into account. In equilibrium, it must be that taxes equal government spending, $\tau_t = G_t$, and that output equals private and public demand, $y_t = c_t + G_t$. So, equilibrium output is given by:

$$y_t = G_t + (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}[(1+\text{CD})\cdot\overline{\lambda}^Y y_{t+h} - \overline{\lambda}^\tau G_{t+h}] + \sigma\sum_{h=0}^{\infty}\beta^h r^n_{t+h}. \tag{3.64}$$

The government-spending multiplier, $dy_t/dG_t$, can be computed recursively using:

$$\frac{dy_t}{dG_t} = 1 + (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\left[(1+\text{CD})\cdot\overline{\lambda}^Y\frac{dy_{t+h}}{dG_{t+h}} - \overline{\lambda}^\tau\right]\frac{dG_{t+h}}{dG_t}. \tag{3.65}$$

This equation relates the time $t$ government-spending multiplier to people's beliefs about future spending multipliers. First, suppose that individuals are fully attentive. It follows that the spending multiplier is equal to one, $dy_t/dG_t = 1$, as in the FIRE analysis conducted in Woodford (2011) and Bilbiie (2011). When agents are inattentive to income and taxes, this multiplier is modified to take into account how expectations of future disposable income affect consumption choices today. The expectations of future disposable income are a function of expectations for future taxes and the effect that higher future spending has on future incomes, i.e., the future government-spending multipliers. All else equal, higher future spending multipliers increase the spending multiplier at time $t$, and higher correlated disagreement or greater attention to income also increases the spending multiplier if future spending multipliers are positive. The intuition for these results is that if agents expect future income to be higher, they start consuming more today, resulting in a larger spending multiplier. Instead, all else equal, a higher level of attention to taxes implies that the spending multiplier is lower because it reduces people's perceived disposable income and leads them to curtail private spending.

**Proposition 12.** *Suppose that $dG_t = \rho_G^t dG_0$, then the government-spending multiplier is given by:*

$$\frac{dy_t}{dG_t} = \frac{1-\varrho_G\overline{\lambda}^\tau}{1-\varrho_G\cdot(1+CD)\cdot\overline{\lambda}^Y}, \tag{3.66}$$

*where $\varrho_G \equiv (1-\beta)\rho_G/(1-\beta\rho_G) \in (0,1)$. It follows that:*

1. *The government-spending multiplier is increasing in correlated disagreement.*

*2. The government-spending multiplier is larger than under FIRE if and only if*

$$(1 + CD) \cdot \overline{\lambda}^{Y} \geq \overline{\lambda}^{\tau}.$$

Proposition 12 shows that the spending multiplier is constant over time and depends on the average level of attention to taxes, correlated disagreement, and the average level of attention to income changes. Other things equal, a higher level of attention to taxes decreases the multiplier, while a higher level of attention to income or higher correlated disagreement increases the spending multiplier.

Suppose that people are fully attentive to taxes, $\overline{\lambda}^{\tau} = 1$. The spending multiplier is always lower than obtained under full information and rational expectations. Because people are fully attentive to taxes, they immediately react by decreasing consumption in expectation of higher future taxes. Because they are inattentive to income changes, they do not fully incorporate how, in general equilibrium, future higher spending translates into higher future income. In other words, the positive general-equilibrium effect of future government spending on consumption is dampened. The net effect is a lower government-spending multiplier than under FIRE. This result has been previously emphasized by Farhi et al. (2020) and Bianchi-Vimercati et al. (2021).

Instead, suppose the average attentions to taxes and income are the same. Then, the spending multiplier is larger than the one obtained under FIRE if and only if correlated disagreement is positive. While people are heterogeneously exposed to changes in income, they are equally exposed to an increase in the tax rate. It follows that the response of aggregate demand to higher taxes is captured by the economy-wide average level of attention to taxes, while the response to higher income must take into account correlated disagreement. If the average attention to taxes and income are equal, but correlated disagreement is positive, then the relevant attention to income is higher, generating a larger spending multiplier.

**Targeted spending** But what if the government can affect the composition of spending by directly eliciting labor from different groups? How does the composition of spending affect the spending multiplier?

Note that the analysis above implicitly assumes that government purchases in goods produced from each member of group $g$ are given by:

$$G_{g,t} = \frac{\Gamma_g(Y_t)}{Y_t} G_t.$$

Suppose, instead, that the government can affect the composition of spending so that

$$G_{g,t} = \left( \frac{\Gamma_g(Y_t)}{Y_t} + \omega_g \right) G_t,$$

where $\omega_g$ are the targeting parameters which satisfy $\sum_g \pi_g \omega_g = 0$. This section investigates the impact of targeting via $\omega_g$ for the size of the spending multiplier. In doing so, I assume that this is a one-time unanticipated policy, so I keep the people's level of attention unchanged. See Remark 7 for a discussion.

In this case, the individual demand function (3.62) continues to hold, but now the individual's change in income is given by:

$$y_{g,t} = \gamma_g y_t + \omega_g G_t.$$

It follows that equilibrium output can be written as:

$$y_t = G_t + (1-\beta) \sum_{h=1}^{\infty} \beta^{h-1} \left[ (1 + \text{CD}) \cdot \overline{\lambda}^Y y_{t+h} + \left( \text{TC} \cdot \overline{\lambda}^Y - \overline{\lambda}^\tau \right) G_{t+h} \right] + \sigma \sum_{h=0}^{\infty} \beta^h r^n_{t+h}, \quad (3.67)$$

where $\text{TC} \equiv \mathbb{C}\text{ov} \left( \omega_g, \lambda^Y_g / \overline{\lambda}^Y \right)$ captures the covariance between the targeting parameters $\omega_g$ and the attention parameters $\lambda^Y_g$. Using this equation, we can compute the spending multiplier recursively using the following relationship:

$$\frac{dy_t}{dG_t} = 1 + (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\left[(1+\mathrm{CD})\cdot\overline{\lambda}^Y\frac{dy_{t+h}}{dG_{t+h}} + \mathrm{TC}\cdot\overline{\lambda}^Y - \overline{\lambda}^\tau\right]\frac{dG_{t+h}}{dG_t}. \tag{3.68}$$

Compared to equation (3.65), equation (3.68) displays a new term, $\mathrm{TC} \equiv \mathbb{C}\mathrm{ov}\left(\omega_g, \lambda_g^Y/\overline{\lambda}^Y\right)$, which captures the covariance between the targeting parameters $\omega_g$ and the level of attention of the group $\lambda_g^Y$. This term captures the fact that by changing the composition of spending, the government increases the incomes of certain groups more than others. To the extent that these groups have different levels of attention, they will also react heterogeneously to this income increase. People with higher income cyclicality are more attentive and will react more to the increase in income which results from higher spending. Instead, people with lower income cyclicality are less attentive and will react less to the increase in income which results from higher spending.

It follows that if the government targets the most cyclical/most attentive workers, i.e., if $\mathrm{TC} > 0$, then spending increases the income of people with high cyclicality and attention. Because these workers are more attentive, it follows that they will increase consumption by more in response to higher spending, leading to a larger spending multiplier than without targeting. Instead, if the government targets the most cyclical workers, i.e., if $\mathrm{TC} < 0$, then the opposite happens, and the resulting spending multiplier is lower.

**Proposition 13.** *Suppose that $dG_t = \rho_G^t dG_0$, then the government-spending multiplier is given by:*

$$\frac{dy_t}{dG_t} = \frac{dy_t^u}{dG_t} + \frac{\varrho_G\cdot TC\cdot\overline{\lambda}^Y}{1-\varrho_G\cdot(1+CD)\cdot\overline{\lambda}^Y}, \tag{3.69}$$

*where $\frac{dy_t^u}{dG_t}$ denotes the untargeted government-spending multiplier, defined in equation (3.66). It follows that:*

1. *With homogeneous attention, $\lambda_g^Y = \overline{\lambda}^Y$, then targeting is irrelevant since $TC = 0$.*

2. *With heterogeneous attention, the government-spending multiplier increases if the government targets the most cyclical workers.*

Proposition 13 computes the government-spending multiplier with targeting. It shows that the government spending is equal to the untargeted spending multiplier plus an additional term which accounts for how targeting correlates with heterogeneous attention.

With homogeneous attention or with FIRE, targeted spending does not affect the spending multiplier. In this model, since all workers share the same marginal propensity to consume out of income, then targeting would be irrelevant.[20] Instead, with disagreement, targeting the most attentive workers increases the government-spending multiplier. This result follows from the fact that targeting highly attentive workers magnifies the positive effect that government spending has on aggregate demand.

The spending multiplier exceeds the one obtained under FIRE if and only if:

$$\mathrm{TC} \geq \frac{\overline{\lambda}^{\tau} - (1 + \mathrm{CD}) \cdot \overline{\lambda}^{Y}}{\overline{\lambda}^{Y}}. \tag{3.70}$$

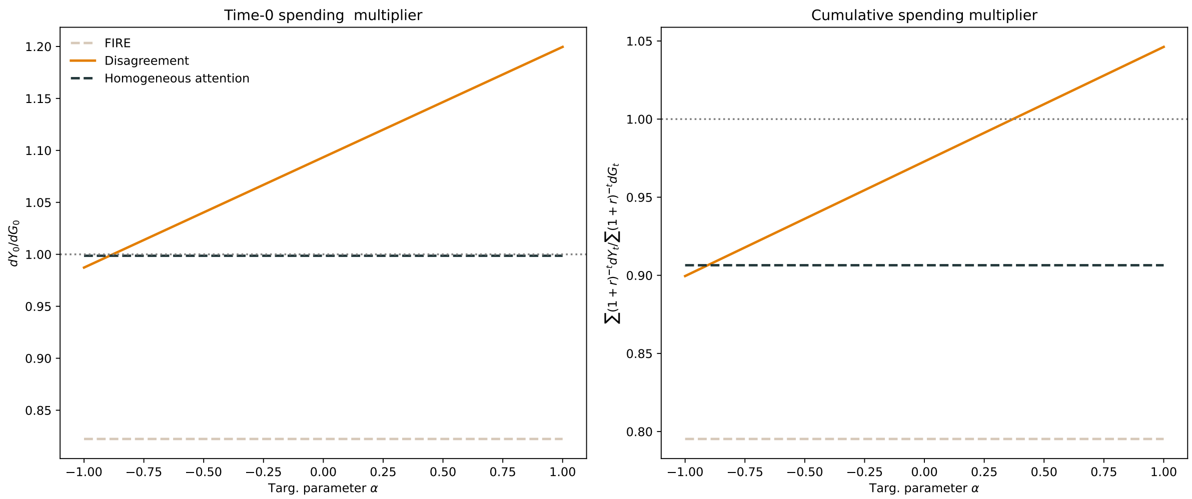
A large multiplier is possible if the government targets the most attentive workers. Note that, even if there is full attention to taxes $\overline{\lambda}^{\tau} = 1$, it is possible to obtain a multiplier exceeding the FIRE multiplier by appropriately targeting spending.

**Remark 7.** *The analysis in this section assumes that attention is not reoptimized after the government policy change. So, this section can best be considered analyzing a one-time unanticipated policy. If the government were to adopt a policy that would systematically alter people's income processes, they would eventually reoptimize their levels of attention in a way that may affect the conclusions derived in this section.*

[20]Instead, if marginal propensities to consume are heterogeneous across groups, targeting would affect the spending multiplier in a way that is similar to how heterogeneous attention affects the spending multiplier. See Baqaee and Farhi (2018) and Flynn et al. (2021).

### 3.5.2 Targeted spending in the quantitative model

Figure 3.9: Targeted spending multipliers




*Notes:* This figure shows the impact and cumulative spending multiplier as a function of the targeting parameter $\alpha$ for the baseline economy, FIRE, and for the economy with homogeneous attention. See text for more details.

In this section, I evaluate the quantitative implications of targeted spending for the government-spending multiplier. I extend the model in Section 3.4 to allow for targeted spending in the same way as in the simple model above. This assumption implies that:

$$dY_{g,t} = \gamma_g dY_t + \omega_g dG_t, \tag{3.71}$$

where $\omega_g$ captures the targeting parameters which satisfy $\sum_g \pi_g \omega_g = 0$. Furthermore, I assume that

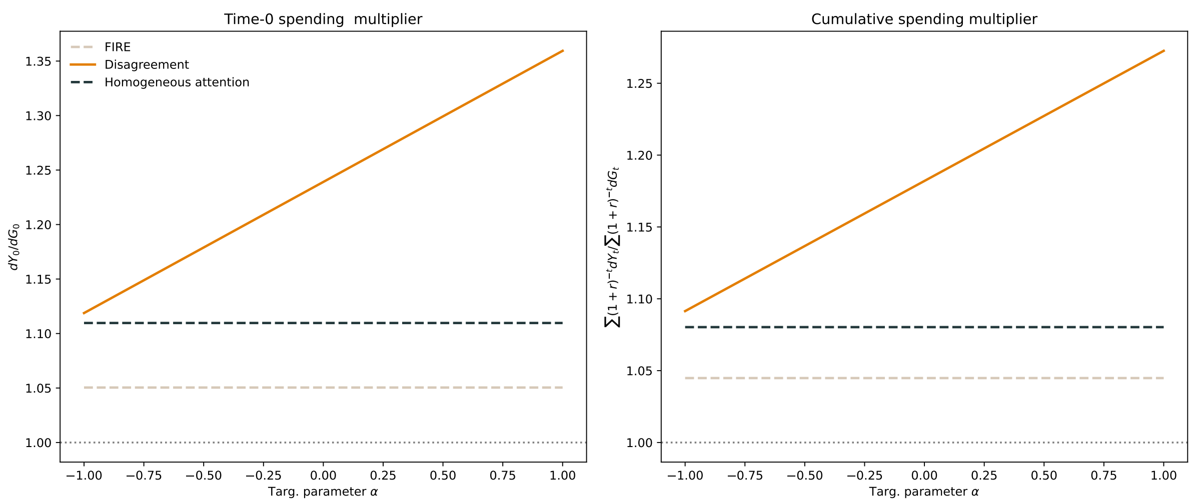
$$\omega_g = \alpha \cdot (\gamma_g - 1). \tag{3.72}$$

This expression implies that the targeting of a particular group $g$ is proportional to the difference between their income cyclicality, $\gamma_g$, to the average income cyclicality 1. The proportionality parameter, $\alpha$, captures the strength of targeting in this policy. If $\alpha >$

0, spending targets the most cyclical groups, while if $\alpha < 0$, spending targets the least cyclical groups. The higher the targeting parameter, the higher the level of targeting to high-cyclicality workers.

Figure 3.9 displays the government-spending multiplier as a function of the targeting parameter $\alpha$. The persistence of government spending is calibrated in the same way as in Section 3.4. I plot the impact and cumulative multipliers on the left and right panels, respectively, for the baseline economy, FIRE, and the counterfactual economy with homogeneous attention. The impact multiplier is defined as $dY_0/dG_0$ while the cumulative multiplier is defined as $\sum_t (1+r)^{-t} dY_t / \sum_t (1+r)^{-t} dG_t$.

Figure 3.10: Targeted spending multipliers: The role of monetary policy




*Notes:* This figure shows the impact and cumulative spending multiplier as a function of the targeting parameter $\alpha$ assuming that the Taylor parameter is $\phi_\pi = 1.1$ instead of $\phi_\pi = 1.5$. See text for more details.

Figure 3.9 shows that under homogeneous attention or under FIRE, the spending multiplier is not affected by targeting. This result follows directly from the fact that group heterogeneity does not affect aggregate outcomes in this economy without belief heterogeneity. Instead, the spending multiplier is increasing in the targeting parameter in the economy with disagreement. The more spending targets highly cyclical workers, the larger the spending multiplier. In this model, we see that moving from $\alpha = -1$ to $\alpha = 1$ increases the impact spending multiplier by more than 0.20 and similarly for the cumulative multiplier.

Figure 3.10 redoes the same exercise but assumes a weaker response of monetary policy to inflation, $\phi_\pi = 1.1$. As in the analysis of business-cycle amplification, Figure 3.10 shows that the results for the spending multiplier are magnified if the monetary policy response is weaker.

These results show that the government-spending multiplier can depend substantially on which groups see their incomes rising. The power of targeted spending is higher the more accommodative monetary policy is.

## 3.6 Conclusion

This paper studies the aggregate implications of belief disagreement for the transmission of business cycles and fiscal spending policy. In particular, I study the impact of belief disagreement in shaping how aggregate demand responds to macroeconomic shocks and policies. I conduct this analysis through the lens of standard New Keynesian models with two sources of heterogeneity: heterogeneous beliefs about future income and heterogeneous income cyclicality.

The results establish the determinant role of *correlated disagreement* (CD) in shaping aggregate demand. This statistic summarizes the covariance between individual income cyclicality and heterogeneity in the response of beliefs about future income. In other words, CD summarizes the covariance between exposure and attention to shocks. I show that CD affects the general-equilibrium channel from higher future income, feeding into an expansion of aggregate demand contemporaneously, i.e., the effective marginal propensity to consume (MPC) out of future aggregate income. I show that when CD is positive, this channel is magnified relative to a counterfactual economy without heterogeneous beliefs but the same average level of attention, i.e., the effective MPC out of future income is higher. Instead, when CD is negative, the channel is dampened relative to that counterfactual economy, i.e., the effective MPC out of future income is lower.

When CD is positive, business-cycle shocks can be amplified relative to the homogeneous attention economy. I show that amplification is more significant the more persistent shocks are. This result follows from the fact that more persistence shocks attribute higher quantitative importance to expectations about the future in determining consumption and savings decisions. Instead, when CD is negative, these results are reversed.

I then endogenize beliefs via behavioral inattention as in Gabaix (2014). This model allows us to establish theoretical predictions for the sign of correlated disagreement. I show that endogeneizing beliefs implies that the sign of correlated disagreement is positive because people who are more exposed to the shock choose to pay more attention. Because they are more exposed, people with higher income cyclicality see their incomes varying more following changes in macroeconomic conditions. So, the benefit of paying attention to these shocks is higher for these individuals. It follows that attention is positively related to income exposure, implying a positive sign for correlated disagreement. I show that this implication has empirical support. Using survey data on beliefs, I compute average forecast errors in predicting income growth at the state level. I show that the magnitude of these forecast errors decreases with state average income cyclicality. Through the lens of the model, this result must be a consequence of rising levels of attention as income exposure increases.

I quantify the relevance of this propagation mechanism in a quantitative model in the Heterogeneous Agent New Keynesian tradition with countercyclical income risk, incomplete markets, and borrowing constraints. I leverage the recent computational advances to show how the model can be written in a computationally tractable way despite the large extent of income, wealth, and belief heterogeneity. I show that the correlated disagreement mechanism can lead to substantial business cycle amplification. For example, an oil shock may be propagated by as much as 16 percent. The amplification of oil shocks rises above 30 percent if monetary policy is less reactive to inflation.

Finally, I turn to a fiscal policy application. I analyze how the composition of government spending affects the fiscal spending multiplier. I show that this multiplier is higher the more the government targets workers with more cyclical incomes. If government spending targets the services of high-income cyclicality workers, it will increase the income of the most attentive people. Because they are more attentive, these workers respond more to the increase in incomes generated by government spending, leading to a more significant increase in aggregate demand than would occur if spending was targeted towards people with low income cyclicality. It follows that the spending multiplier becomes larger by targeting highly-attentive individuals. I show that, quantitatively, the differences in spending multipliers can be quite substantial as a function of the level of targeting.

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# Appendix A

# Appendix to Chapter One

## A.1 Appendix to section 1.2

### A.1.1 Proof of proposition 1

We can solve for the government spending multiplier using

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t},$$

where the level-1 government spending multiplier is given by

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+s}}{\Delta G_t}$$

Suppose that $0 \leq \eta < 1$. Note that since $\Delta G_{t+s}/\Delta G_t > 0$, then $\Delta Y_t^1/\Delta G_t \leq 1$ for all $t$. By induction, suppose that $\Delta Y_t^{k-1}/\Delta G_t \leq 1$ for all $t$, then

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \underbrace{\frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1}_{\leq 0} \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1,$$

for all $t$. The first result follows.

Furthermore, if $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \geq 0$ for all $t$, then

$$\frac{\Delta Y_t^1}{\Delta G_t} = \left\{ 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\} + \eta \left\{ \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\} > \eta$$

for all $t$. Note that, with this assumption,

$$\frac{\Delta Y_t^2}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^1}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 - \Omega_t \sum_{s=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^1}{\Delta G_t}.$$

By induction, suppose that $\Delta Y_t^k / \Delta G_t \geq \Delta Y_t^{k-1} / \Delta G_t$, then

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_t^k}{\Delta G_t} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_t^{k-1}}{\Delta G_t} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

Then the second result follows.

Now, suppose that $\eta = 1$, then

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+s}}{\Delta G_t} = 1.$$

It then follows that if $\Delta Y_t^{k-1} / \Delta G_t = 1$ for all $t$, then

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} [1 - 1] \frac{\Delta G_{t+s}}{\Delta G_t} = 1.$$

Suppose that $\eta > 1$. Note that since $\Delta G_{t+s} / \Delta G_t > 0$, then $\Delta Y_t^1 / \Delta G_t \geq 1$ for all $t$. By induction, suppose that $\Delta Y_t^{k-1} / \Delta G_t \geq 1$ for all $t$, then

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \underbrace{\frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1}_{\geq 0} \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1,$$

for all $t$. The first result follows.

Furthermore, if $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \geq 0$ for all $t$, then

$$\frac{\Delta Y_t^1}{\Delta G_t} = \left\{ 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\} + \eta \left\{ \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\} < \eta$$

for all $t$. Note that, with this assumption,

$$\frac{\Delta Y_t^2}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^1}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1 - \Omega_t \sum_{s=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^1}{\Delta G_t}.$$

By induction, suppose that $\Delta Y_t^k / \Delta G_t \leq \Delta Y_t^{k-1} / \Delta G_t$, then

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_t^k}{\Delta G_t} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_t^{k-1}}{\Delta G_t} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

Then the second result follows.

### A.1.2 Proof of proposition 2

(1) As we show in the main text, for any level of cognitive sophistication, setting

$$1 + \tau_{T-1} = (1 + \tau)\, e^{-(\chi - \rho)} \tag{A.1}$$

implements $Y_{T-1}^k = 1$ for all $k$. Note that for any $t$ and $k$, the equilibrium level of output at time $t$ is a function only of current and future consumption taxes plus beliefs about future output:

$$Y_t = \left( \frac{1 + \tau^c}{1 + \tau_t^c} \right)^{\sigma} \frac{(1 - \beta) \sum_{s=1}^{T-t-1} \left( \frac{1+\tau_{t+s}^c}{1+\tau^c} \right) Y_{t+s}^e + 1}{(1 - \beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi - \rho)s} \left[ \frac{1+\tau_{t+s}^c}{1+\tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi - \rho)}}.$$

As a result, for any cognitive level $k$, $Y^{e,k}_{t+s}$ is independent of $\tau_t$. This means that, for a fixed $k$, we can construct the policy as follows.

Set $\tau_{T-1}$ to the value implied by (A.1). Then, proceed recursively from that date. For each $t \leq T-2$, fix $\tau_{t+s}$ for $s \geq 1$. These imply a path for $Y^{k-1}_{t+s}$ for $s \geq 1$. Let us choose $\tau_t$ so that

$$\left(\frac{1+\tau^c}{1+\tau^c_t}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau^c_{t+s}}{1+\tau^c}\right)Y^{e,k}_{t+s}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau^c_{t+s}}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}=1$$

or, equivalently,

$$1+\tau^c_t=(1+\tau^c)\left(\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau^c_{t+s}}{1+\tau^c}\right)Y^{e,k}_{t+s}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau^c_{t+s}}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}\right)^{1/\sigma}.$$

This implies that

$$Y^k_t=1$$

for all $t$.

(2) Suppose that $Y^{e,1}_t=1$. Then,

$$Y^1_t=\left(\frac{1+\tau^c}{1+\tau^{c,*}_t}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau^{c,*}_{t+s}}{1+\tau^c}\right)Y^{e}_{t+s}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau^{c,*}_{t+s}}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}=1.$$

This implies that $Y^{e,k}_t=Y^{k-1}=1$ for all $k$, and then $Y^k_t=1$ for all $t$ and $k$.

### A.1.3 Rules-based equilibrium

Under a rules-based policy, the temporary equilibrium is given by

$$\mathcal{Y}_t\left(\{Y^e_{t+s}\}\right)=\frac{\sum_{s=1}^{T-t-1}Q^e_{t,t+s}\left(\frac{1+\tau^{c,e}_{t+s}}{1+\tau^{c,e}_t}\right)Y^e_{t+s}+\sum_{s=T-t}^{\infty}Q^e_{t,t+s}\left(\frac{1+\tau^{c,e}_{t+s}}{1+\tau^{c,e}_t}\right)}{\sum_{s=1}^{T-t-1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^\sigma\left[Q^e_{t,t+s}\frac{1+\tau^{c,e}_{t+s}}{1+\tau^{c,e}_t}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}\sum_{s=T-t}^{\infty}\beta^{\sigma(s-(T-t))}\left[Q^e_{t,t+s}\frac{1+\tau^{c,e}_{t+s}}{1+\tau^{c,e}_t}\right]^{1-\sigma}}$$

where

$$Q^e_{t,t+s}\frac{1+\tau^{c,e}_{t+s}}{1+\tau^{c,e}_t}=\begin{cases}\beta^s\prod_{\tau=t}^{t+s-1}\left(Y^e_\tau\right)^{-\phi_y} & \text{if } s\leq T-t-1\\ \beta^s\prod_{\tau=t}^{T-1}\left(Y^e_\tau\right)^{-\phi_y} & \text{if } s\geq T-t.\end{cases}$$

Assuming that $Y^{e,1}_t=1$, implies that

$$Q^e_{t,t+s}\frac{1+\tau^{c,e}_{t+s}}{1+\tau^{c,e}_t}=\beta^s$$

for all $t$ and $s$. This implies that,

$$\mathcal{Y}_t\left(\left\{Y^{e,1}_{t+s}\right\}\right)=\frac{e^{-\frac{\sigma\chi}{1+\sigma\phi_y}}}{\left[(1-\beta)\,\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}}+e^{(T-t-1)(\sigma\chi-\rho)}\right]^{\frac{1}{1+\sigma\phi_y}}},$$

or in logs:

$$y^1_t\equiv\log Y^1_t=-\frac{\sigma\chi+\varphi_t}{1+\sigma\phi_y},$$

where $\varphi_t\equiv\log\left((1-\beta)\,\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}}+e^{(T-t-1)(\sigma\chi-\rho)}\right)$.

### A.1.4 Proof of proposition 3

**Targets-based policy** Note that, under rational expectations, the targets based policy with $\{\tau^{c,r}_t\}$ implements the same equilibrium

$$y^*_t=-\frac{\chi}{\phi_y}\left[1-\frac{1}{\left(1+\sigma\phi_y\right)^{T-t}}\right]<0.$$

Now, suppose that the government announces the sequence of policies $R_t = 1$ for $t \leq T-1$, $R_t = \beta^{-1}$ for $t \geq T$, and $\{\tau_t^{c,r}\}$. Then, the level-1 equilibrium is given by

$$y_t^1 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}} e^{y_{t+s}^{e,1}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\},$$

where $y_t^{e,1} \equiv \log Y_t^{e,1}$. Since $y_t^{e,1} = 0 \geq y_t^*$, then

$$y_t^1 \geq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}} e^{y_{t+s}^{*}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^*.$$

**Rules-based policy** The basic proof is constructed as follows. First, we note that if $\chi = 0$, then $y_t^* = y_t^1 = 0$. Second, we show that both $y_t^*$ and $y_t^1$ are decreasing in $\chi$. Third, $y_t^*$ is linear in $\chi$, while $y_t^1$ is concave in $\chi$. Fourth, we show that $dy_t^1/d\chi < dy_t^*/d\chi$ as long as $\beta \geq (1+\sigma\phi_y)^{-1}$. The collection of these results finally implies that

$$y_t^1 \leq \frac{dy_t^1}{d\chi}|_{\chi=0}\cdot\chi \leq \frac{dy_t^*}{d\chi}|_{\chi=0}\cdot\chi = y_t^*.$$

Log-output under rational expectations in the rules equilibrium is given by:

$$y_t^* = -\frac{\chi}{\phi_y}\left[1 - \frac{1}{(1+\sigma\phi_y)^{T-t}}\right],$$

and the level-1 equilibrium is given by:

$$y_t^1 = -\frac{\sigma\chi + \log\left((1-\beta)\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}} + e^{(T-t-1)(\sigma\chi-\rho)}\right)}{1+\sigma\phi_y}.$$

(1) For any $t$, if the shock is zero then output stays at steady state, i.e., if $\chi = 0$, then

using the expressions above it is clear that

$$y_t^* = y_t^1 = 0.$$

(2) Furthermore, the effects of $\chi$ on $y_t^*$ and $y_t^1$ are given by

$$\frac{dy_t^*}{d\chi} = -\frac{1}{\phi_y}\left[1 - \frac{1}{(1+\sigma\phi_y)^{T-t}}\right] < 0,$$

and since $\frac{1-\left(e^{\sigma\chi-\rho}\right)^{T-t-1}}{1-e^{\sigma\chi-\rho}} = \sum_{s=0}^{T-t-2} e^{s(\sigma\chi-\rho)}$, we can write

$$\frac{dy_t^1}{d\chi} = -\frac{\sigma}{1+\sigma\phi_y}\left[1 + \frac{(1-\beta)\sum_{s=0}^{T-t-2} se^{s(\sigma\chi-\rho)} + (T-t-1)\left(e^{\sigma\chi-\rho}\right)^{T-t-1}}{(1-\beta)\sum_{s=0}^{T-t-2} e^{s(\sigma\chi-\rho)} + \left(e^{\sigma\chi-\rho}\right)^{T-t-1}}\right] < 0. \quad \text{(A.2)}$$

(3) The rational-expectations equilibrium in this economy is exactly log-linear as a function of the shock, which implies that

$$y_t^* = \left\{\frac{dy_t^*}{d\chi}|_{\chi=0}\right\} \cdot \chi.$$

However, the same is not true under bounded rationality. To show this note that, for $t \leq T-2$,

$$\frac{d^2y_t^1}{d\chi^2} = -\frac{\sigma^2}{1+\sigma\phi_y}\left[\frac{(1-\beta)\sum_{s=0}^{T-t-2} s^2e^{s(\sigma\chi-\rho)} + (T-t-1)^2\left(e^{\sigma\chi-\rho}\right)^{T-t-1}}{\overline{\mu}_t}\right]$$
$$+ \frac{\sigma}{1+\sigma\phi_y}\frac{\left\{(1-\beta)\sum_{s=0}^{T-t-2} se^{s(\sigma\chi-\rho)} + (T-t-1)\left(e^{\sigma\chi-\rho}\right)^{T-t-1}\right\}^2}{\overline{\mu}_t^2}$$

where $\overline{\mu}_t \equiv (1-\beta)\sum_{s=0}^{T-t-2} e^{s(\sigma\chi-\rho)} + \left(e^{\sigma\chi-\rho}\right)^{T-t-1} > 0$. Define $\mu_{t,s} \equiv \frac{(1-\beta)e^{s(\sigma\chi-\rho)}}{\overline{\mu}_t}$ if $s < T-t-1$ and $\mu_{t,T-t-1} \equiv \frac{e^{(T-t-1)(\sigma\chi-\rho)}}{\overline{\mu}_t}$, and note that: $\mu_{t,s} > 0$, $\sum_{s=0}^{T-t-1} \mu_{t,s} = 1$. Using these definitions, we can rewrite the derivative as follows:

$$\frac{d^2 y_t^1}{d\chi^2} = -\frac{\sigma^2}{1+\sigma\phi_y}\left\{\sum_{s=0}^{T-t-1}\mu_{t,s}s^2 - \left(\sum_{s=0}^{T-t-1}\mu_{t,s}s^2\right)^2\right\}.$$

$$= -\frac{\sigma^2}{1+\sigma\phi_y}\sum_{s=0}^{T-t-1}\mu_{t,s}\left(s - \sum_{s=0}^{T-t-1}\mu_{t,s}s\right)^2 < 0$$

This shows that log-output in the level-1 equilibrium is concave in $\chi$.

(4) Evaluating (A.2) at $\chi = 0$ we obtain:

$$\frac{dy_t^1}{d\chi}\Big|_{\chi=0} = -\frac{\sigma}{1+\sigma\phi_y} - \frac{\sigma}{1+\sigma\phi_y}\left[(1-\beta)\sum_{s=0}^{T-t-2} se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right].$$

We want to show that $\frac{dy_t^1}{d\chi}\Big|_{\chi=0} < \frac{dy_t^*}{d\chi}\Big|_{\chi=0}$, which is equivalent

$$-\frac{\sigma}{1+\sigma\phi_y}\left[1+(1-\beta)\sum_{s=0}^{T-t-2} se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] \leq -\frac{\sigma}{1+\sigma\phi_y}\left[1+\frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}\right]$$

$$\Leftrightarrow \left[(1-\beta)\sum_{s=0}^{T-t-2} se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] \geq \frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}$$

Define

$$\Delta_t \equiv \left[(1-\beta)\sum_{s=0}^{T-t-2} se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] - \frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}.$$

The desired inequality follows if $\Delta_t \geq 0$. First, let us note that this is true for $t = T-1$ because:

$$\Delta_{T-2} = \beta - \frac{1}{1+\sigma\phi_y} \geq 0,$$

by assumption. Then, for any $t \leq T-2$ note that:

$$\Delta_{t-1} - \Delta_t = \left[ (1-\beta) \sum_{s=0}^{T-t-1} s e^{-s\rho} + (T-t)\,\beta^{T-t} \right] - \frac{\sum_{s=0}^{T-t-1} \left(1+\sigma\phi_y\right)^{-s}}{1+\sigma\phi_y}$$
$$- \left[ (1-\beta) \sum_{s=0}^{T-t-2} s e^{-s\rho} + (T-t-1)\,\beta^{T-t-1} \right] + \frac{\sum_{s=0}^{T-t-2} \left(1+\sigma\phi_y\right)^{-s}}{1+\sigma\phi_y}$$

$$\Leftrightarrow \Delta_{t-1} - \Delta_t = (1-\beta)(T-t-1)\,\beta^{(T-t-1)} + (T-t)\,\beta^{T-t} - (T-t-1)\,\beta^{T-t-1} - \frac{\left(1+\sigma\phi_y\right)^{-(T-t-1)}}{1+\sigma\phi_y}$$

$$\Leftrightarrow \Delta_{t-1} - \Delta_t = \beta^{T-t} - \left(1+\sigma\phi_y\right)^{-(T-t)}.$$

This implies that, under the same assumption, $\Delta_{t-1} \geq \Delta_t$. Since $\Delta_{T-2} \geq 0$, it follows that $\Delta_t \geq \Delta_{T-2} \geq 0$ for all $t$ and the result follows. In addition, this logic also delivers the fact that $y_t^* - y_t^1$ decreases with $t$ and increases with $\chi$.

### A.1.5 Proof of proposition 4

As described above, for level-1 we find that $y_t^1 \geq y_t^*$. Furthermore,

$$y_t^1 = \log \left\{ \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}} e^{y_t^{e,1}}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}} \right\} \leq y_t^{e,1}.$$

Since $y_t^{e,k} = y_t^{k-1}$ and

$$y_t^k = \log \left\{ \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}} e^{y_{t+s}^{e,k}}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}} \right\},$$

then,

$$y_t^2 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,2}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\}$$

$$\leq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^1$$

with strict inequality if $t \leq T-2$. Also, because $y_t^{e,2} \geq y_t^*$ then

$$y_t^2 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,2}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\}$$

$$\geq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{*}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^*.$$

This shows that $y_t^2 \in [y_t^*, y_t^1]$, and $y_t^2 < y_t^1$ if $y_t^1 \neq y_t^*$,i.e., if $t \leq T-2$.

For each $k$, suppose that $y_t^{e,k} = y_t^{k-1} \in \left[y_t^*, y_t^{e,k-1}\right]$, with $y_t^{k-1} < y_t^{e,k-1}$ if $y_t^{e.k-1} \neq y_t^*$. Then,

$$y_t^k = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\}$$

$$\leq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k-1}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^{k-1},$$

with strict inequality if $y_{t+s}^{e,k} \neq y_{t+s}^*$ for some $s \geq 1$. Also,

$$y_t^k = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\}$$

$$\geq \log \left\{ \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau^{c,r}_{t+s}}{1+\tau^{c,r}_{t}} e^{y^*_{t+s}}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau^{c,r}_{t+s}}{1+\tau^{c,r}_{t}}\right]^{1-\sigma}} \right\} = y^*_t.$$

This shows that $y^k_t$ forms a decreasing sequence in $k$, $y^k_t \leq y^{k-1}_t$, and $y^k_t \to y^*_t$ as $k \to \infty$.

## A.2 Appendix: Bounded rationality – alternative models

In the benchmark model, we assume that people are standard level-$k$ thinkers. However, our results do not depend crucially on the specific assumptions underlying this model of bounded rationality. In this appendix, we show that the main results of our model continue to hold under alternative models of bounded rationality. We first derive the benchmark model under a *generalized level-k thinking* model based on Camerer et al. (2004). Second, we show that our results are also robust to assuming that people have *reflective expectations* as in García-Schmidt and Woodford (2019). Finally, we also show that our results hold under the *shallow reasoning* model of Angeletos and Sastry (2021). For simplicity, we show this for the benchmark model without inflation, but these same principles hold more generally.

### A.2.1 Generalized level-$k$ thinking

In this section, we show that our results for the standard level-$k$ thinking in the benchmark model go through in the generalized level-$k$ thinking model. We restrict our analysis to the case in which policies are announced as targets, since we already discuss the implications of this model under rules in the main text.

While in standard level-$k$ thinking, an individual with ability $k$ believes that everyone else is level $k-1$, the generalized model allows individuals to conjecture that the population is distributed across all lower cognitive levels. Formally, we assume that individuals with ability $k$ believe that a fraction $f_k(j)$ of the population is level $j = 0, 1, ..., k-1$. The reasoning process is initialized with some equilibrium if the economy is populated by level-0 agents, $Y_t^0$. For technical reasons, it is useful to define the beliefs $\left\{Y_t^{e,0}\right\}$ which justify $Y_t^0 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,0}\right\}_{s\geq 1}\right)$ for all $t$.

Level-1 agents believe that everyone is level 0, i.e., $f_1(0) = 1$, and so they believe that output is given by:

$$Y_t^{e,1} = Y_t^0.$$

The equilibrium in an economy where all individuals are level-1 is given by

$$Y_t^1 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,1}\right\}_{s\geq 1}\right).$$

Level-2 people believe that a fraction $f_2(0)$ and $f_2(1)$ are level 0 and 1, respectively. Under the assumptions discussed in section 1.2.2, we can write their beliefs as

$$Y_t^{e,2} = \sum_{j=0}^{1} f_2(j)\, Y_t^j.$$

More generally, the level-$k$ beliefs can be constructed recursively

$$Y_t^{e,k} = \sum_{j=0}^{1} f_2(j)\, Y_t^k.$$

We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. Let $\gamma_k \equiv f_k(k-1)$ for all $k$. Then assumption (1.29) implies that $f_k(j) = (1-\gamma_k) f_{k-1}(j)$ for $j \leq k-2$. We can write the expectation of level-$k$ individuals as follows:

$$Y_t^{e,k} = (1-\gamma_k)\, Y_t^{e,k-1} + \gamma_k Y_t^{k-1}. \tag{A.3}$$

Intuitively, the beliefs of a level-$k$ thinker are given by a weighted average of the beliefs of level $k-1$ agents and the temporary equilibrium that would arise under those beliefs. Standard level-$k$ thinking corresponds to the case of $\gamma_k = 1$. By varying $\gamma_k$, we can control the intensity of learning across level-$k$ iterations.

While the standard level-$k$ thinking model assumes that everyone is level $k$, the generalized level-$k$ thinking model also allows for heterogeneity cognitive abilities. We let $f(k)$ for $k = 0, 1, ...$ denote the share of individuals who are level $k$ in the economy. The observed equilibrium path is thus given by

$$Y_t = \sum_{k=0}^{\infty} f(k) Y_t^k. \tag{A.4}$$

**Government spending multipliers**

We continue to define the level-$k$ multiplier as $\Delta Y_t^k / \Delta G_t$ which is given by

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t},$$

where

$$\frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} = (1-\gamma_k) \frac{\Delta Y_{t+s}^{e,k-1}}{\Delta G_{t+s}} + \gamma_k \frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}}$$

for $k \geq 2$.The observed government spending multiplier is given by:

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t}.$$

Suppose that $\Delta Y_t^{e,1} / \Delta G_t = \Delta Y_{t+s}^0 / \Delta G_{t+s} = \eta$, this implies that

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+s}}{\Delta G_t}.$$

If $\eta < 1$, then $\Delta Y_t^1/\Delta G_t \leq 1$ which implies that$\Delta Y_t^{e,2}/\Delta G_t \leq 1$. For any $k$, if $\Delta Y_t^{e,k}/\Delta G_t \leq 1$ then $\Delta Y_t^k/\Delta G_t \leq 1$, which implies that $\Delta Y_t^{e,k+1}/\Delta G_t \leq 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} \leq 1.$$

If $\eta = 1$, then $\Delta Y_t^1/\Delta G_t = 1$ which implies that$\Delta Y_t^{e,2}/\Delta G_t = 1$. For any $k$, if $\Delta Y_t^{e,k}/\Delta G_t = 1$ then $\Delta Y_t^k/\Delta G_t = 1$ for all $k$, which implies that $\Delta Y_t^{e,k+1}/\Delta G_t = 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} = 1,$$

for all $f(k)$.

If $\eta > 1$, then $\Delta Y_t^1/\Delta G_t \geq 1$ which implies that$\Delta Y_t^{e,2}/\Delta G_t \geq 1$. For any $k$, if $\Delta Y_t^{e,k}/\Delta G_t \geq 1$ then $\Delta Y_t^k/\Delta G_t \geq 1$, which implies that $\Delta Y_t^{e,k+1}/\Delta G_t \geq 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} \geq 1.$$

Suppose that $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} > 0$. Note that:

$$\frac{\Delta Y_t^1}{\Delta G_t} = \left\{ 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\} + \eta \left\{ \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\}.$$

If $\eta < 1$, then $\Delta Y_t^1/\Delta G_t \geq \eta$ and $\Delta Y_t^{e,2}/\Delta G_t \geq \Delta Y_t^{e,1}/\Delta G_t = \eta$. This immediately implies that $\Delta Y_t^2/\Delta G_t \geq \Delta Y_t^1/\Delta G_t$. We now show that $\Delta Y_t^{e,k}/\Delta G_t$ and $\Delta Y_t^k/\Delta G_t$ are increasing in $k$. To see this, suppose that $\Delta Y_t^j/\Delta G_t \geq \Delta Y_t^{j-1}/\Delta G_t$ for all $j \leq k$ then this implies that $\Delta Y_t^{e,k+1}/\Delta G_t \geq \Delta Y_t^{e,k}/\Delta G_t$. Furthermore,

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{e,k+1}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

This shows that $\Delta Y_t^k/\Delta G_t$ is increasing in individual cognitive ability $k$. But the equilibrium spending multiplier depends on the full distribution $f(k)$. The analog statement to proposition 1 requires assumptions on the distribution $f(k)$. When comparing to economies, we say that one economy is strictly more sophisticated than another if its distribution of cognitive abilities first-order dominates the distribution of the second one. Formally, consider two economies with distributions $f^A(k)$ and $f^B(k)$. Suppose that $\sum_{s=0}^{k} f^A(s) \leq \sum_{s=0}^{k} f^B(s)$ for all $k$. Then, the government spending multiplier is higher in economy $B$ than economy $A$.

If $\eta > 1$, then $\Delta Y_t^1/\Delta G_t \leq \eta$ and $\Delta Y_t^{e,2}/\Delta G_t \leq \Delta Y_t^{e,1}/\Delta G_t = \eta$. This immediately implies that $\Delta Y_t^2/\Delta G_t \leq \Delta Y_t^1/\Delta G_t$. We now show that $\Delta Y_t^{e,k}/\Delta G_t$ and $\Delta Y_t^k/\Delta G_t$ are decreasing in $k$. To see this, suppose that $\Delta Y_t^j/\Delta G_t \leq \Delta Y_t^{j-1}/\Delta G_t$ for all $j \leq k$ then this implies that $\Delta Y_t^{e,k+1}/\Delta G_t \leq \Delta Y_t^{e,k}/\Delta G_t$. Furthermore,

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{e,k+1}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

This shows that $\Delta Y_t^k/\Delta G_t$ is increasing in individual cognitive ability $k$. But the equilibrium spending multiplier depends on the full distribution $f(k)$. The analog statement to proposition 1 requires assumptions on the distribution $f(k)$. When comparing to economies, we say that one economy is strictly more sophisticated than another if its distribution of cognitive abilities first-order dominates the distribution of the second one. Formally, consider two economies with distributions $f^A(k)$ and $f^B(k)$. Suppose that $\sum_{s=0}^{k} f^A(s) \leq \sum_{s=0}^{k} f^B(s)$ for all $k$. Then, the government spending multiplier is lower in economy $B$ than economy $A$.

**Consumption-tax policy**

The equilibrium in this economy is given by

$$Y_t = \left(\frac{1+\tau^c}{1+\tau_t^c}\right)^{\sigma} \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)\sum_{k=0}^{\infty} f(k)\, Y_{t+s}^{e,k} + 1}{(1-\beta)\sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}}.$$

As before, beliefs about future output $Y_{t+s}^{e,k}$ for any $k$ is only a function of future tax policy, which implies that the analog construction of tax policy $\tau_t^c$ implements $Y_t = 1$. Note, however, that this policy may now imply consumption heterogeneity across different cognitive levels, because they may have different beliefs about future output. As it turns out, this is not the case if $Y_t^{e,1} = 1$. We show this next.

Suppose now that $Y_t^{e,1} = 1$. Then, announcing the tax policy $\tau_t^{c,*}$ implies that $Y_t^1 = 1$. It then follows that $Y_t^{e,k} = Y_t^k = 1$ for all $k$. As a result,

$$Y_t = 1$$

for any $f(k)$. This shows that proposition 2 continues to hold.

## A.2.2 Reflective expectations

García-Schmidt and Woodford (2019) describe a different process of belief formation which they call *reflective expectations.* This process allows cognitive ability to vary continuously but is otherwise similar in spirit to level $k$. Indexing beliefs by the cognitive ability $n$, García-Schmidt and Woodford (2019) assume that beliefs evolve according to

$$\frac{dY_t^{e,n}}{dn} = Y_t^n - Y_t^{e,n},$$

for $n \geq 0$ and starting from the initial expectations $Y_t^{e,0}$, where $Y_t^n$ denotes the equilibrium in an economy with level-$n$ people.We use superscript $k$ to denote equilibria and beliefs under level-$k$ thinking and superscript $n$ to denote equilibria and beliefs under reflective expectations.

García-Schmidt and Woodford (2019) show that the beliefs of a level-$n$ individual with reflective expectations are equivalent to a convex combination of standard level-$k$ beliefs determined by a Poisson distribution with mean $n$, i.e.,

$$Y_t^{e,n} = \sum_{k=1}^{\infty} \frac{n^{k-1} e^{-n}}{(k-1)!} Y_t^{e,k}, \tag{A.5}$$

where $Y_t^{e,k}$ denote the beliefs that standard level-$k$ thinkers have, which we develop in section 1.2. Equation (A.5) can be used to analyze the relationship between the equilibrium properties of standard level-$k$ thinking and reflective expectations economies.

### Government spending multipliers

For the case of the government spending multiplier, the beliefs of a level $n$ individual can be computed from the beliefs under level-$k$ thinking as follows:

$$\frac{\Delta Y_t^{e,n}}{\Delta G_t} = \sum_{k=1}^{\infty} \frac{n^{k-1} e^{-n}}{(k-1)!} \frac{\Delta Y_t^{e,k}}{\Delta G_t}.$$

Suppose $\eta < 1$. Since $\Delta Y_t^k / \Delta G_t \leq 1$ for all $k$, then $\Delta Y_t^{e,n} / \Delta G_t \leq 1$ for all $n$. Also, since the level-$k$ multiplier increases with $k$, then so does the level-$n$ belief over the multiplier. Suppose $\eta = 1$. Since $\Delta Y_t^k / \Delta G_t = 1$ for all $k$, then $\Delta Y_t^{e,n} / \Delta G_t = 1$ for all $n$. Suppose $\eta > 1$. Since $\Delta Y_t^k / \Delta G_t \geq 1$ for all $k$, then $\Delta Y_t^{e,n} / \Delta G_t \geq 1$ for all $n$. Also, since the level-$k$ multiplier decreases with $k$, then so does the level-$n$ belief over the multiplier.

The equilibrium spending multiplier under reflective expectations is given by:

$$\frac{\Delta Y_t^n}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[ \frac{\Delta Y_{t+s}^{e,n}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t}.$$

This relationship follows directly from Lemma 1. If $\eta < 1$ then since $\Delta Y_t^{e,n}/\Delta G_t \leq 1$ for all $t$, then $\Delta Y_t^n/\Delta G_t \leq 1$ for all $t$. Also, since the $\Delta Y_t^{e,n}/\Delta G_t$ is increasing with $n$, then $\Delta Y_t^n/\Delta G_t$ is increasing in $n$. If $\eta = 1$ then since $\Delta Y_t^{e,n}/\Delta G_t = 1$ for all $t$, then $\Delta Y_t^n/\Delta G_t = 1$ for all $t$. If $\eta > 1$ then since $\Delta Y_t^{e,n}/\Delta G_t \geq 1$ for all $t$, then $\Delta Y_t^n/\Delta G_t \geq 1$ for all $t$. Also, since the $\Delta Y_t^{e,n}/\Delta G_t$ is decreasing with $n$, then $\Delta Y_t^n/\Delta G_t$ is decreasing in $n$.

### Consumption-tax policy

The temporary equilibrium with reflective expectations is given by:

$$Y_t^n = \left( \frac{1+\tau^c}{1+\tau_t^c} \right)^\sigma \frac{(1-\beta) \sum_{s=1}^{T-t-1} \left( \frac{1+\tau_{t+s}^c}{1+\tau^c} \right) Y_{t+s}^{e,n} + 1}{(1-\beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s} \left[ \frac{1+\tau_{t+s}^c}{1+\tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}},$$

where

$$\frac{dY_t^{e,n}}{dn} = Y_t^n - Y_t^{e,n}.$$

As it turns out, the results of Proposition 2 extend to the model with reflective expectations. We prove this result below.

Set $\tau_{T-1}^c$ to the value implied by (A.1). Then, proceed recursively from that date. For each $t \leq T-2$, fix $\tau_{t+s}^c$ for $s \geq 1$. These imply a path for $Y_{t+s}^{e,n}$ for $s \geq 1$. Let us choose $\tau_t^c$ so that

$$\left( \frac{1+\tau^c}{1+\tau_t^c} \right)^\sigma \frac{(1-\beta) \sum_{s=1}^{T-t-1} \left( \frac{1+\tau_{t+s}^c}{1+\tau^c} \right) Y_{t+s}^{e,n} + 1}{(1-\beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s} \left[ \frac{1+\tau_{t+s}^c}{1+\tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}} = 1$$

or, equivalently,

$$1+\tau_t^c = (1+\tau^c)\left(\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)Y_{t+s}^{e,k}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}\right)^{1/\sigma}.$$

This implies that

$$Y_t^n = 1$$

for all $t$.

Suppose that $Y_t^{e,0} = 1$ and

$$\tau_t^c = \tau_t^{c,*} = (1+\tau^c)\, e^{-(T-t)(\chi-\rho)} - 1.$$

Then,

$$Y_t^0 = \left(\frac{1+\tau^c}{1+\tau_t^{c,*}}\right)^{\sigma}\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^{c,*}}{1+\tau^c}\right)+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}} = 1$$

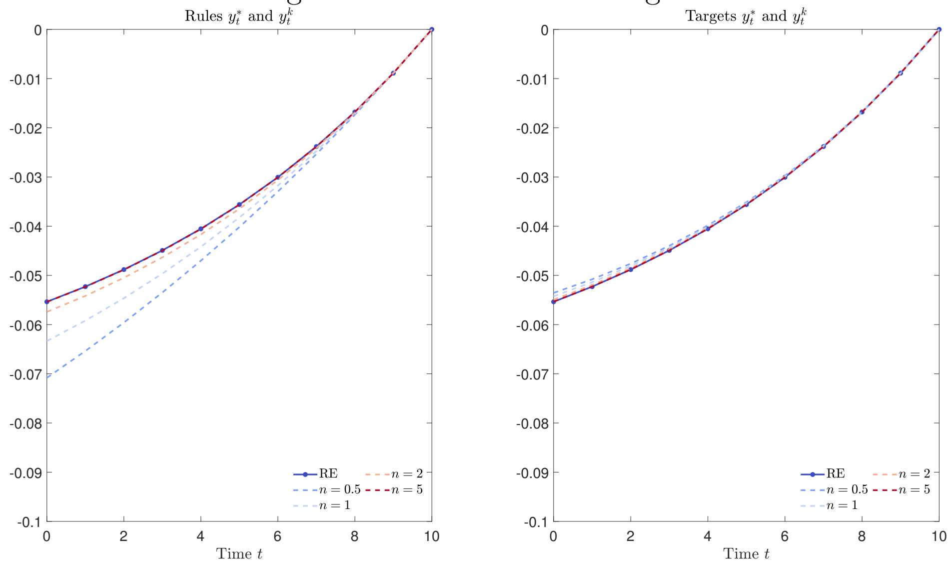
and

$$\frac{dY_t^{e,n}}{dn}|_{n=0} = Y_t^0 - Y_t^{e,0} = 1-1 = 0,$$

which implies that $dY_t^n/dn = 0$ for all $n$ and then $Y_t^n = Y_t^0 = 1$ for all $n$.

**Rules versus targets** Figure A.1 shows the reflective equilibria for different levels of $n$ both for rules-based communication and targets-communication in the left and right panels, respectively. Consistent with the results for the generalized level-$k$ model, output contracts more sharply for lower levels of cognitive ability. As highlighted by Angeletos and Sastry (2021), the peculiar oscillatory feature that is present under standard level-$k$ thinking does not arise under reflective expectations. We see that as cognitive ability rises, output converges to that under rational expectations. Also in line with the results in the baseline model,

Figure A.1: Rules versus targets




we see that, with targets, output contracts less with lower levels of cognitive sophistication and the level of output also converges to the rational expectations equilibrium as $n$ increases.

This confirms the claim in the paper that all the results in the benchmark model extend to the reflective expectations model.

### A.2.3 Shallow reasoning

Angeletos and Sastry (2021) describe a different process of belief formation which they refer to as *shallow reasoning*. In this model it is assumed that everyone is rational and attentive, knows that everyone else is rational but believe that only a fraction $\lambda$ are attentive to changes in the economic environment. For simplicity, we work with the linearized equilibrium relation. The consumption of individual $i$ can be written as follows:

$$c_{i,t} = (1-\beta) \sum_{s=0}^{T-1-(t-s)} \beta^s \frac{Y}{C} \left[\mathbb{E}_i y_{t+s} - g_{t+s}\right] - \sigma\beta \sum_{s=0}^{T-1-t} \beta^s \left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\},$$

where $\mathbb{E}_i[y_t]$ denotes individual $i$'s expectation of output. Lower-case letters denote log-deviations from steady-state values, except for $g_t = G_t/Y$. Market clearing requires $y_t = \frac{C}{Y}\int c_{i,t}di + g_t$. Individual $i$ fully understands that other individuals have the same policy function, conditional on their beliefs. Using the market clearing condition we can write

$$y_t = g_t + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\overline{\mathbb{E}}y_{t+s} - g_{t+s}\right] - \frac{C}{Y}\sigma\sum_{s=0}^{T-1-t}\beta^{s}\left\{r_{t+s} - \Delta\hat{\tau}^{c}_{t+s+1} + \chi_{t+s}\right\},$$

where $\overline{\mathbb{E}}[y_t] \equiv \int_0^1 \mathbb{E}_i[y_t]\,di$ denotes the average expectation in the economy. Let

$$\Psi_t \equiv g_t - (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}g_{t+s} - \frac{C}{Y}\sigma\beta\sum_{s=0}^{T-1-t}\beta^{s}\left\{r_{t+s} - \Delta\hat{\tau}^{c}_{t+s+1} + \chi_{t+s}\right\}$$

We can write

$$\boldsymbol{y} = (1-\beta)\,\boldsymbol{M}\overline{\mathbb{E}}[\boldsymbol{y}] + \boldsymbol{\Psi}$$

where

$$\boldsymbol{y} \equiv \begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_{T-1} \end{bmatrix}, \quad \boldsymbol{M} \equiv \begin{bmatrix} 0 & 1 & \beta & \dots & \beta^{T-1} \\ 0 & 0 & 1 & \dots & \beta^{T-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \boldsymbol{\Psi} \equiv \begin{bmatrix} \Psi_0 \\ \Psi_1 \\ \dots \\ \Psi_{T-1} \end{bmatrix}.$$

This implies that

$$\overline{\mathbb{E}}[\boldsymbol{y}] = (1-\beta)\,\boldsymbol{M}\overline{\mathbb{E}}^2[\boldsymbol{y}] + \overline{\mathbb{E}}[\boldsymbol{\Psi}],$$

where $\overline{\mathbb{E}}^h[\cdot] \equiv \overline{\mathbb{E}}\left[\overline{\mathbb{E}}^{h-1}[\cdot]\right]$. Note that the law of iterated expectations does not apply for the average expectation. Then, iterating on this relation and using the fact that $\boldsymbol{M}^h$ converges to a zero matrix as $h$ goes to infinity, we obtain

$$\overline{\mathbb{E}}[\boldsymbol{y}] = \sum_{h=1}^{\infty}\left\{(1-\beta)\,\boldsymbol{M}\right\}^{h-1}\overline{\mathbb{E}}^h[\boldsymbol{\Psi}].$$

Following Angeletos and Sastry (2021), the behavioral assumptions imply that $\overline{\mathbb{E}}^h[\boldsymbol{\Psi}] = \lambda^h \boldsymbol{\Psi}$, and so

$$\overline{\mathbb{E}}[\boldsymbol{y}] = \lambda \left[\boldsymbol{I} - (1-\beta)\boldsymbol{M}\lambda\right]^{-1} \boldsymbol{\Psi} = \lambda \boldsymbol{y},$$

where the last equality follows from the fact that

$$\begin{aligned}\boldsymbol{y} &= (1-\beta)\boldsymbol{M}\overline{\mathbb{E}}[\boldsymbol{y}] + \boldsymbol{\Psi} = (1-\beta)\boldsymbol{M}\lambda\left[\boldsymbol{I} - (1-\beta)\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi} + \boldsymbol{\Psi} \\ &= \left[\boldsymbol{I} - (1-\beta)\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi}\end{aligned}$$

As a result, we can write the equilibrium relation as:

$$y_t = g_t + (1-\beta)\sum_{s=1}^{T-1-t} \beta^{s-1}\left[\lambda y_{t+s} - g_{t+s}\right] - \frac{C}{Y}\sigma\beta \sum_{s=0}^{T-1-t} \beta^s \left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\}. \quad \text{(A.6)}$$

**Government spending multipliers**

Using the equilibrium relation (A.6), we find that the fiscal spending multiplier solves the following recursion:

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t} \beta^{s-1}\left[\lambda \frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t}. \quad \text{(A.7)}$$

For consistency with earlier results, the multiplier is expressed in terms of levels of $Y_t$ and $G_t$. As in the benchmark model, the date $T-1$ fiscal multiplier is the same as the rational expectations fiscal multiplier:

$$\frac{\Delta Y_{T-1}}{\Delta G_{T-1}} = 1.$$

This then implies that

$$\frac{\Delta Y_{T-2}}{\Delta G_{T-2}} = 1 - (1-\beta)\left[1-\lambda\right]\frac{\Delta G_{T-1}}{\Delta G_{T-2}}.$$

Since $\lambda < 1$, then $\Delta Y_{T-2}/\Delta G_{T-2} < 1$. As $\lambda \to 1$ then $\Delta Y_{T-2}/\Delta G_{T-2} \to 1$ which coincides with the rational expectations multiplier. We can also see that the fiscal multiplier is monotonically increasing in $\lambda$,

$$\frac{d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}}{d\lambda} = (1-\beta)\frac{\Delta G_{T-1}}{\Delta G_{T-2}} > 0,$$

so as $\lambda$ increases the multiplier gets closer to the rational expectations multiplier. Via standard inductive arguments these properties extend to all time $t$ multipliers. To see this result, note that for $\lambda < 1$, if $\Delta Y_{t+s}/\Delta G_{t+s} \leq 1$ for all $s \geq 1$ then

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t} < 1.$$

Furthermore,

$$\lim_{\lambda\to 1}\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lim_{\lambda\to 1}\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t} = 1$$

as long as $\lim_{\lambda\to 1}\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} = 1$. This result shows that all time $t$ spending multipliers converge to the rational expectations multipliers as $\lambda$ goes to one. Furthermore, under the assumption that

$$1 - (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\frac{\Delta G_{t+s}}{\Delta G_t} > 0, \tag{A.8}$$

we find that $\Delta Y_t/\Delta G_t > 0$ for all $t$. Differentiating (A.7) with respect to $\lambda$, we obtain:

$$\frac{d\frac{\Delta Y_t}{\Delta G_t}}{d\lambda} = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} + \lambda\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\lambda}\right]\frac{\Delta G_{t+s}}{\Delta G_t}.$$

Under assumption (A.8), we know that $\Delta Y_{t+s}/\Delta G_{t+s} > 0$. Then, if

$$\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\lambda} > 0,$$

then $d\frac{\Delta Y_t}{\Delta G_t}/d\lambda > 0$. Since we have shown that $d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}/d\lambda > 0$, then it is true that $d\frac{\Delta Y_t}{\Delta G_t}/d\lambda > 0$ for all $t$. This confirms that the shallow reasoning spending multiplier is increasing in the sophistication parameter $\lambda$.

Finally, suppose that $\Delta G_t = \zeta^t \Delta G_0$ for $\zeta > 0$, then

$$\frac{\Delta Y_{T-2}}{\Delta G_{T-2}} = 1 - (1-\beta)\left[1-\lambda\right]\zeta \Rightarrow d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}/d\zeta < 0$$

and

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t} \beta^{s-1}\left[\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\zeta^s. \tag{A.9}$$

$$d\frac{\Delta Y_t}{\Delta G_t}/d\zeta = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\zeta}\zeta^s + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\underbrace{\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1}_{<0}\right]s\zeta^{s-1} < 0$$

as long as $d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}/d\zeta < 0$. As a result, the spending multiplier is decreasing in the persistence of government spending.

### Consumption-tax policy

Suppose that $g_t = 0$ for all $t$ and for simplicity suppose that $Y = C$. Interest rates are at the ZLB for $t \leq T-1$, and go back to steady state levels for $t \geq T$:

$$r_t = \log R_t - \rho = \begin{cases} -\rho & \text{if } t \leq T-1 \\ 0 & \text{if } t \geq T. \end{cases}$$

Then, we find that for $t \geq T$ output is back to steady state $y_t = 0$. However, for $t \leq T-1$ output solves the fixed-point system of equations of $\{y_t\}_{t=0}^{T-1}$:

$$y_t = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s} - \sigma\sum_{s=0}^{T-1-t}\beta^s\left\{(\chi-\rho) - \left(\hat{\tau}^c_{t+s+1} - \hat{\tau}^c_{t+s}\right)\right\}. \tag{A.10}$$

Then, consider the policy that implements full stabilization under rational expectations:

$$1+\tau_t^c = (1+\tau^c)\,e^{-(T-t)(\chi-\rho)}$$

which implies that

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c} = e^{-(\chi-\rho)} \Rightarrow \hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \chi-\rho.$$

Replacing these consumption taxes in the equilibrium relation (A.10), we obtain

$$y_t = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s},$$

which implies that $y_t = 0$ for all $t$ is a shallow reasoning equilibrium under this policy. In sum, the same policy that implements the flexible-price allocation under rational expectations also implements the flexible-price allocation irrespective of the degree of rationality $\lambda$.

**Rules versus targets** Consider now the case in which policy is designed as rules, i.e., such that interest rates and consumption taxes are set so that

$$r_t = \max\left\{\phi_y y_t, -\rho\right\},$$

and

$$\hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \min\left\{\phi_y y_t + \rho, 0\right\}$$

which implies that:

$$r_t + \hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \phi_y y_t.$$

The shallow reasoning equilibrium is a solution to the fixed point system of equations given by:

$$y_t = -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s}.$$

As before, if $\lambda = 1$, then $y_t = -\frac{\chi}{\phi_y}\left[1-(1+\sigma\phi_y)^{-(T-t)}\right] = y^*_t < 0$ which is the rational expectations equilibrium. Furthermore, note that for $t = T-1$:

$$y_{T-1} = -\frac{\sigma\chi}{1+\sigma\phi_y} = y^*_{T-1} < 0$$

for any $\lambda$. Next, we show that, if $\beta > (1+\sigma\phi_y)^{-1}$, for $\lambda < 1$, $y_t < y^*_t$ for all $t \leq T-2$.

Output at time $t = T-2$ is given by

$$\begin{aligned} y_{T-2} &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\lambda y_{T-1} \\ &< -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right) y^*_{T-1} = y^*_{T-2}, \end{aligned}$$

which shows that $y_{T-2} < y^*_{T-2}$. Furthermore, we also find that $\lambda y_{T-2} > y^*_{T-2}$, which follows from the fact that:

$$\begin{aligned} \lambda y_{T-2} - y^*_{T-2} &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\left(\lambda^2 y_{T-1} - y^*_{T-1}\right) \\ &= (\lambda-1)\left\{-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)(\lambda+1)\, y^*_{T-1}\right\} \\ &> (\lambda-1)\left\{-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right) y^*_{T-1}\right\} = (\lambda-1)\, y^*_{T-2} > 0. \end{aligned}$$

Therefore, we find that $y_{T-2} < y^*_{T-2}$, but $\lambda y_{T-2} > y^*_{T-2}$, i.e., $y_{T-2} \in (\lambda^{-1}y^*_{T-2}, y^*_{T-2})$. For any $t$, suppose that $y_{t+s} \in (\lambda^{-1} y^*_{t+s}, y^*_{t+s}]$ for all $s = 1, ..., T-t-1$, then

$$\begin{aligned} y_t &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s} \\ &< -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\lambda^{-1} y^*_{t+s} = y^*_t. \end{aligned}$$

Furthermore, we also find that $\lambda y_t > y^*_t$, which follows from the fact that

$$\begin{aligned} \lambda y_t - y^*_t &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\left(\lambda^2 y_{t+s} - y^*_{t+s}\right) \\ &> -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\left(\lambda^2 y^*_{t+s} - y^*_{t+s}\right) \\ &= (\lambda-1)\left[-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}(\lambda+1)\, y^*_{t+s}\right] \\ &> (\lambda-1)\left[-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1} y^*_{t+s}\right] \\ &> (\lambda-1)\, y^*_t > 0. \end{aligned}$$

Then, by induction, we find that $y_t \in (\lambda^{-1} y^*_t, y^*_t]$, which shows that the stabilizing power of fiscal policy under rules becomes weaker.

Suppose now, that the policy is communicated as targets. We show that under targets-based communication $y_t \geq y^*_t$ for all $t$. First, using (A.10) we find that:

$$\lim_{\lambda\to 0} y_t = -\sigma \sum_{s=0}^{T-1-t}\beta^s\left\{(\chi-\rho) - \left(\hat{\tau}^{c,r}_{t+s+1} - \hat{\tau}^{c,r}_{t+s}\right)\right\} < 0,$$

and

$$\frac{dy_{T-2}}{d\lambda} = (1-\beta)\, y^*_{T-1} < 0 \Rightarrow y_{T-2} < 0,$$

for all $\lambda$. Now, note that

$$\frac{dy_t}{d\lambda} = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}y_{t+s} + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\frac{dy_{t+s}}{d\lambda}.$$

So, as long as $y_{t+s} \leq 0$ and $dy_{t+s}/d\lambda \leq 0$ for all $s \geq 1$, with one strict inequality, then we find that $dy_t/d\lambda < 0$ and $y_t < 0$. Furthermore, to show that $y_t > y_t^*$, note that

$$y_t - y_t^* = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left\{\lambda y_{t+s} - y_{t+s}^*\right\}.$$

As before, this implies that $y_{T-1} = y_{T-1}^*$. Now, evaluating time $t = T-1$, we see that

$$y_{T-2} - y_{T-2}^* = (1-\beta)\left\{\lambda - 1\right\}y_{T-1}^* > 0 \Rightarrow y_{T-2} > y_{T-2}^*.$$

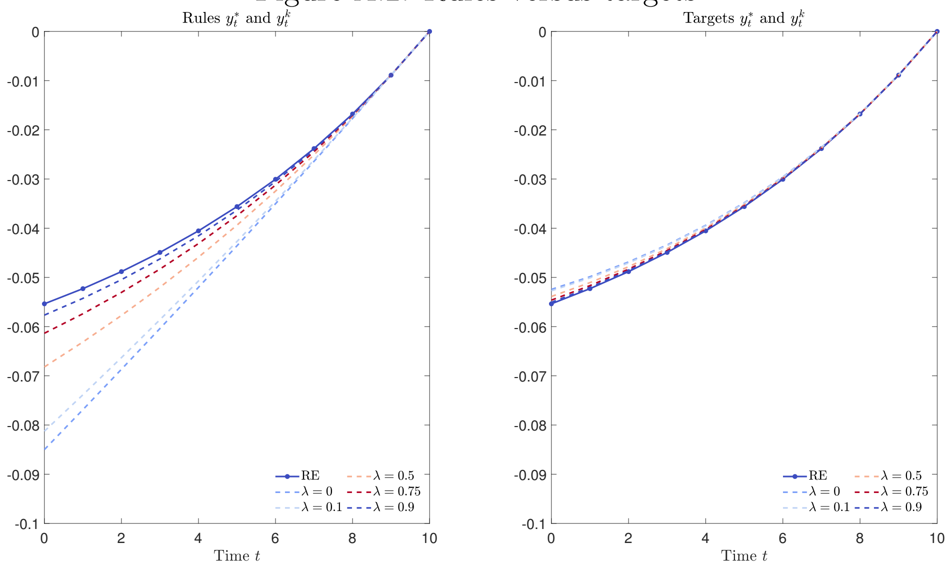
This result serves as the base for the inductive argument. Suppose that $0 > y_{t+2} > y_{t+s}^*$ for all $s$, then

$$y_t - y_t^* = \sum_{s=1}^{T-1-t}\beta^{s-1}\left\{\lambda y_{t+s} - y_{t+s}^*\right\} > 0.$$

Figure A.2 shows the equilibrium path for log-output in the economy with shallow reasoning for different levels of $\lambda$. As highlighted by Angeletos and Sastry (2021), the peculiar oscillatory feature that is present under simple level-$k$ thinking does not arise under reflective expectations. We see that as cognitive ability rises, output converges to that under rational expectations. Also in line with the results in the baseline model, we see that, with targets, output contracts less with lower levels of cognitive sophistication and the level of output also converges to the rational expectations equilibrium as $\lambda$ increases.

This confirms the claim in the paper that all the results in the benchmark model extend to the shallow reasoning model.

Figure A.2: Rules versus targets




# A.3 Appendix to section 1.3

## A.3.1 Consumption function

The household's optimal consumption plan satisfies:

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} \left\{ \left(1 - \tau^n_{t+s}\right) W^e_{t+s} N^e_{t+s} + \Omega^e_{t+s} - T^e_{t+s} \right\} + R_{t-1} B_t}{P_t \left(1 + \tau_t\right) \left[ 1 + \sum_{s\geq 1} \left( \beta^s \frac{\xi_{t+s}}{\xi_t} \right)^\sigma \left[ Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t(1+\tau_t)} \right]^{1-\sigma} \right]}.$$

Given their beliefs for output, the household's expectations for lump-sum taxes are given by 1.36. Replacing beliefs for lump-sum taxes, we obtain:

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} \left\{ W^e_{t+s} N^e_{t+s} + \tau^c_{t+s} P^e_{t+s} C^e_{t+s} + \Omega^e_{t+s} - P^e_{t+s} G^e_{t+s} \right\}}{P_t \left(1 + \tau^c_t\right) \left[ 1 + \sum_{s\geq 1} \left( \beta^s \frac{\xi_{t+s}}{\xi_t} \right)^\sigma \left[ Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t(1+\tau^c_t)} \right]^{1-\sigma} \right]}.$$

Using the fact that

$$Y^e_{t+s} = \frac{W^e_{t+s}}{P^e_{t+s}} N^e_{t+s} + \frac{\Omega^e_{t+s}}{P^e_{t+s}}$$

and

$$C^e_{t+s} = Y^e_{t+s} - G_{t+s}$$

we can write the consumption function as

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} P^e_{t+s} \left\{Y^e_{t+s} - G_{t+s} + \tau^c_{t+s}\left(Y^e_{t+s} - G_{t+s}\right)\right\}}{P_t\left(1+\tau^c_t\right)\left[1 + \sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\right]^{1-\sigma}\right]},$$

or equivalently

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\left[Y^e_{t+s} - G_{t+s}\right]}{1 + \sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\right]^{1-\sigma}},$$

## A.3.2 Unions and wage setting

In this appendix we solve the problem of the union and derive the wage equation 1.38. The problem of a union that gets to reset its wage is

$$\max_{w_{u,t},\{\tilde{n}_{u,t+s}\}} \sum_{s\geq 0} (\beta\lambda)^s \left\{u'\left(C^e_{t+s}\right)\frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}}\frac{w_{u,t}\tilde{n}_{u,t+s}}{P^e_{t+s}} - v'\left(L^e_{t+s}\right)\tilde{n}_{u,t+s}\right\}$$

subject to the constraint

$$\tilde{n}_{u,t+s} = \left(\frac{w_{u,t}}{W^e_{t+s}}\right)^{-\theta} N^e_{t+s}.$$

Because every union represents an infinitesimal number of workers in each household, the union does not directly affect aggregate consumption, $C_t$, hours worked by the household, $L_t$, the composite labor input, $N_t$, aggregate wages, $W_t$, and prices, $P_t$. As discussed in the main text, we assume that the union has rational expectations with respect to the exogenous variables, but is boundedly rational with respect to future endogenous variables.

The optimal reset wage $W_t^*$ solves the following first order condition:

$$\sum_{s\geq 0}(\beta\lambda)^s\left\{-(\theta-1)\,u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}\frac{W_t^*\left(\frac{W_t^*}{W_{t+s}^e}\right)^{-\theta}N_{t+s}^e}{P_{t+s}^e}+\theta v'\left(L_{t+s}^e\right)\left(\frac{W_t^*}{W_{t+s}^e}\right)^{-\theta}N_{t+s}^e\right\}=0$$

which can be equivalently written as follows:

$$\frac{W_t^*}{P_t}=\frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e v'\left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}.$$

### A.3.3 Sufficient conditions for equilibrium and the linearized system

Given beliefs, a temporary equilibrium denotes a solution to the following system of equations:

1. The consumption function

$$C_t=\frac{\sum_{s=1}^{\infty}Q_{t,t+s}\frac{P_{t+s}^e\left(1+\tau_{t+s}^c\right)}{P_t\left(1+\tau_t^c\right)}\left\{Y_{t+s}^e-G_{t+s}\right\}}{\sum_{s=1}^{\infty}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P_{t+s}^e\left(1+\tau_{t+s}^c\right)}{P_t\left(1+\tau_t^c\right)}\right]^{1-\sigma}},$$

   where we have imposed market clearing, $C_t=Y_t-G_t$.

2. Unions optimal wage setting

$$\frac{W_t^*}{P_t}=\frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e v'\left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}$$

and the aggregate wage is

$$W_t = \left[\lambda W_{t-1}^{1-\theta} + (1-\lambda)\left(W_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

3. Real wages are equal to the marginal productivity of labor

$$\frac{W_t}{P_t} = (1-\alpha) A \left(\frac{\overline{K}}{N_t}\right)^{\alpha}.$$

4. Output is given by

$$Y_t = A\overline{K}^{\alpha} N_t^{1-\alpha},$$

where

$$L_t = \mu_t N_t$$

$$\mu_t = \int_0^1 \left(\frac{w_{u,t}}{W_t}\right)^{-\theta} du = \lambda \mu_{t-1}\left(\frac{W_{t-1}}{W_t}\right)^{-\theta} + (1-\lambda)\left(\frac{W_t^*}{W_t}\right)^{-\theta}$$

where $\mu_{-1} = 1$.

5. Market clearing

$$C_t + G_t = Y_t.$$

For each quantity and price $X_t$ we denote their log-linear deviation from steady state by $x_t \equiv \log X_t - \log X$, except for $g_t = G_t/Y$. For taxes we denote their log-linear deviation by $\hat{\tau}_t^c = \log(1+\tau_t^c) - \log(1+\tau^c)$ and $\hat{\tau}_t^n = -\{\log(1-\tau_t^n) - \log(1-\tau^n)\}$. Finally, $\log \xi_{t+1}/\xi_t = \chi_t$, where $\chi_t = \chi > 0$ for $t \leq T-1$ and $\chi_t = 0$ for $t \geq T$. The log-linear system can be written as follows.

Consumption is given by

$$c_t = \frac{(1-\beta)}{\beta}\sum_{s=1}^{\infty}\beta^s \frac{Y}{C}\left\{y^e_{t+s} - g_{t+s}\right\} - \sigma\sum_{s=0}^{\infty}\beta^s\left\{r_{t+s} - \pi^e_{t+s+1} - \left(\hat{\tau}^c_{t+s+1} - \hat{\tau}^c_{t+s}\right) + \chi_{t+s}\right\}. \tag{A.11}$$

Wage inflation $\pi^w_t = w_t - w_{t-1}$ is given by

$$\pi^w_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\sum_{s\geq 0}^{\infty}(\beta\lambda)^s\left\{\varphi n^e_{t+s} + \sigma^{-1}c^e_{t+s} + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} + \alpha n^e_{t+s}\right\} \tag{A.12}$$

$$+ \frac{1-\lambda}{\lambda}\sum_{s\geq 1}^{\infty}(\beta\lambda)^s\,\pi^{w,e}_{t+s}. \tag{A.13}$$

Below, we show how to derive these two equations below.

Price inflation $\pi_t = p_t - p_{t-1}$ is given by

$$\pi_t = \pi^w_t + \alpha\Delta n_t. \tag{A.14}$$

Finally, output is given by

$$y_t = (1-\alpha)\,n_t, \tag{A.15}$$

and the market clearing condition is

$$\frac{C}{Y}c_t + g_t = y_t. \tag{A.16}$$

To first order, $n_t = l_t$.

**Log-linearized wage inflation** Wage setting is given by

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta} \left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e v' \left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1} \left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e u' \left(C_{t+s}^e\right) \frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}$$

and the aggregate wage is

$$W_t = \left[\lambda W_{t-1}^{1-\theta} + (1-\lambda) \left(W_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Log-linearizing the wage setting condition we obtain

$$w_t^* - p_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c\right\}$$

$$+ \sum_{s=1}^{\infty} (\beta\lambda)^s p_{t+s}^e - \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} p_{t+s}^e$$

$$\begin{aligned} \Leftrightarrow w_t^* - w_t =& (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c\right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s p_{t+s}^e \\ &- \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} p_{t+s}^e + p_t - w_t \end{aligned}$$

or equivalently,

$$\begin{aligned} w_t^* - w_t =& (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c - \left(w_{t+s}^e - \hat{p}_{t+s}^e\right)\right\} \\ &+ \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e} \end{aligned}$$

since $w^e_{t+s} - \hat{p}^e_{t+s} = -\alpha n^e_{t+s}$ then

$$w^*_t - w_t = (1-\beta\lambda)\sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n^e_{t+s} + \sigma^{-1} c^e_{t+s} + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} + \alpha n^e_{t+s}\right\} + \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi^{w,e}_{t+s}.$$

Log-linearizing the aggregate wage condition we obtain

$$w_t = \lambda w_{t-1} + (1-\lambda) w^*_t.$$

Now, define $\pi^w_t = w_t - w_{t-1}$, we can use the equation above to show that

$$\lambda \pi^w_t = (1-\lambda)(w^*_t - w_t) \Leftrightarrow \pi^w_t = \frac{1-\lambda}{\lambda}(w^*_t - w_t).$$

Replacing $w^*_t - w_t$ we find that

$$\pi^w_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n^e_{t+s} + \sigma^{-1} c^e_{t+s} + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} + \alpha n^e_{t+s}\right\}$$

$$+\frac{1-\lambda}{\lambda}\sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi^{w,e}_{t+s}.$$

### A.3.4 Proof of proposition 5

**Part 1** The proof strategy is as follows. First, we show that if level-1 people believe that the economy will stay at steady state for $t \geq T$, then all level-$k$ beliefs and corresponding equilibria feature output, consumption, labor and wage inflation remaining at their steady state levels from $t \geq T$, and price inflation is zero for $t \geq T+1$. Second, we note that beliefs about future output, inflation, consumption, and labor are a function only of future tax rates and policies. Finally, for a given level $k$, we recursively construct a sequence of policies $\left\{\hat{\tau}_t^{c,k}, \hat{\tau}_t^{n,k}\right\}$ which implements the flexible-price allocation and always features zero inflation for all $t$.

(1) Suppose that $y_t^e = c_t^e = n_t^e = 0$ and $\pi_{t+1}^{w,e} = \pi_{t+1}^e = 0$ if $t \geq T$. Then, setting $g_t = \hat{\tau}_t^c = \hat{\tau}^n = r_t = 0$ for all $t \geq T$, implies that consumption, output, and labor for $t \geq T$ are given by

$$c_t = \frac{(1-\beta)}{\beta} \sum_{s=1}^{\infty} \beta^s \frac{Y}{C} y_{t+s}^e = 0,$$

$$y_t = \frac{C}{Y} c_t = 0,$$

and

$$n_t = \frac{y_t}{1-\alpha} = 0,$$

respectively. Then, wage inflation for $t \geq T$ is given by

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \left\{\varphi n_t + \sigma^{-1} c_t + \alpha n_t\right\} = 0.$$

Finally, this implies that price inflation is

$$\pi_t = \pi_t^w + \alpha \Delta n_t = 0$$

for $t \geq T+1$, and $\pi_T = -\alpha n_{T-1}$. This then shows the initial beliefs $y_t^{e,1} = c_t^{e,1} = n_t^{e,1} = \pi_t^{w,e,1} = \pi_{t+1}^{e,1} = 0$ are consistent with what happens in equilibrium. This result implies that all level-$k$ people believe $y_t^{e,k} = c_t^{e,k} = n_t^{e,k} = \pi_t^{w,e,k} = \pi_{t+1}^{e,k} = 0$ for $t \geq T$.

(2) Recall that the temporary equilibrium for time $t$ solves the system of equations (A.11)-(A.16). This equilibrium does not depend on policies before time $t$. So, for each $t$, level-$k$

beliefs are unaffected by past policies, $\left\{\hat{\tau}_s^{c,k}, \hat{\tau}_s^{n,k}\right\}_{s=0}^{t-1}$.

(3) For $t = T-1$, the level-$k$ equilibrium levels of consumption and wage inflation solve

$$c_{T-1}^k = -\sigma\left\{-\pi_T^{e,k} + \hat{\tau}_{T-1}^{c,k} + \chi - \rho\right\},$$

and

$$\pi_{T-1}^{w,k} = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\left\{\varphi n_{T-1}^k + \sigma^{-1}c_{T-1}^k + \hat{\tau}_{T-1}^{n,k} + \hat{\tau}_{T-1}^{c,k} + \alpha n_{T-1}^k\right\}.$$

Note that by setting $\hat{\tau}_{t+s}^{c,k} = \rho + \pi_T^{e,k} - \chi$, then $c_{T-1}^k = 0$. Since consumption remains at its steady-state level, then $y_{T-1}^k = n_{T-1}^k = 0$. Setting $\hat{\tau}_{T-1}^{n,k} = -\hat{\tau}_{T-1}^{c,k}$, implies that $\pi_{T-1}^{w,k} = 0$. Furthermore, since $\pi_T^k = -\alpha n_{T-1}^k$ then this policy also implies that $\pi_T^k = 0$.

We now proceed recursively. At time $t$, fix the future policies $\left\{\hat{\tau}_{t+s}^{c,k}, \hat{\tau}_{t+s}^{n,k}\right\}_{s\geq 1}$ and the implied beliefs $\left\{y_{t+s}^{e,k}, c_{t+s}^{e,k}, n_{t+s}^{e,k}, \pi_{t+s}^{w,e,k}, \pi_{t+s}^{e,k}\right\}_{s\geq 1}$. Consumption at time $t$ is given by we set $\hat{\tau}_t^{c,k}$ so that

$$c_t^k = \frac{(1-\beta)}{\beta}\sum_{s=1}^{\infty}\beta^s\frac{Y}{C}y_{t+s}^{e,k} - \sigma\sum_{s=0}^{\infty}\beta^s\left\{r_{t+s} - \pi_{t+s+1}^{e,k} - \left(\hat{\tau}_{t+s+1}^{c,k} - \hat{\tau}_{t+s}^{c,k}\right) + \chi_{t+s}\right\}.$$

We set $\hat{\tau}_t^{c,k}$ such that $c_t^k = 0$, which implies

$$\begin{aligned}\hat{\tau}_t^{c,k} = {} & \frac{(1-\beta)}{\beta\sigma}\sum_{s=1}^{T-t-1}\left[\beta^s\frac{Y}{C}y_{t+s}^{e,k}\right] - \left\{-\pi_{t+1}^{e,k} - \hat{\tau}_{t+1}^{c,k} + \chi - \rho\right\} \\ & - \sum_{s=1}^{\infty}\beta^s\left\{-\pi_{t+s+1}^{e,k} - \left(\hat{\tau}_{t+s+1}^{c,k} - \hat{\tau}_{t+s}^{c,k}\right) + \chi_{t+s} - \rho\right\}.\end{aligned}$$

Since $c_t^k = 0$, it follows from (A.15) and (A.16) that $n_t^k = y_t^k = 0$. Wage inflation is given by

$$\begin{aligned}\pi_t^{w,k} =& \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi n_{t+s}^{e,k} + \sigma^{-1} c_{t+s}^{e,k} + \hat{\tau}_{t+s}^{n,k} + \hat{\tau}_{t+s}^{c,k} + \alpha n_{t+s}^{e,k} \right\} \\ &+ \frac{1-\lambda}{\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e,k}.\end{aligned}$$

We set $\hat{\tau}_t^{n,k}$ such that $\pi_t^{w,k} = 0$, which implies

$$\begin{aligned}\hat{\tau}_t^{n,k} =& - \hat{\tau}_t^{c,k} - \sum_{s=1}^{\infty} (\beta\lambda)^s \left\{ \varphi n_{t+s}^{e,k} + \sigma^{-1} c_{t+s}^{e,k} + \hat{\tau}_{t+s}^{n,k} + \hat{\tau}_{t+s}^{c,k} + \alpha n_{t+s}^{e,k} \right\} \\ &- \frac{1}{1-\beta\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e,k}.\end{aligned}$$

These policies implement an allocation in which $n_t^k = 0$ and $\pi_t^{w,k} = 0$ for all $t$. It follows (A.14) from then $\pi_t^k = 0$ for all $t$.

**Part 2** Suppose that beliefs are anchored at the initial steady state. Consider setting taxes on consumption and labor such that

$$\tau_t^c = (1+\tau^c)\, e^{-(T-t)(\chi-\rho)} - 1.$$

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}.$$

Then, consumption is given by

$$C_t = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c} \{Y-G\}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right]^{1-\sigma}} = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}}{\sum_{s\geq 1} \beta^s \frac{\xi_{t+s}}{\xi_t}} \{Y-G\} = C.$$

This implies that

$$Y_t = C_t + G = C + G = Y,$$

and then

$$N_t = \left(\frac{Y}{A\overline{K}^{\alpha}}\right)^{\frac{1}{1-\alpha}} = N.$$

The reset wage is:

$$\begin{aligned}\frac{W_t^*}{P_t} &= \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{W}{P}\right)^{\theta}Nv'(L)}{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{W}{P}\right)^{\theta}Nu'(C)\,\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}} \\ &= \frac{\theta}{\theta-1}\frac{1+\tau^c}{1-\tau^n}\frac{v'(L)}{u'(C)} = \frac{W}{P}.\end{aligned}$$

Then, from the first-order condition of the firm we see that $W_t/P_t$ is constant

$$\frac{W_t}{P_t} = (1-\alpha)\,A\left(\frac{\overline{K}}{N}\right)^{\alpha} = \frac{W}{P}$$

which, combined with

$$\frac{W_t}{P_t} = \left[\lambda\left(\frac{W_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{1-\theta} + (1-\lambda)\left(\frac{W_t^*}{P_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

implies that $P_t = P_{t-1}$ for all $t$. Finally, this implies that $\mu_t = 1$ for all $t$ and $N_t = L_t = N$. Since this result holds for the non-linear model, it trivially extends to the linearized model.

## A.4 Appendix: A model with sticky prices

In this appendix, we present an alternative New Keynesian model with sticky prices instead of sticky wages and show that our main results continue to hold for this alternative specification. We assume that households have the same utility function as the one in our benchmark model, see (1.1).

The final good is produced using a continuum of intermediate inputs $y_{u,t}$ for $u \in [0,1]$

according to the technology:

$$Y_t = \left[\int_0^1 y_{u,t}^{\frac{\theta-1}{\theta}} du\right]^{\frac{\theta}{\theta-1}}.$$

Each variety $u$ is produced by a monopolistic firm using the technology:

$$y_{u,t} = An_{u,t}^{1-\alpha}.$$

The good market clearing condition is still given by (1.3). We assume that the government has access to the same monetary and fiscal instruments as in section 1.3.

**Final goods firms** The representative final goods producer maximizes profits

$$P_t Y_t - \int_0^1 p_{u,t} y_{u,t} du,$$

which implies that demand for the intermediate input is given by

$$y_{u,t} = \left(\frac{p_{u,t}}{P_t}\right)^{-\theta} Y_t.$$

The aggregate price level satisfies:

$$P_t = \left[\int_0^1 p_{u,t}^{1-\theta} du\right]^{\frac{1}{1-\theta}}.$$

**Intermediate goods producers** Each intermediate good $u$ is produced by a monopolist. Producers set prices subject to Calvo frictions. At time $t$, a fraction $1-\lambda$ can reset their price. As is standard, it is optimal for producers to choose the same reset price, $P_t^*$. The optimal reset price is the solution to:

$$\max_{P_t^*} \sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} \frac{P_{t+s}^e}{P_t} \left\{ \left(\frac{P_t^*}{P_{t+s}^e}\right)^{1-\theta} Y_{t+s}^e - \frac{W_{t+s}^e}{P_{t+s}^e A^{\frac{1}{1-\alpha}}} \left(\frac{P_t^*}{P_{t+s}^e}\right)^{-\frac{\theta}{1-\alpha}} \left(Y_{t+s}^e\right)^{\frac{1}{1-\alpha}} \right\}.$$

We assume that the monopolist has rational expectations with respect to exogenous variables, but is boundedly rational with respect to endogenous variables. In particular, we assume that the firm forms beliefs about future aggregate prices, $P_t^e$, wages, $W_t^e$,and output $Y_t^e$ using level-$k$ thinking.

The first-order condition implies that:

$$\frac{P_t^*}{P_t} = \left\{ \frac{\theta}{(\theta-1)(1-\alpha)} \frac{\sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} \frac{P_{t+s}^e}{P_t} \frac{W_{t+s}^e}{P_{t+s}^e} \frac{1}{A^{\frac{1}{1-\alpha}}} \left(\frac{P_{t+s}^e}{P_t}\right)^{\frac{\theta}{1-\alpha}} \left(Y_{t+s}^e\right)^{\frac{1}{1-\alpha}}}{\sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} \frac{P_{t+s}^e}{P_t} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1} Y_{t+s}^e} \right\}^{\frac{1-\alpha}{1-\alpha(1-\theta)}}. \quad \text{(A.17)}$$

Let lower case letters denote the log-deviation of a variable from its steady-state value, $x_t \equiv \log X_t - \log X$. Using (A.17) we obtain

$$p_t^* - p_t = \zeta (1-\lambda\beta) \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ w_{t+s}^e - p_{t+s}^e + \frac{\alpha}{1-\alpha} y_{t+s}^e \right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s \pi_{t+s}^e, \quad \text{(A.18)}$$

where $\zeta \equiv \frac{1-\alpha}{1-\alpha(1-\theta)}$.

The price level is given by

$$P_t = \left[\lambda P_{t-1}^{1-\theta} + (1-\lambda)\left(P_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

so

$$p_t = \lambda p_{t-1} + (1-\lambda) p_t^* \Leftrightarrow \pi_t = \frac{1-\lambda}{\lambda} \left(p_t^* - p_t\right). \quad \text{(A.19)}$$

Combining (A.18) and (A.19) we obtain:

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ w_{t+s}^e - p_{t+s}^e + \frac{\alpha}{1-\alpha} y_{t+s}^e \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi_{t+s}^e, \quad \text{(A.20)}$$

where $\kappa \equiv \xi \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}$.

**Household** The household chooses consumption and labor to maximize:

$$\max \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[ u\left(\tilde{C}_{t+s}\right) - v\left(\tilde{N}_{t+s}\right)\right]$$

$$\sum_{s=0}^{\infty} Q_{t,t+s} P^e_{t+s} \left(1+\tau^c_{t+s}\right) \tilde{C}_{t+s} = \sum_{s=0}^{\infty} Q^e_{t,t+s} \left[\left(1-\tau^n_{t+s}\right) W^e_{t+s} \tilde{N}_{t+s} + \Omega^e_{t+s} - T^e_{t+s}\right] + R_{t-1} B_t.$$

The solution to this problem implies

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} \left\{\left(1-\tau^n_{t+s}\right) W^e_{t+s} \tilde{N}_{t+s} + \Omega^e_{t+s} - T^e_{t+s}\right\} + R_{t-1} b_{i,t}}{\left(1+\tau_t\right) P_t \left[1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau_t\right)}\right]^{1-\sigma}\right]},$$

where

$$\tilde{N}^{\varphi}_{t+s} = \frac{\left(1-\tau^n_{t+s}\right) W^e_{t+s}}{\left(1+\tau^c_{t+s}\right) P^e_{t+s}} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{-1} \frac{Q_{t,t+s} P^e_{t+s} \left(1+\tau^c_{t+s}\right)}{P_t \left(1+\tau^c_t\right)} C_t^{-\sigma^{-1}}. \tag{A.21}$$

Using people's beliefs about the government budget constraint, (1.36), and the aggregate resource constraint, (1.3), we obtain

$$C_t = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)} \left\{\left(\frac{1-\tau^n_{t+s}}{1-\tau^c_{t+s}}\right) \frac{W^e_{t+s}}{P^e_{t+s}} \left\{\tilde{N}_{t+s} - N^e_{t+s}\right\} + Y^e_{t+s} - G_{t+s}\right\}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau_t\right)}\right]^{1-\sigma}}. \tag{A.22}$$

Log-linearizing equations A.21and A.22 yields:

$$\begin{aligned} \tilde{n}_{t+s} &= -\varphi^{-1}\left(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s}\right) + \varphi^{-1}\left(w^e_{t+s} - p^e_{t+s}\right) \\ &\quad - \varphi^{-1} \sum_{m=0}^{s-1} \left(r_{t+m} - \pi^e_{t+m+1} - \Delta\hat{\tau}^c_{t+m} + \chi_{t+m}\right) - (\varphi\sigma)^{-1} c_t \end{aligned} \tag{A.23}$$

and

$$c_t = \frac{1-\beta}{\beta}\sum_{s\geq 1}\beta^s\frac{Y}{C}\left\{y^e_{t+s} - g_{t+s} - \omega_N n^e_{t+s}\right\} + \frac{1-\beta}{\beta}\sum_{s\geq 1}\beta^s\frac{Y}{C}\omega_N\tilde{n}_{t+s} \qquad \text{(A.24)}$$
$$- \sigma\sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - \pi^e_{t+s+1} - \Delta\hat{\tau}^c_{t+s} + \sigma\chi_{t+s}\right\}$$

where $\omega_N = \left(\frac{1-\tau^n}{1-\tau^c}\right)\frac{W}{P}\frac{N}{Y}$. Replacing (A.23) in (A.24), we obtain:

$$c_t = \psi\sum_{s\geq 1}\beta^s\frac{Y}{C}\left\{y^e_{t+s} - g_{t+s} - \omega_N n^e_{t+s} + \varphi^{-1}\left\{w^e_{t+s} - p^e_{t+s} - \left(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s}\right)\right\}\right\}$$
$$- \sigma\sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - \pi^e_{t+s+1} - \Delta\hat{\tau}^c_{t+s} + \sigma\chi_{t+s}\right\}$$

where $\psi \equiv \frac{\sigma}{\sigma+\frac{Y}{C}\omega_N\varphi^{-1}}\frac{1-\beta}{\beta}$.

**Equilibrium** In equilibrium, labor-market clearing, $N_t = \int n_{u,t}du$, implies that:

$$N_t = \int n_{u,t}du = \int\left(\frac{y_{u,t}}{A}\right)^{\frac{1}{1-\alpha}}du = \int\left(\frac{Y_t}{A}\right)^{\frac{1}{1-\alpha}}\left(\frac{p_{u,t}}{P_t}\right)^{-\frac{\theta}{1-\alpha}}du$$

which implies that

$$Y_t = \mu_t^{\alpha-1}AN_t^{1-\alpha} = C_t + G_t,$$

where $\mu_t = \int\left(\frac{p_{u,t}}{P_t}\right)^{-\frac{\theta}{1-\alpha}}$ denotes the standard price distortion. Starting from an non-distorted steady state implies $\mu_{-1} = 1$ and to first order the price distortion is zero.

The temporary equilibrium conditions are as follows.

1. Consumption is given by

$$c_t = \psi \sum_{s\geq 1} \beta^s \frac{Y}{C} \left\{ y^e_{t+s} - g_{t+s} - \omega_N n^e_{t+s} + \varphi^{-1} \left\{ w^e_{t+s} - p^e_{t+s} - \left( \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} \right) \right\} \right\} \tag{A.25}$$
$$- \sigma \sum_{m=0}^{\infty} \beta^s \left\{ r_{t+s} - \pi^e_{t+s+1} - \Delta \hat{\tau}^c_{t+s} + \chi_{t+s} \right\}.$$

2. Inflation is given by

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ \left( \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} \right) + \varphi n^e_{t+s} + \sigma^{-1} c^e_{t+s} + \frac{\alpha}{1-\alpha} y^e_{t+s} \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi^e_{t+s}. \tag{A.26}$$

3. Output is given by

$$y_t = (1-\alpha)\, n_t. \tag{A.27}$$

4. Market clearing implies

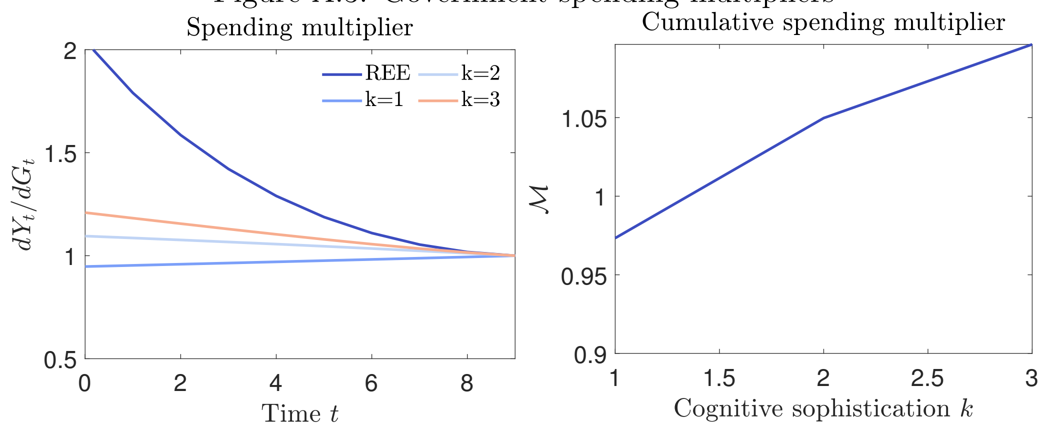
$$y_t = \frac{C}{Y} c_t + g_t. \tag{A.28}$$

Note that we assume that the beliefs that firms have about the real wage are consistent with household labor supply. An equilibrium is a solution to this system along with a specification of belief formation corresponding to level-$k$ thinking.

### A.4.1 Government spending multipliers

In this section we briefly illustrate the analog to Proposition 1 for the case in which tax rates are constant and government spending rises by $\Delta G$ during the ZLB period.

Figure A.3: Government spending multipliers




Comparing figures 1.3 and A.3, we see that the implications of level-$k$ thinking for the government multiplier are essentially the same, regardless of whether Calvo frictions apply to wages and prices.

## A.4.2 Consumption-tax policy

Proposition 5 continues to hold for the economy in which prices, rather than wages, are subject to Calvo frictions.

*Proof.* (Part 1) The proof strategy is as follows. Fix a $k$. First, we show that if the level-1 believe that the economy will stay at steady state for $t \geq T$, then this implies that all level-$k$ beliefs and equilibrium feature output, consumption, labor and wage inflation remaining at their steady state levels from $t \geq T$, and price inflation becoming zero from $t \geq T+1$ on. Second, we note that beliefs about future output, inflation, consumption, and labor are a function only of future tax rates and policies. (3) Finally, we recursively construct a sequence of policies $\left\{\hat{\tau}_t^{c,k}, \hat{\tau}^{n,k}\right\}$ which implements the flexible-price allocation and always features zero inflation. □

(1) Suppose that $y_t^e = c_t^e = n_t^e = 0$ and $\pi_t^e = 0$ if $t \geq T$, then the policies $g_t = \hat{\tau}_t^{c,k} =$

$\hat{\tau}^{n,k} = r_t = 0$ for all $t \geq T$ imply that consumption, output, and labor for $t \geq T$ are given by

$$c_t = \psi \sum_{s \geq 1} \beta^s \frac{Y}{C} \left\{ y^e_{t+s} - \omega_N n^e_{t+s} + \varphi^{-1} \left\{ w^e_{t+s} - p^e_{t+s} \right\} \right\} = 0.$$

$$y_t = \frac{C}{Y} c_t = 0,$$

and

$$n_t = \frac{y_t}{1-\alpha} = 0,$$

respectively. Finally, inflation is given by

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ \varphi n^e_{t+s} + \sigma^{-1} c^e_{t+s} + \frac{\alpha}{1-\alpha} y^e_{t+s} \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi^e_{t+s} = 0.$$

This then shows that starting from the initial beliefs $y^{e,1}_t = c^{e,1}_t = n^{e,1}_t = 0$ and $\pi^{e,1}_t = 0$ implies that the same holds for all $k$.

(2) Note that the temporary equilibrium for time $t$, which solves the system of equations (A.25)-(A.28) does not depend on policies before time $t$. This implies that for each $t$, $y^{e,k}_t$ is unaffected by policies $\left\{ \hat{\tau}^{c,k}_s, \hat{\tau}^{n,k}_s \right\}_{s=0}^{t-1}$.

(3) We now proceed recursively. At time $t$, given policies $\left\{ \hat{\tau}^{c,k}_{t+s}, \hat{\tau}^{n,k}_{t+s} \right\}_{s \geq 1}$ and beliefs $\left\{ y^{e,k}_{t+s}, c^{e,k}_{t+s}, n^{e,k}_{t+s}, \pi^{e,k}_{t+s} \right\}_{s \geq 1}$, we set the consumption tax $\hat{\tau}^{c,k}_t$ so that

$$\begin{aligned} \hat{\tau}^{c,k}_t = \frac{\psi}{\sigma} \sum_{s \geq 1} \beta^s \frac{Y}{C} \left\{ y^{e,k}_{t+s} - \omega_N n^e_{t+s} + \varphi^{-1} \left\{ w^{e,k}_{t+s} - p^{e,k}_{t+s} - \left( \hat{\tau}^{n,k}_{t+s} + \hat{\tau}^{c,k}_{t+s} \right) \right\} \right\} \\ - \left\{ -\pi^{e,k}_{t+1} - \hat{\tau}^{c,k}_{t+1} + \chi - \rho \right\} - \sum_{s=1}^{\infty} \beta^s \left\{ r_{t+s} - \pi^{e,k}_{t+s+1} - \Delta \hat{\tau}^{c,k}_{t+s} + \chi_{t+s} \right\}, \end{aligned}$$

which implies that $c^k_t = 0$. It then follows that $n^k_t = y^k_t = 0$. Then, setting $\hat{\tau}^{n,k}_{t+s}$ such that

$$\hat{\tau}_t^{n,k} = -\hat{\tau}_t^{c,k} - \sum_{s=1}^{\infty} (\beta\lambda)^s \left\{ \left(\hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c\right) + \varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \frac{\alpha}{1-\alpha} y_{t+s}^e \right\} \\ - \frac{1-\lambda}{\lambda\kappa} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi_{t+s}^e$$

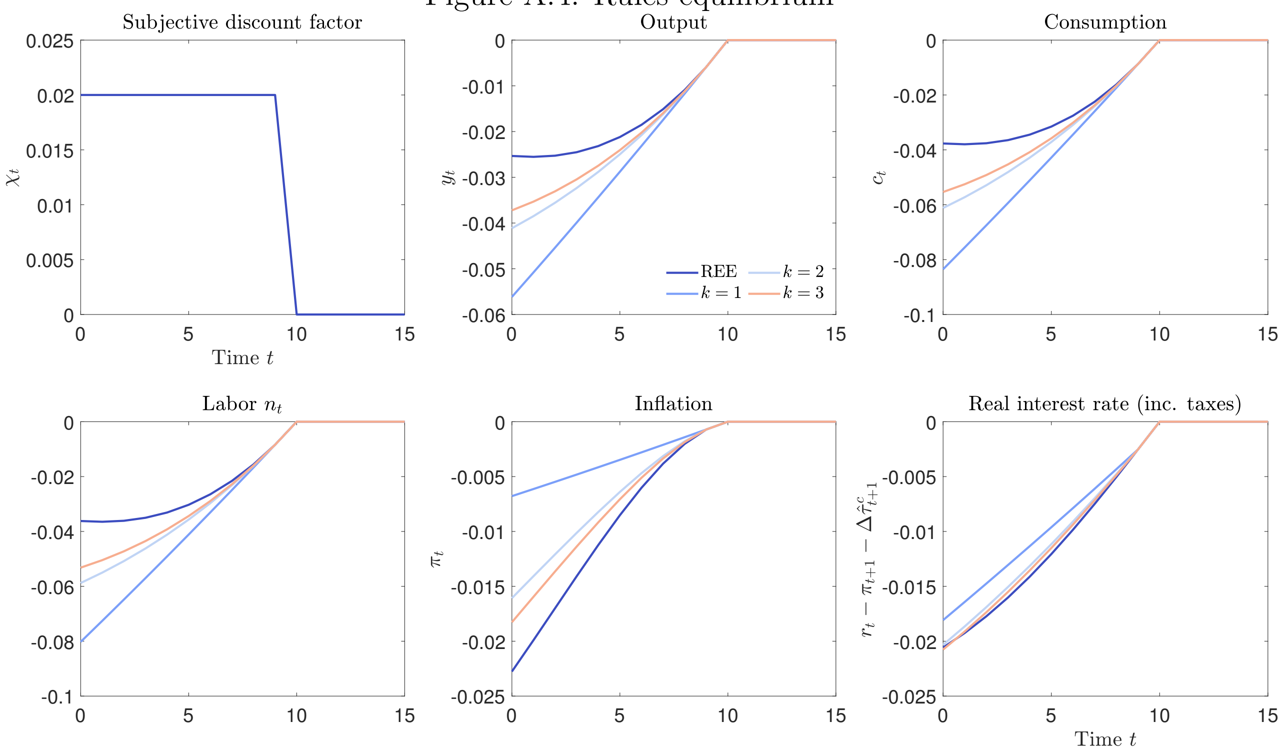
which implies that $\pi_t^k = 0$.

*Proof.* (Part 2) Under this assumption, the consumption function still implies that $C_t^1 = C$, which implies that $N_t^1 = N$ and $Y_t^1 = Y$, i.e., both consumption, labor, and output in the level-1 economy stay at their steady state levels. Using the fact that $(1 - \tau_t^n) / (1 + \tau_t^c) = (1 - \tau^n) / (1 + \tau^c)$, this implies that the relative wage $W_t^1 / P_t^1$ remains at its pre-shock steady state as well. Finally, this implies that $p_t^{*,1} = p_t$ and so inflation is always zero. The same argument then holds for $k > 1$. □

**Rules versus targets**

Figure A.4 is the analog to Figure 1.5, assuming prices are subject to Calvo-style frictions. We see that the implications of level-$k$ thinking for the efficacy of fiscal policy when prices, rather than wages, are subject to Calvo-style frictions are essentially the same.

Figure A.4: Rules equilibrium



# Appendix B

# Appendix to Chapter Two

## B.1 Appendix to section 2

### B.1.1 Deriving the equilibrium

We solve the model backwards, starting at the end of period 1. At this point, households observe the relevant equilibrium outcomes $\{b_1, \tau_1\}$ as well as their employment status $e_1 \in \{E, U\}$. Their problem is

$$V_1(e_1, a_0) = \max_{c_1, a_1} u(c_1) \qquad \text{s.t. } c_1 + a_1 = (1+r)a_0 + \mathbf{1}_{\{e_t = E\}} + \mathbf{1}_{\{e_1 = U\}} \cdot b_1 - \tau_1 \qquad a_1 \geq 0 \tag{B.1}$$

where we already imposed $w_1 = 1$. Clearly, the optimal decision is to consume the entire cash on hand

$$a_1(e_1, a_0) = 0 \tag{B.2}$$

$$c_1(e_1, a_0) = (1+r)a_0 + \mathbf{1}_{\{e_1 = E\}} + \mathbf{1}_{\{e_1 = U\}} \cdot b_1 - \tau_1 \tag{B.3}$$

Since assets are in zero net supply and borrowing is not allowed, all workers have zero assets in equilibrium, $a_0 = 0$. Given the policies $\{b_1, M_1\}$, the time-1 equilibrium can be computed recursively as

$$N_1 = M_1 \tag{B.4}$$

$$\tau_1 = (1 - N_1)b_1 \tag{B.5}$$

$$c_1(E) = 1 - \tau_1 \tag{B.6}$$

$$c_1(U) = b_1 - \tau_1 \tag{B.7}$$

Note that time-1 equilibrium is independent of what happens in period 0, including the beliefs $\{N_1^e, b_1^e, \tau_1^e\}$ that households hold in period 0.

Let's turn to period 0. Combining the consumption policy function (B.3) with the fact that the probability of employment is iid, the expected continuation value at the end of period 0 can be written as

$$V_1^e(a_0) = N_1^e \cdot u\Big((1+r)a_0 + 1 - \tau_1^e\Big) + (1 - N_1^e) \cdot u\Big((1+r)a_0 + b_1 - \tau_1\Big) \tag{B.8}$$

At time $t = 0$, households solve

$$\max_{c_0, a_0} u(c_0) + \beta V_1^e(a_0) \qquad \text{s.t. } c_0 + a_0 = \mathbf{1}_{\{e_0 = E\}} + \mathbf{1}_{\{e_0 = U\}} \cdot b_0 - \tau_0 \qquad a_0 \geq 0 \tag{B.9}$$

Taking FOCs yields the Euler equation (2.5) in the main text

$$u'(c_0) \geq \beta (V_1^e)'(a_0) \tag{B.10}$$

$$= \beta(1+r)\Big[N_1^e \cdot u'\Big(1 - \tau_1^e + (1+r)a_0\Big) + (1 - N_1^e) \cdot u'\Big(b_1^e - \tau_1^e + (1+r)a_0\Big)\Big] \tag{B.11}$$

Note that the right-hand side does not depend on time-0 employment status. Since there can't be any saving in equilibrium ($a_0 = 0$), households consume their income in period

0, which is higher for employed households. Given that $u'(\bullet)$ is decreasing, this implies that either both types of households are borrowing constrained or only unemployed workers are constrained. We assume that $\beta$ and $r$ is such that only unemployed households are constrained.

Then, given beliefs $\{N_1^e, \tau_1^e, b_1^e\}$ and policies $\{b_0, r\}$, the time-0 equilibrium can be computed recursively as follows

$$u'\Big(c_0(E)\Big) = \beta(1+r)\Big[N_1^e \cdot u'\Big(1-\tau_1^e\Big) + (1-N_1^e)\cdot u'\Big(b_1^e - \tau_1^e\Big)\Big] \tag{B.12}$$

$$\tau_0 = 1 - c_0(E) \tag{B.13}$$

$$c_0(U) = b_0 - \tau_0 \tag{B.14}$$

$$N_0 = 1 - \frac{\tau_0}{b_0} \tag{B.15}$$

$$M_0 = N_0 \tag{B.16}$$

### B.1.2 Proof of proposition 1

*Proof.* Consider an infinitesimal shock to money supply in period 1, $dM_1$. Differentiating the time-1 equilibrium (B.12)–(B.16) yields

$$dN_1 = dM_1 \tag{B.17}$$

$$d\tau_1 = (1-N_1)db_1 - dN_1 \cdot b_1 \tag{B.18}$$

$$dc_1(E) = -d\tau_1 \tag{B.19}$$

$$dc_1^U = db_1 - d\tau_1 \tag{B.20}$$

Since we assumed that both UI extension regimes implement the same benefits $db_1$, the time-1 responses are the same under both regimes. Given our model of beliefs (2.10), this implies that expectations of employment and taxes are the same

$$dN_1^{e,rule} = dN_1^{e,*} = \lambda \cdot dN_1 \qquad \text{and} \qquad d\tau_1^{e,rule} = d\tau_1^{e,*} = \lambda \cdot d\tau_1 \tag{B.21}$$

but the expectation of unemployment benefits may differ

$$db_1^{e,rule} = -\zeta_b \cdot dN_1^{e,rule} = -\zeta_b \lambda \cdot dN_1 \leq -\zeta_b \cdot dN_1 = db_1^{e,*} \tag{B.22}$$

These expectations are relevant for pinning down $dc_0(E)$ through the Euler equation of employed workers (B.12). To first order after the shock, the Euler equation reads as

$$\begin{aligned} u''(1-\tau_0) \cdot dc_0(E) = \beta(1+r)\Big[ & dN_1^e \cdot u'\Big(1-\tau_1^e\Big) - N_1^e \cdot u''\Big(1-\tau_1^e\Big) d\tau_1^e \\ & - dN_1^e u'\Big(b_1^e - \tau_1^e\Big) + (1-N_1^e) \cdot u''\Big(b_1^e - \tau_1^e\Big)(db_1^e - d\tau_1^e)\Big] \end{aligned} \tag{B.23}$$

So the difference in consumption under the two UI extension regimes is

$$dc_0(E)^{\text{rule}} - dc_0(E)^* = \underbrace{\frac{\beta(1+r)(1-N_1^e) \cdot u''(b_1^e - \tau_1^e)}{u''(1-\tau_0)}}_{\equiv M_b} \cdot \Big(db_1^{e,rule} - db_1^{e,*}\Big) \tag{B.24}$$

where $M_b \in [0,1]$ can be interpreted as the marginal propensity to consume out of anticipated UI benefits. Note that the $dN_1^e$ and $d\tau_1^e$ terms cancel because these expectations are independent of the UI extension regime.

Differentiating the rest of the time-0 equilibrium conditions (B.13)–(B.16) gives us

$$d\tau_0 = -dc_0(E) \tag{B.25}$$

$$dc_0(U) = db_0 - d\tau_0 \tag{B.26}$$

$$dN_0 = -\frac{d\tau_0}{b_0} + \frac{\tau_0}{b_0^2} db_0 \tag{B.27}$$

$$dM_0 = dN_0 \tag{B.28}$$

Let's assume that UI benefits respond only in period 1, $db_0 = 0$, in order to isolate the impact of precautionary behavior. Combining the perturbed time-0 equilibrium conditions proves the proposition

$$dY_0^{\text{rule}} - dY_0^* = \frac{1}{b_0} \cdot M_b \cdot (1 - \lambda) \cdot dM_1 \tag{B.29}$$

To interpret the $1/b_0$ term, note that the aggregate consumption function of this economy is

$$C_0 = N_0 \cdot c_0(E) + (1 - N_0)(b_0 - \tau_0) \tag{B.30}$$

According to (B.12), the consumption choice of employed workers $c_0(E)$ does not depend on $N_0$. Also recall that, in equilibrium, $c_0(E) = 1 - \tau_t$. So

$$\frac{\partial C_0}{\partial N_0} = c_0(E) - (b_0 - \tau_0) = 1 - b_0$$

This implies that

$$\frac{1}{b_0} = \frac{1}{1 - \frac{\partial C_0}{\partial N_0}} \equiv \mathcal{M} > 0 \tag{B.31}$$

is a standard Keynesian multiplier. □

## B.2 Appendix to section 3

### B.2.1 Proof of Proposition 7

We consider a generic representation of a heterogeneous-agent problem as a mapping from some input $X_t$ to a time-path of aggregates $C_t$. Following Auclert et al. (2021), a generic representation of a heterogeneous-agent problem is a mapping between aggregate inputs $\boldsymbol{X}_t$, a time path f aggregate outputs $\boldsymbol{Y}_t$. Assume that there are $n_x$ inputs and $n_y$ outputs, and that the distribution if discretized on $n_g$ points. Let $\boldsymbol{D}_t$ denote the $n_g \times 1$ distribution of agents. Then let $\boldsymbol{y}_t$ be the $n_g \times n_y$ matrix of individual outcomes.

$$\boldsymbol{v}_t = v\left(\boldsymbol{v}^e_{t+1}, \boldsymbol{X}_t\right) \tag{B.32}$$

$$\boldsymbol{v}^e_t = v\left(\boldsymbol{v}^e_{t+1}, \boldsymbol{X}^{e,0}_t\right) \tag{B.33}$$

$$\boldsymbol{D}_{t+1} = \Lambda\left(\boldsymbol{v}^e_{t+1}, \boldsymbol{X}_t\right)' \boldsymbol{D}_t \tag{B.34}$$

$$\boldsymbol{Y}_t = y\left(\boldsymbol{v}^e_{t+1}, \boldsymbol{X}_t\right)' \boldsymbol{D}_t \tag{B.35}$$

Let $(\boldsymbol{Y}, \boldsymbol{v}, \boldsymbol{v}^e, \boldsymbol{D})$ denote the steady state which satisfies $\boldsymbol{X}^e = \boldsymbol{X}$. This immediately implies that $\boldsymbol{v} = \boldsymbol{v}^e$. For convenience, let $\Lambda_{ss} \equiv \Lambda\left(\boldsymbol{v}^e, \boldsymbol{X}\right)$. Consider transitions of length T that satisfy $\boldsymbol{X}_{t-1} = \boldsymbol{X}$ and $\boldsymbol{v}^e_T = \boldsymbol{v}_T = \boldsymbol{v}$. The initial distribution $\boldsymbol{D}_0$ is given and we assume that $\boldsymbol{D}_0 = \boldsymbol{D}$.

Given all of this, this defines a $T \times n_y$ vector of stacked outputs

$$\boldsymbol{Y} = h\left(\boldsymbol{X}, \boldsymbol{X}^{e,0}\right).$$

Assume that all functions are differentiable, then so it $h$. We want to characterize the Jacobian $\mathcal{J}$ of $h$ evaluated at the stead state with respect to variables $X$ and $X^e$.

**Responde to $dX_s$** Consider a change to input $X$ at time $s$, $dX_s$, with $dX_t = 0$ for all $t \neq s$. It follows immediately that

$$v_t^e = v^e = v,$$

for all $t$, and $v_t = v$ for all $t \neq s$. Furthermore, it follows that, for all $t \neq s$, $\boldsymbol{y}_t \equiv y\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right) = \boldsymbol{y}$ and $\boldsymbol{\Lambda}_t \equiv \Lambda\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right) = \boldsymbol{\Lambda}$, so $d\boldsymbol{y}_t = 0$ and $d\boldsymbol{\Lambda}_t = 0$.

Note that, by the chain rule, we find that:

$$d\boldsymbol{Y}_t = d\boldsymbol{y}_t'\boldsymbol{D} + \boldsymbol{y}d\boldsymbol{D}_t$$

and

$$d\boldsymbol{D}_{t+1} = d\boldsymbol{\Lambda}_t'\boldsymbol{D} + \boldsymbol{\Lambda}'d\boldsymbol{D}_t.$$

Using these expressions and the results above, it follows that $dY_t = 0$ and $d\boldsymbol{D}_{t+1} = 0$ for all $t < s$. Furthermore, for $t = s$, we obtain

$$d\boldsymbol{D}_{s+1} = d\boldsymbol{\Lambda}_s'\boldsymbol{D}, \quad \text{and} \quad d\boldsymbol{Y}_s = d\boldsymbol{y}_s'\boldsymbol{D},$$

and for $t > s$ we find that

$$\boldsymbol{D}_t = \boldsymbol{\Lambda}'d\boldsymbol{D}_{t-1} = \left(\boldsymbol{\Lambda}'\right)^{t-(s+1)} d\boldsymbol{D}_{s+1}, \quad \text{and} \quad d\boldsymbol{Y}_t = \boldsymbol{y}d\boldsymbol{D}_t$$

Finally, note that $d\boldsymbol{y}_s$ does not depend on the time $s$, but rather than the shock happens at that moment and is not anticipated. It immediately follows that

$$\frac{\partial Y_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s \\ \frac{\partial Y_{t-s}}{\partial X_0}, & \text{if } t \geq s \end{cases} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0}, & \text{if } t \geq s, \end{cases} \tag{B.36}$$

where $\mathcal{J}$ denotes the FIRE Jacobian.

**Response to** $dX_s^e$ Note that, $v_{s+t}^e = v_{t+1} = v$ for all $t \geq s$, which implies that $\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}$ and $\boldsymbol{y}_t = \boldsymbol{y}$. It follows that, for $t > s$,

$$d\boldsymbol{Y}_t = \boldsymbol{y} d\boldsymbol{D}_t$$

and

$$d\boldsymbol{D}_{t+1} = \boldsymbol{\Lambda}' d\boldsymbol{D}_t$$

where $d\boldsymbol{D}_t = \boldsymbol{\Lambda}' d\boldsymbol{D}_{t-1} = (\boldsymbol{\Lambda}')^{t-(s+1)} d\boldsymbol{D}_{s+1}$.

So, for $t < s$, the response is exactly the same as that which would be obtained under FIRE, i.e., $dY_t = \mathcal{J}_{t,s}$. For $t = s$, we find that $v_{s+1}^e = v$ and since $X_s = X$, then $\boldsymbol{y}_s = \boldsymbol{y}$ and $\boldsymbol{\Lambda}_s = \boldsymbol{\Lambda}$. It follows that

$$d\boldsymbol{D}_{s+1} = \boldsymbol{\Lambda} d\boldsymbol{D}_s^* = d\boldsymbol{D}_{s+1}^* - (d\boldsymbol{\Lambda}_s^*)' \boldsymbol{D} = d\boldsymbol{D}_{s+1}^* - d\boldsymbol{D}_{s+1}^0$$

and

$$d\boldsymbol{Y}_s = \boldsymbol{y} d\boldsymbol{D}_s^* = d\boldsymbol{Y}_s^* - (d\boldsymbol{y}_s^*)' \boldsymbol{D} = d\boldsymbol{Y}_s^* - d\boldsymbol{Y}_s^0,$$

where $dD_s^*$ and $d\Lambda_s^*$ denote the response under FIRE, i.e., $dX_s^e = dX_s$, and $dD_s^0$ and $d\Lambda_s^0$ denote the responses to an unanticipated change $dX_s \neq 0$ with $dX_s^{e,0} = 0$. Finally, for $t > s$, we also find that $\boldsymbol{v}_{t+1}^e = \boldsymbol{v}$ and $\boldsymbol{X}_t = \boldsymbol{X}$ so that decisions and transitions do not change. As a result,

$$d\boldsymbol{D}_t = (\boldsymbol{\Lambda}')^{t-(s+1)} d\boldsymbol{D}_{s+1} = (\boldsymbol{\Lambda}')^{t-(s+1)} \left(d\boldsymbol{D}_{s+1}^* - d\boldsymbol{D}_{s+1}^0\right) = d\boldsymbol{D}_t^* - d\boldsymbol{D}_t^0$$

$$d\boldsymbol{Y}_t = \boldsymbol{y} d\boldsymbol{D}_t = \boldsymbol{y} d\boldsymbol{D}_t^* - \boldsymbol{y} d\boldsymbol{D}_t^0 = d\boldsymbol{Y}_t^* - d\boldsymbol{Y}_t^0.$$

As a result,

$$\frac{dY_t}{dX_s^{e,0}} = \begin{cases} \mathcal{J}_{t,s} & \text{if } t < s \\ \mathcal{J}_{t,s} - \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases} \tag{B.37}$$

**Putting it together** Define

$$\mathcal{E} \equiv \begin{bmatrix} \mathcal{J}_{0,0} & 0 & 0 & ... \\ \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & 0 & ... \\ \mathcal{J}_{2,0} & \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & ... \\ ... & ... & ... & ... \end{bmatrix}, \tag{B.38}$$

then we can summarize these results in the following expressions

$$d\boldsymbol{Y} = (\mathcal{J} - \mathcal{E}) \cdot d\boldsymbol{X}^{e,0} + \mathcal{E} \cdot d\boldsymbol{X}^0 = \mathcal{J} \cdot d\boldsymbol{X}^{e,0} + \mathcal{E} \cdot (d\boldsymbol{X}^0 - d\boldsymbol{X}^{e,0}). \tag{B.39}$$

### B.2.2 Proof of proposition 8

Let $\left\{\{X_s^{e,t}\}_{s=t+1}^{T-1}\right\}_{t=0}^{T-1}$ denote their beliefs at each point in time, then the representation is

$$\boldsymbol{v}_s^{e,t} = v\left(\boldsymbol{v}_{s+1}^{e,t}, \boldsymbol{X}_s^{e,t}\right), \quad s = 0, ..., T-1, t = 0, ..., T-1 \tag{B.40}$$

$$\boldsymbol{v}_t = v\left(\boldsymbol{v}_{t+1}^{e,t}, \boldsymbol{X}_t\right) \tag{B.41}$$

$$\boldsymbol{D}_{t+1} = \Lambda\left(\boldsymbol{v}_{t+1}^{e,t}, \boldsymbol{X}_t\right)' \boldsymbol{D}_t \tag{B.42}$$

$$\boldsymbol{Y}_t = y\left(\boldsymbol{v}_{t+1}^{e,t}, \boldsymbol{X}_t\right)' \boldsymbol{D}_t, \tag{B.43}$$

where $v_{T+1}^{e,t} = v$ and $X_{T+1}^{e,t} = X$ for all t. The rational expectations are captured by $\mathcal{J}$ and the Jacobian with respect to $d\boldsymbol{X}_s$ is still $\mathcal{J}_{t-s,0}$ for $t \geq s$.

Now, the main observation is that, when considering a partial change $dX_s^{e,\tau}$, nothing changes for $t < \tau$ and so

$$\frac{\partial Y_t}{\partial X_s^{e,\tau}} = \frac{\partial Y_{t-\tau}}{\partial X_{s-\tau}^{e,0}}. \tag{B.44}$$

Note that if we change $X_s^{e,0}$, then $v_s^{e,t} = v$, $y_t = y$, $\Lambda_t = \Lambda$ for $t > 0$. Then,

$$dY_0 = dy_0' D = (dy_0^*)' D$$

$$dD_1 = (d\Lambda_0^*)' D = dD_1^*$$

$$dD_{t+1} = \Lambda' dD_t = (\Lambda')^t dD_1 = (\Lambda')^t dD_1^*$$

$$dY_t = y' dD_t$$

Note that $dY_0$ is exactly the same as under rational expectations. For $t \leq s$, we have that

$$dD_t = (\Lambda')^{t-1} dD_1^*$$

$$dY_t = y' dD_t.$$

Futhermore, define the FIRE response as

$$dD_t^* = \left(d\Lambda_{t-1}^*\right)' D + \Lambda' dD_{t-1}^* = \sum_{m=0}^{t-2} (\Lambda')^m \left(d\Lambda_{t-1-m}^*\right)' D + (\Lambda')^{t-1} dD_1^*$$

$$dY_t^* = (dy_t^*)' D + y' dD_t^*$$

It is useful to put superscripts for the time of the shock, $s$. For example, define the FIRE response as:

$$dD_t^{*,s} = \sum_{m=0}^{t-2} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D + (\Lambda')^{t-1} dD_1^{*,s}$$

$$dY_t^{*,s} = (dy_t^{*,s})' D + y' dD_t^{*,s}$$

Now, note that

$$dD_t^s = dD_t^{*,s} - \sum_{m=0}^{t-2} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D$$

$$dY_t = dY_t^* - (dy_t^{*,s})' D + y' (dD_t^s - dD_t^{*,s}).$$

Note furthermore, that

$$\begin{aligned} dD_t^{*,s-1} &= \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \Lambda' dD_{t-1}^{*,s-1} \\ &= \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \Lambda' \left(d\Lambda_{t-1}^{*,s-1}\right)' D + (\Lambda')^2 dD_{t-2}^{*,s-1} \\ &= \sum_{m=0}^{t-1} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s-1}\right)' D + (\Lambda')^t \underbrace{dD_0^{*,s-1}}_{=0} \\ &= \sum_{m=0}^{t-1} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s-1}\right)' D \end{aligned}$$

and now using the fact that

$$d\Lambda_{t-1-m}^{*,s-1} = d\Lambda_{t-m}^{*,s}$$

we can write

$$dD_t^{*,s-1} = \sum_{m=0}^{t-1} (\Lambda')^m (d\Lambda_{t-m}^{*,s})' D$$

$$dD_{t-1}^{*,s-1} = \sum_{m=0}^{t-2} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D.$$

As a result, $dD_t^s = dD_t^{*,s} - dD_{t-1}^{*,s-1}$. Finally, we can write $dY_t^s = dY_t^{s,*} - (dy_t^{*,s})' D - y' dD_{t-1}^{*,s-1}$ and since $dy_t^{*,s} = dy_{t-1}^{*,s-1}$ then

$$dY_t^s = dY_t^{s,*} - \left(dy_{t-1}^{*,s-1}\right)' D - y' dD_{t-1}^{*,s-1} = dY_t^{s,*} - dY_{t-1}^{s-1,*}.$$

For $t \geq s$, we still find that

$$dD_t = (\Lambda')^{t-s-1} dD_{s+1}$$

$$dY_t = y' dD_t$$

while the FIRE response would have been

$$dD_t^* = \Lambda' dD_{t-1}^* = (\Lambda')^{t-s-1} dD_{s+1}^*$$

$$dY_t^* = y' dD_t^*.$$

Once again, we can write

$$dD_{s+1}^{*,s} = \sum_{m=0}^{s-1} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D + (\Lambda')^s dD_1^{*,s}$$

and

$$dD_{s+1}^s = (\Lambda')^s dD_1^{*,s}.$$

It follows that

$$dD_{s+1}^s - dD_{s+1}^{s,*} = -\sum_{m=0}^{s-1} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D = -dD_s^{*,s-1}$$

and, as a result,

$$dY_t^s = y' dD_t^s = dY_t^{*,s} + y' \left(dD_t^s - dD_t^{*,s}\right) = dY_t^{*,s} + y' (\Lambda')^{t-s-1} \left(dD_{s+1}^s - dD_{s+1}^{*,s}\right),$$

$$dY_t^s = dY_t^{*,s} - y' (\Lambda')^{t-s-1} dD_s^{*,s-1}$$

and

$$dY_t^s = dY_t^{*,s} - dY_{t-1}^{*,s-1}.$$

It thus follows that

$$\frac{\partial Y_t}{\partial X_s^{e,0}} = \mathcal{J}_{t,s} - \mathcal{J}_{t-1,s-1}. \tag{B.45}$$

**Putting everything together** We have thus found that

$$\frac{\partial Y_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases}$$

and

$$\frac{\partial Y_t}{\partial X_s^{e,\tau}} = \begin{cases} 0 & \text{if } t < \tau \text{ or } s \leq \tau \\ \mathcal{J}_{t-\tau,s-\tau} - \mathcal{J}_{t-\tau-1,s-\tau-1} & \text{if } t > \tau \text{ and } s > \tau \\ \mathcal{J}_{0,s-t} & \text{if } t = \tau \text{ and } s > \tau = t. \end{cases}$$

Putting everything together we can write

$$dY_t = \sum_{s=0}^{t} \mathcal{J}_{t-s,0} \cdot dX_s + \sum_{\tau=0}^{t-1} \sum_{s=\tau+1}^{\infty} \left(\mathcal{J}_{t-\tau,s-\tau} - \mathcal{J}_{t-\tau-1,s-\tau-1}\right) \cdot dX_s^{e,\tau} + \sum_{s=t+1}^{\infty} \mathcal{J}_{0,s-t} \cdot dX_s^{e,t} \tag{B.46}$$

$$= \sum_{\tau=1}^{t} \sum_{s=\tau}^{\infty} \mathcal{J}_{t-\tau,s-\tau} \left(dX_s^{e,\tau} - dX_s^{e,\tau-1}\right) + \sum_{s=0}^{\infty} \mathcal{J}_{t,s} dX_s^{e,0} \tag{B.47}$$

from where equation (2.21) follows immediately.

### B.2.3 Special cases

Throughout, we maintain the following notation. $E_t[dX_{t+h}]$ denotes the agent's time-$t$ expectations about the variable at horizon $h$. $\mathbb{E}_t[dX_{t+h}]$ denotes the full-information and rational expectation for the same variable.

**Shallow reasoning**

(Angeletos and Sastry, 2021). $E_t[dX_{t+h}] = \lambda \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.48}$$

**Cognitive discounting**

(Gabaix, 2020). $E_t[dX_{t+h}] = \lambda^h \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & 0 & \lambda^2 & 0 & \dots \\ 0 & 0 & 0 & \lambda^3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.49}$$

**Sticky expectations**

(Mankiw and Reis, 2002; Carroll, Crawley, Slacalek, Tokuoka, and White, 2018). A date-0 shock $\epsilon$ causes a sequence of disturbances $\{dX_t\}$. At each date $t \geq 0$, some agents learn about $\epsilon$ and deduce $\{dX_{t+h}\}$ for all $h \geq 0$. The probability of learning $\epsilon$ is $1-\lambda$ for every agent who hasn't learned it already. Thus the share of ignorant agents at date $t$ is $\lambda^{t+1}$. They believe that the disturbances observed so far were special events, and don't expect any disturbances in the future. This setup implies that average expectations are $E_t[dX_{t+h}] = (1-\lambda^{t+1}) \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1-\lambda & 0 & 0 & \dots \\ 0 & 0 & 1-\lambda & 0 & \dots \\ 0 & 0 & 0 & 1-\lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1-\lambda^2 & 0 & \dots \\ 0 & 0 & 0 & 1-\lambda^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.50}$$

**Noisy information and rational expectations**

(Angeletos and Huo, 2021). A date-0 shock $\epsilon$ causes a sequence of disturbances $\{dX_t\}$ according to an MA process

$$dX_t = M_t \epsilon \tag{B.51}$$

Suppose that agents know the MA coefficients, $M_t$, but they don't observe $\epsilon$. Their prior is that $\epsilon$ is distributed $\mathcal{N}(0, 1/\tau_\epsilon)$. At each date $t \geq 0$, agents receive independent private signals $\epsilon + \nu_t$, where $\nu_t \sim \mathcal{N}(0, 1/\tau_\nu)$. Bayesian updating implies that the average posterior belief is

$$E_t[\epsilon] = \frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}\epsilon \tag{B.52}$$

Then, the average expectation of $dX_{t+h}$ at date $t$ is

$$E_t[dX_{t+h}] = M_{t+h} E_t[\epsilon] = M_{t+h} \underbrace{\left( \frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} \right)}_{\lambda_t} \epsilon = \lambda_t dX_{t+h} \tag{B.53}$$

Thus the associated $\Lambda_t$ matrices are

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda_0 & 0 & 0 & \dots \\ 0 & 0 & \lambda_0 & 0 & \dots \\ 0 & 0 & 0 & \lambda_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda_1 & 0 & \dots \\ 0 & 0 & 0 & \lambda_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.54}$$

### Extrapolation

Geometric extrapolation. $E_t[dX_{t+h}] = \lambda^h dX_t$. First example of non-diagonal $\Lambda$ matrices.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \lambda & 0 & 0 & 0 & \dots \\ \lambda^2 & 0 & 0 & 0 & \dots \\ \lambda^3 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & \lambda^2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.55}$$

### Adaptive expectations

(Cagan, 1956; Friedman, 1957). $E_t[dX_{t+h}] = \lambda^h \kappa \sum_{\tau=0}^{\infty} \lambda^\tau dX_{t-\tau}$, where $\kappa > 0$ scales the geometric sum.

$$\Lambda_0 = \kappa \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \lambda & 0 & 0 & 0 & \dots \\ \lambda^2 & 0 & 0 & 0 & \dots \\ \lambda^3 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \kappa \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ \lambda^2 & \lambda & 0 & 0 & \dots \\ \lambda^3 & \lambda^2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.56}$$

**Diagnostic expectations**

(Bordalo, Gennaioli, and Shleifer, 2018; Bianchi, Ilut, and Saijo, 2021). Let $E_t^r[dX_{t+h}]$ denote a reference expectation for the variable $h$ periods ahead. Then, the diagnostic expectation with parameter $\theta$ is given by:

$$E_t[dX_{t+h}] = \mathbb{E}_t[dX_{t+h}] + \theta\left(\mathbb{E}_t[dX_{t+h}] - E_t^r[dX_{t+h}]\right).$$

Bordalo et al. (2018) assume that $E_t^r[dX_{t+h}] = \mathbb{E}_{t-1}[dX_{t+h}]$. In this case,

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1+\theta & 0 & 0 & \dots \\ 0 & 0 & 1+\theta & 0 & \dots \\ 0 & 0 & 0 & 1+\theta & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_t = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.57}$$

for $t \geq 1$.

Bianchi et al. (2021) develop a generalization of this framework to allow for long memory, which assumes that $E_t^r[dX_{t+h}] = \sum_{j=1}^{\infty} \alpha_j \mathbb{E}_{t-j}[dX_{t+h}]$. With this assumption, we find

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1+\theta & 0 & 0 & \dots \\ 0 & 0 & 1+\theta & 0 & \dots \\ 0 & 0 & 0 & 1+\theta & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1+\theta(1-\alpha_1) & 0 & \dots \\ 0 & 0 & 0 & 1+\theta(1-\alpha_1) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{B.58}$$

and, for any $t$,

$$\Lambda_t = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 1+\theta(1-\sum_{j=1}^{t}\alpha_j) & 0 & \dots \\ 0 & 0 & 0 & 1+\theta(1-\sum_{j=1}^{t}\alpha_j) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{B.59}$$

**Noisy information and diagnostic expectations**

(Bordalo, Gennaioli, Ma, and Shleifer, 2020) As in noisy information and rational expectations, the agent observes a signal $\epsilon + \nu_t$, where $\nu_t \sim \mathcal{N}(0, 1/\tau_\nu)$. However, the forecaster then overweighs representative states by using the distorted posterior

$$f^\theta(\epsilon|S_t^i) = f(\epsilon|S_t^i)R_t^i(\epsilon)^\theta \frac{1}{Z_t} \tag{B.60}$$

Bordalo et al. (2020) assume that $R_t^i(\epsilon) = f(\epsilon|S_t^i)/f(\epsilon|S_{t-1}^i \cup \{\mathbb{E}_{i,t-1}[\epsilon]\})$. This assumption implies that the mean of the distorted posterior is given by:

$$E_{i,t}^\theta[\epsilon] = \mathbb{E}_{i,t}\left[\epsilon\right] + \theta\left(\mathbb{E}_{i,t}\left[\epsilon\right] - \mathbb{E}_{i,t-1}\left[\epsilon\right]\right) \tag{B.61}$$

where $\mathbb{E}_{i,t}[\epsilon]$ denotes the time-$t$ rational expectation with information set $S_t^i$. It follows that the average expectation is given by

$$\overline{E}_t^\theta[\epsilon] = \underbrace{\left[\frac{\left(\frac{t+1+\theta}{t+1}\right)\tau_\epsilon/\tau_\nu + t}{\tau_\epsilon/\tau_\nu + t}\right]\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}}_{\equiv \lambda_t}\epsilon \tag{B.62}$$

Thus the associated $\Lambda_t$ matrices are

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda_0 & 0 & 0 & \dots \\ 0 & 0 & \lambda_0 & 0 & \dots \\ 0 & 0 & 0 & \lambda_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda_1 & 0 & \dots \\ 0 & 0 & 0 & \lambda_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{B.63}$$

Analogously to Bianchi et al. (2021), we can extend this model to include long-memory as follows. Assume that $R_t^\theta(\epsilon) = f(\epsilon|S_t^i)/f^*(\epsilon|S_t^i)$, where $\epsilon \sim_{f^*|S_t^i} \mathcal{N}(E_t^r[\epsilon], \tau_\epsilon + (t+1)\tau_\nu)$ and $E_t^r[\epsilon] = \sum_{j=1}^t \alpha_j \mathbb{E}_{i,t-j}[\epsilon]$. It follows that

$$E_{i,t}^\theta[\epsilon] = \mathbb{E}_{i,t}[\epsilon] + \theta\left(\mathbb{E}_{i,t}[\epsilon] - E_{i,t}^r[\epsilon]\right). \tag{B.64}$$

As a result, the average expectation is given by

$$\overline{E}_t^\theta[\epsilon] = \left[(1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} - \theta\sum_{j=1}^t \alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right)\right]\epsilon, \tag{B.65}$$

and defining now $\lambda_t \equiv (1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu+t+1} - \theta\sum_{j=1}^t \alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu+t+1-j}\right)$ we obtain the analogous $\Lambda_t$ matrices as above.

## B.3 Appendix to section 4

### B.3.1 Financial intermediary

Set up decision problem formally and derive no-arbitrage conditions.

### B.3.2 Retailers

The Bellman equation of firm $j$ is

$$J_t(p_{jt-1}) = \max_{k_{jt},l_{jt},y_{jt},p_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - h_t l_{jt} - r_t^K k_{jt} - \frac{\psi_p}{2} \left[ \log\left( \frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t + \frac{J_{t+1}(p_{jt})}{1+r_t^e} \right\}$$

$$\text{s.t. } y_{jt} = F_t(k_{jt}, l_{jt})$$

$$y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$$

Substitute the production function and write the problem as

$$J_t(p_{jt-1}) = \max_{k_{jt},l_{jt},p_{jt}} \left\{ \frac{p_{jt}}{P_t} F_t(k_{jt}, l_{jt}) - h_t l_{jt} - r_t^K k_{jt} - \frac{\psi_p}{2} \left[ \log\left( \frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t + \frac{J_{t+1}(k_{jt}, p_{jt})}{1+r_t^e} \right\}$$

$$\text{s.t. } \frac{p_{jt}}{P_t} Y_t = \left( \frac{F_t(k_{jt}, l_{jt})}{Y_t} \right)^{-\frac{1}{\epsilon}} Y_t$$

Let $\eta_{jt}$ denote the Lagrange multiplier on the constraint. The FOCs with respect to $p_{jt}$ and $p_{jt-1}$ are

$$0 = \frac{1}{P_t} F_t(k_{jt}, l_{jt}) - \psi_p \log\left( \frac{p_{jt}}{p_{jt-1}} \right) \frac{Y_t}{p_{jt}} - \eta_{jt} \frac{Y_t}{p_{jt}} + \frac{\partial_p J_{t+1}(k_{jt}, p_{jt})}{1+r_t^e} \tag{B.66}$$

$$\partial_p J_t(k_{jt-1}, p_{jt-1}) = \psi_p \log\left( \frac{p_{jt}}{p_{jt-1}} \right) \frac{Y_t}{p_{jt-1}} \tag{B.67}$$

In symmetric equilibrium, the FOCs simplify to

$$0 = \frac{1}{P_t} F(u_t k_{t-1}, L_t) - \psi_p \log \left( \frac{P_t}{P_{t-1}} \right) \frac{Y_t}{P_t} - \eta_t \frac{Y_t}{P_t} + \frac{1}{1 + r_t^e} \psi_p \log \left( \frac{P_{t+1}}{P_t} \right) \frac{Y_{t+1}}{P_t} \tag{B.68}$$

$$0 = Y_t - \psi_p \log \left( \frac{P_t}{P_{t-1}} \right) Y_t - \eta_t Y_t + \frac{1}{1 + r_t^e} \psi_p \log \left( \frac{P_{t+1}}{P_t} \right) Y_{t+1} \tag{B.69}$$

$$\log(1 + \pi_t) = \frac{1}{\psi_p}(1 - \eta_t) + \frac{1}{1 + r_t^e} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}) \tag{B.70}$$

where $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate. Define the real marginal cost as $mc_t \equiv (\epsilon - \eta_t)/\epsilon$. Then the equilibrium conditions can be summarized as

- Phillips curve:

$$\log(1 + \pi_t) = \frac{\psi_p}{\epsilon} \left( mc_t - \frac{\epsilon - 1}{\epsilon} \right) + \frac{1}{1 + r_t^e} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}) \tag{B.71}$$

- Labor demand:

$$h_t = mc_t \cdot \partial F_L(\tilde{K}_t, L_t) = mc_t (1 - \alpha) \frac{Y_t}{L_t} \tag{B.72}$$

- Capital demand:

$$r_t^K = mc_t \cdot \partial F_K(\tilde{K}_t, L_t) = mc_t \alpha \frac{Y_t}{\tilde{K}_t} \tag{B.73}$$

- Production:

$$Y_t = F_t(\tilde{K}_t, L_t) = \Theta_t \tilde{K}_t^{\alpha} L_t^{1-\alpha} \tag{B.74}$$

- Price adjustment cost:

$$\Psi_t = \frac{\psi_p}{2} \Big[ \log(1 + \pi_t) \Big]^2 Y_t \tag{B.75}$$

- Dividends:

$$d_t^R = Y_t - h_t L_t - r_t^K \tilde{K}_t - \Psi_t \tag{B.76}$$

### B.3.3 Capital producer

The Bellman equation is

$$J_t(K_{t-1}, I_{t-1}) = \max_{K_t, I_t} \left\{ r_t^K K_{t-1} - I_t + \frac{J_{t+1}(K_t, I_t)}{1+r_t} \right\}$$
$$\text{s.t. } K_t = (1-\delta)K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{B.77}$$

Let's define Tobin's $Q$ as the marginal value of capital at the end of period $t$

$$Q_t \equiv \frac{\partial_K J_{t+1}(K_t, I_t)}{1+r_t} \tag{B.78}$$

The FOC with respect to $K_{t-1}$ is

$$\partial_K J_t(K_{t-1}, I_{t-1}) = r_t^K + \frac{\partial_K J_{t+1}^K(K_t, I_t)}{1+r_t}(1-\delta) \tag{B.79}$$

$$Q_t(1+r_t) = r_{t+1}^K + Q_{t+1}(1-\delta) \tag{B.80}$$

The FOC with respect to $I_{t-1}$ is

$$\partial_I J_t(K_{t-1}, I_{t-1}) = \mu_t Q_t \left(\frac{I_t}{I_{t-1}}\right)^2 S'\left(\frac{I_t}{I_{t-1}}\right) \tag{B.81}$$

The FOC with respect to $I_t$ is

$$0 = -1 + Q_t\mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \frac{\partial_I J_{t+1}}{1+r_t^e} \tag{B.82}$$

To summarize, the equilibrium conditions of the capital producer are

- Valuation:

$$1 + r_t = \frac{r_{t+1}^K + Q_{t+1}(1-\delta)}{Q_t} \tag{B.83}$$

- Investment:

$$1 = Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \frac{\mu_{t+1} Q_{t+1}}{1 + r_t^e} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \tag{B.84}$$

- Capital law of motion:

$$K_t = (1-\delta) K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{B.85}$$

- Dividends:

$$d_t^K = r_t^K K_{t-1} - I_t \tag{B.86}$$

For concreteness, let the $S(\bullet)$ be quadratic

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{B.87}$$

$$S'\left(\frac{I_t}{I_{t-1}}\right) = \psi\left(\frac{I_t}{I_{t-1}} - 1\right) \tag{B.88}$$

### B.3.4 Labor agency

The Bellman equation is

$$\begin{aligned} J_t(N_{t-1}) &= \max_{N_t, v_t} \left\{ (h_t - w_t) N_t - (\kappa_v + \kappa_h q_t) v_t + \frac{J_{t+1}(N_t)}{1 + r_t} \right\} \\ &\text{s.t. } N_t = (1 - s_t) N_{t-1} + q_t v_t \end{aligned} \tag{B.89}$$

Let $\lambda_t$ denote the Lagrange multiplier on the constraint. The FOCs wrt $N_t, v_t$, and $N_{t-1}$ are

$$0 = h_t - w_t - \lambda_t + \frac{J'_{t+1}(N_t)}{1+r_t} \tag{B.90}$$

$$0 = -\kappa_v - \kappa_h q_t + \lambda_t q_t \tag{B.91}$$

$$J'_t(N_{t-1}) = \lambda_t(1 - s_t) \tag{B.92}$$

Combining these yields the job creation curve. In sum, the equilibrium conditions are

- Job creation:

$$\frac{\kappa_v}{q_t} + \kappa_h = h_t - w_t + \frac{1 - s_{t+1}}{1+r_t}\left(\frac{\kappa}{q_{t+1}} + \kappa_h\right) \tag{B.93}$$

- Dividends:

$$d^L_t = (h_t - w_t)N_t - (\kappa_v + \kappa_h q_t)v_t \tag{B.94}$$

## B.4 Appendix to Section 2.5

### B.4.1 Fitting a model of beliefs

Figure B.1: Illustration of parametric belief models

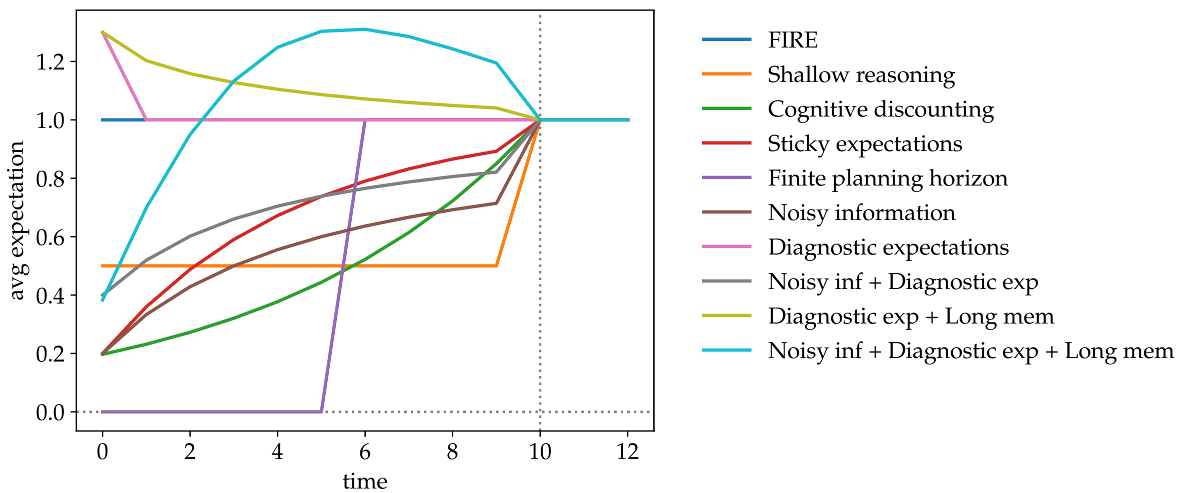
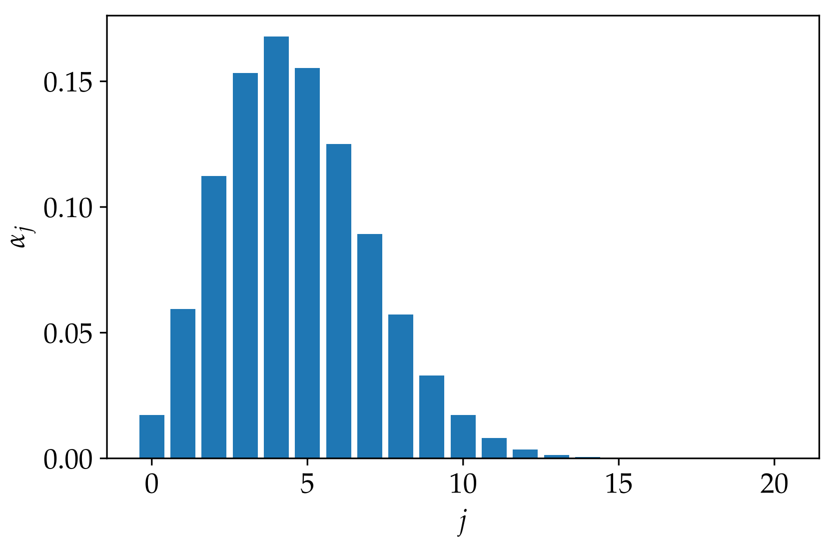


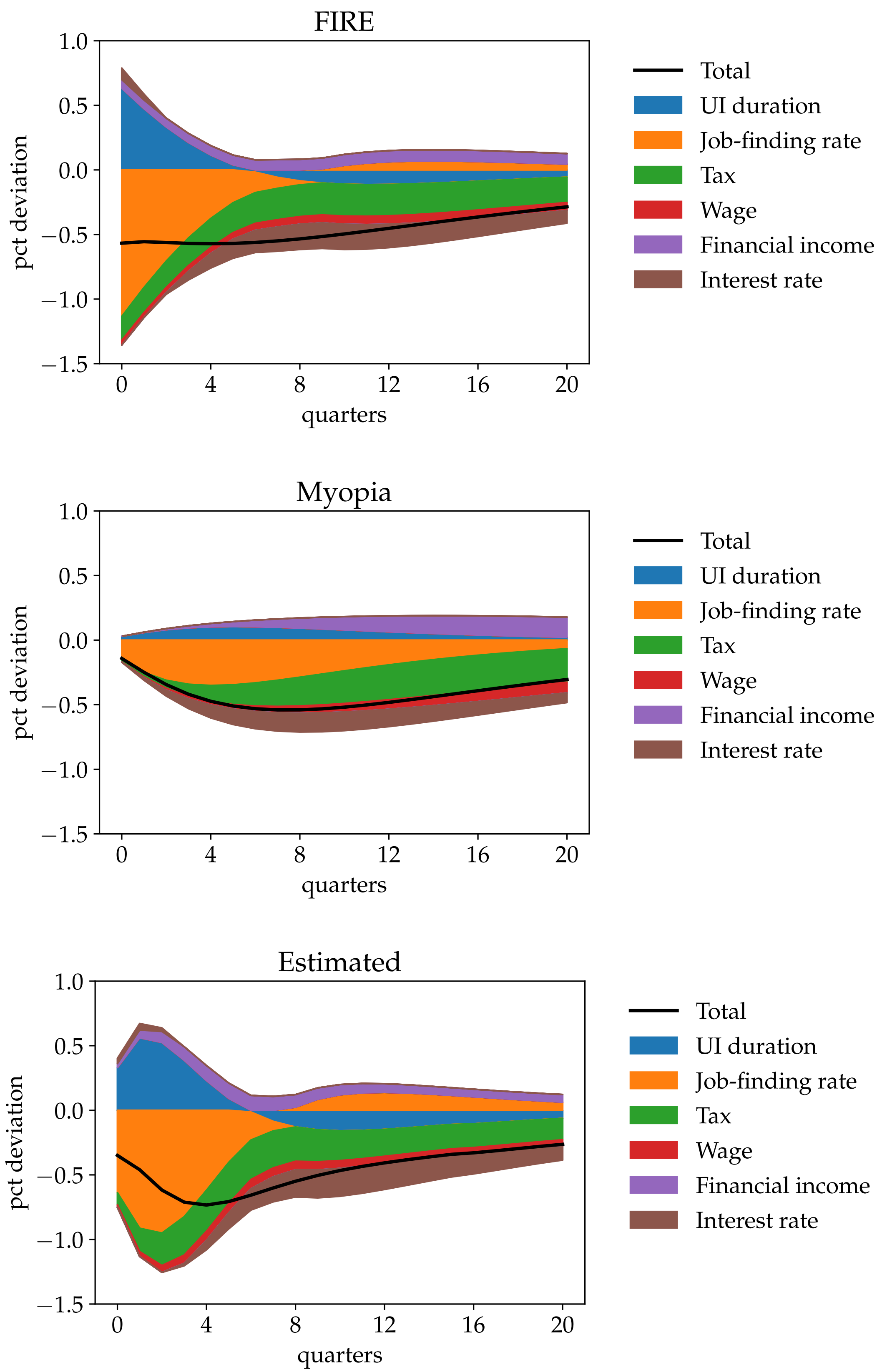

Figure B.2: Estimated memory weights



### B.4.2 Decomposing GE forces

Figure B.3 shows the decomposition of the aggregate consumption response into all the variables that enter the aggregate consumption function directly for each model of belief formation used in our analysis. The job-finding rate and UI duration emerge as the main drivers of consumption response in the first four quarters. The shape of the consumption response to the UI duration is recognizable from the partial equilibrium exercise.

Figure B.3: Decomposition of general equilibrium consumption response



# Appendix C

# Appendix to Chapter Three

## C.1 Appendix to section 3.2

### C.1.1 Individual demand

The household enters time $t$ with assets $A_{i,t}$ and chooses consumption and savings to solve

$$\max E_{i,t} \sum_{h=0}^{\infty} \prod_{s=0}^{h-1} (\beta_{i,t+s}) \left[ u(C_{i,t}) - v\left(N_{i,t}\right) \right], \quad \text{subject to}$$

$$C_{i,t+h} + A_{i,t+h+1} = Y_{g,t+h} + (1+r)A_{i,t+h}.$$

The Euler equation is given by:

$$u'(C_{i,t}) = \prod_{s=0}^{h-1} \beta_{i,t+s}(1+r)E_{i,t}[u'(C_{i,t+h})]. \tag{C.1}$$

I log-linearize the solution to the household problem around a steady-state equilibrium in which $(1+r)\beta = 1$ and all households are symmetrical: $A_i = 0$, $Y_g = Y = 1$, and $C_i = C = Y = 1$. Log-linearizing the Euler equation (C.1) obtains

$$E_{i,t}[c_{i,t+h}] = c_{i,t} - \sigma \sum_{s=0}^{h-1} r^n_{i,t+s}, \tag{C.2}$$

where $c_{i,t} \equiv d\log(C_i, t)$ and $r^n_{i,t} \equiv -d\log(\beta_{i,t})$.

Linearizing the budget constraint, we obtain

$$c_{i,t+h} + a_{i,t+h+1} = y_{g,t+h} + \beta^{-1} a_{i,t+h}, \tag{C.3}$$

where $a_{i,t} = dA_{i,t}$ and $(1 + r) = \beta^{-1}$. Multiplying this equation by $\beta^h$ and iterating forward for each we obtain

$$\sum_{h=0}^{\infty} \beta^h c_{i,t+h} = \sum_{h=0}^{\infty} \beta^h y_{g,t+h} + \beta^{-1} a_{i,t}. \tag{C.4}$$

Taking expectations and replacing equation (C.2) obtains

$$\begin{aligned}
&\sum_{h=0}^{\infty} \beta^h \left[ c_{i,t} - \sigma \sum_{s=0}^{h-1} r^n_{i,t+s} \right] = \sum_{h=0}^{\infty} \beta^h E_{i,t}[y_{g,t+h}] + \beta^{-1} a_{i,t} \\
\Leftrightarrow &\frac{1}{1-\beta} c_{i,t} = \sum_{h=0}^{\infty} \beta^h E_{i,t}[y_{g,t+h}] + \beta^{-1} a_{i,t} + \sum_{s=0}^{\infty} \sum_{h=s+1}^{\infty} \beta^h r^n_{i,t+s} \\
\Leftrightarrow &c_{i,t} = (1-\beta) \sum_{h=0}^{\infty} \beta^h E_{i,t}[y_{g,t+h}] + (1-\beta)\beta^{-1} a_{i,t} + \sum_{h=0}^{\infty} \beta^{h+1} r^n_{i,t+h}.
\end{aligned}$$

### C.1.2 Proof of proposition 9

Computing group-average demand we obtain

$$\overline{c}_{g,t} = (1-\beta) \sum_{h=0}^{\infty} \beta^h \gamma_g \overline{E}_{g,t}[y_{t+h}] + (1-\beta)\beta^{-1} \overline{a}_{g,t} + \sum_{h=0}^{\infty} \beta^{h+1} \overline{r}^n_{g,t+h}.$$

Aggregating across groups, we obtain

$$\begin{aligned}
c_t &= \sum_g \pi_g \overline{c}_{g,t} \\
\Leftrightarrow c_t &= \sum_g \pi_g \left[ (1-\beta) \sum_{h=0}^{\infty} \beta^h \gamma_g \overline{E}_{g,t}[y_{t+h}] + (1-\beta)\beta^{-1} \overline{a}_{g,t} + \sum_{h=0}^{\infty} \beta^{h+1} \overline{r}^n_{g,t+h} \right] \\
\Leftrightarrow c_t &= (1-\beta) \sum_{h=0}^{\infty} \beta^h \sum_g \pi_g \gamma_g \overline{E}_{g,t}[y_{t+h}] + \sum_{h=0}^{\infty} \beta^{h+1} r^n_{t+h},
\end{aligned}$$

where $\sum_g \pi_g \overline{a}_{g,t} = 0$ by asset market clearing and $\sum_g \pi_g \overline{r}^n_{g,t+h} \equiv r^n_{t+h}$. Finally, note that

$$\begin{aligned}
\sum_g \pi_g \gamma_g \overline{E}_{g,t}[y_{t+h}] &= \underbrace{\left[ \sum_g \pi_g \gamma_g \right]}_{=1} \cdot \underbrace{\left[ \sum_g \pi_g \overline{E}_{g,t}[y_{t+h}] \right]}_{=\overline{E}_t[y_{t+h}]} + \mathbb{C}\text{ov}\left( \gamma_g, \overline{E}_{g,t}[y_{t+h}] \right) \\
&= \overline{E}_t[y_{t+h}] + \mathbb{C}\text{ov}\left( \gamma_g, \frac{\overline{E}_{g,t}[y_{t+h}]}{\overline{E}_t[y_{t+h}]} \right) \overline{E}_t[y_{t+h}] = (1 + \text{CD}_{t+h}) \cdot \overline{E}_t[y_{t+h}].
\end{aligned}$$

Aggregate demand is thus given by

$$c_t = (1-\beta) \sum_{h=0}^{\infty} \beta^h (1 + \text{CD}_{t+h}) \cdot \overline{E}_t[y_{t+h}] + \sum_{h=0}^{\infty} \beta^{h+1} r^n_{t+h}. \tag{C.5}$$

Finally, market clearing for goods market requires $c_t = y_t$, and so equilibrium output solves

$$y_t = (1-\beta) y_t + (1-\beta) \sum_{h=1}^{\infty} \beta^h (1 + \text{CD}_{t+h}) \cdot \overline{E}_t[y_{t+h}] + \sum_{h=0}^{\infty} \beta^{h+1} r^n_{t+h} \tag{C.6}$$

$$\Leftrightarrow y_t = (1-\beta) \sum_{h=1}^{\infty} \beta^{h-1} (1 + \text{CD}_{t+h}) \cdot \overline{E}_t[y_{t+h}] + \sum_{h=0}^{\infty} \beta^h r^n_{t+h}. \tag{C.7}$$

Assuming $\overline{E}_{g,t}[y_{t+h}] = \lambda_g y_{t+h}$ and defining $\overline{\lambda} \equiv \sum_g \pi_g \lambda_g$, we can write $\overline{E}_t[y_{t+h}] = \overline{\lambda} y_{t+h}$

and

$$\mathrm{CD}_{t,h} = \mathbb{C}\mathrm{ov}\left(\gamma_g, \frac{\overline{E}_{g,t}[y_{t+h}]}{\overline{E}_t[y_{t+h}]}\right) = \mathbb{C}\mathrm{ov}\left(\gamma_g, \frac{\lambda_g}{\overline{\lambda}}\right) \equiv \mathrm{CD}.$$

Replacing these expressions in the equation above, we finally obtain the following:

$$y_t = (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\cdot(1+\mathrm{CD})\cdot\overline{\lambda}y_{t+h} + \sigma\sum_{h=0}^{\infty}\beta^h r^n_{t+h}. \tag{C.8}$$

### C.1.3 Proof of proposition 10

Note that equation (3.18) can be equivalently written as

$$\begin{aligned} y_t &= (1-\beta)(1+\mathrm{CD})\cdot\overline{\lambda}y_{t+1} + \sigma r^n_t \\ &\quad + \beta\underbrace{\left[(1-\beta)\sum_{h=2}^{\infty}\beta^{h-2}\cdot(1+\mathrm{CD})\cdot\overline{\lambda}y_{t+h} + \sigma\sum_{h=1}^{\infty}\beta^{h-1}r^n_{t+h}\right]}_{=y_{t+1}} \\ \Leftrightarrow y_t &= \left[\beta + (1-\beta)(1+\mathrm{CD})\cdot\overline{\lambda}\right]y_{t+1} + \sigma r^n_t. \end{aligned}$$

If $r^n_t = \rho^t r_0$, then the unique solution to this difference with $\lim_{t\to\infty} y_t = 0$ satisfies $y_t = \rho^t y_0$ and

$$\rho^t y_0 = \left[\beta + (1-\beta)(1+\mathrm{CD})\cdot\overline{\lambda}\right]\rho^{t+1}y_0 + \sigma\rho^t r^n_0 \Leftrightarrow y_0 = \frac{\sigma r^n_0}{1-\rho\left[\beta + (1-\beta)(1+\mathrm{CD})\cdot\overline{\lambda}\right]}. \tag{C.9}$$

The solution with disagreement is

$$y_t = \frac{\sigma\rho^t r^n_0}{1-\rho\left[\beta + (1-\beta)(1+\mathrm{CD})\cdot\overline{\lambda}\right]} \tag{C.10}$$

and with constant attention

$$y_t = \frac{\sigma \rho^t r_0^n}{1 - \rho \left[\beta + (1-\beta) \cdot \overline{\lambda}\right]}. \tag{C.11}$$

Computing amplification we obtain

$$\mathcal{A}_t = \frac{(1-\beta)\rho \cdot \text{CD} \cdot \overline{\lambda}}{1 - \rho \left[\beta + (1-\beta)(1+\text{CD}) \cdot \overline{\lambda}\right]}.$$

Note that

$$\frac{d\mathcal{A}_t}{d\text{CD}} = \frac{\left(1 - \rho \left(\overline{\lambda} + \beta - \beta\overline{\lambda}\right)\right)(1-\beta)\rho \cdot \overline{\lambda}}{\left(1 - \rho \left[\beta + (1-\beta)(1+\text{CD}) \cdot \overline{\lambda}\right]\right)^2} > 0 \tag{C.12}$$

and so amplification is increasing in correlated disagreement.

Furthermore, the comparative static concerning persistence is given by

$$\frac{d\mathcal{A}_t}{d\rho} = \frac{(1-\beta) \cdot \text{CD} \cdot \overline{\lambda}}{\left(1 - \rho \left[\beta + (1-\beta)(1+\text{CD}) \cdot \overline{\lambda}\right]\right)^2} \tag{C.13}$$

This derivative has the same sign as CD.

Finally, the comparative static concerning $\beta$ is given by

$$\frac{d\mathcal{A}_t}{d\beta} = -\frac{\beta\rho\,(1-\rho) \cdot \text{CD} \cdot \overline{\lambda}}{\left(1 - \rho \left[\beta + (1-\beta)(1+\text{CD}) \cdot \overline{\lambda}\right]\right)^2}, \tag{C.14}$$

which has the opposite sign as CD.

### C.1.4 Utility cost of inattention

In this appendix, I derive the expression for the utility costs of attention, equation (3.23). The exposition here translates the discussion in Gabaix (2016) to the setting in this paper. As the main text discusses, individuals face no uncertainty around their forecasts. I define the value function of an individual with full attention as follows:

$$V_{i,t}(A; \{Y_{g,t+h}\}_{h\geq 0}) = \max_C \{u(C) + \beta_{i,t} V_{i,t+1}(Y_{g,t} + (1+r)A - C)\} \quad \text{(C.15)}$$

and the objective function in this problem is

$$v_{i,t}(C) \equiv u(C) + \beta_{i,t} V_{i,t+1}(Y_{g,t} + (1+r)A - C). \quad \text{(C.16)}$$

Instead, the problem of an inattentive individual is given by

$$u(C) + \beta_{i,t} E_{i,t}[V_{i,t+1}(Y_{g,t} + (1+r)A - C)]. \quad \text{(C.17)}$$

Note that the individual acts assuming they will not update their beliefs. First, we want to make a second-order approximation of the objective function around the point in which $C = 1$, $A = 0$, $Y_{g,t} = 1$, i.e., around the unshocked steady-state equilibrium. This approximation yields

$$v_{i,t}(c) = v(0) + \frac{1}{2}\frac{\partial^2 v}{\partial C^2}c^2 + \sum_{h=0}^{\infty}\frac{\partial^2 v}{\partial C \partial Y_h} \cdot c \cdot y_{g,t+h} + \frac{\partial^2 v}{\partial C \partial A} \cdot c \cdot a + \sum_{h=0}^{\infty}\frac{\partial^2 v}{\partial C \partial \beta_{i,h}}\beta \cdot c \cdot r^n_{i,t+h}$$
$$+ \text{terms independent of } C, \quad \text{(C.18)}$$

where $v(0) = \frac{u(1)}{1-\beta}$, and note that because the Euler equation $\frac{\partial v_{i,t}(C)}{\partial C} = 0$ holds, then we can write the following second-order derivatives. First, the curvature in $C$ is given by

$$\frac{\partial^2 v}{\partial C^2} = u''(1) + \beta \frac{\partial V}{\partial A^2} = \beta^{-1} u''(1).$$

because

$$\frac{\partial V}{\partial A^2} = \beta^{-1}\frac{\partial u'(C^*(A))}{\partial A} = u''(1)(1-\beta)\beta^{-2}.$$

By similar logic, we can write

$$\frac{\partial^2 v}{\partial C \partial A} = -\frac{\partial^2 v}{\partial C^2} \underbrace{(1-\beta)\beta^{-1}}_{=\partial C/\partial A},$$

$$\frac{\partial^2 v}{\partial C \partial Y_h} = -\frac{\partial^2 v}{\partial C^2} \underbrace{(1-\beta)\beta^{h}}_{=\partial C/\partial Y_h},$$

and

$$\frac{\partial^2 v}{\partial C \partial \beta_{i,h}} = -\frac{\partial^2 v}{\partial C^2} \underbrace{\sigma\beta^{h}}_{=\partial C/\partial \beta_h},$$

Note that the solution to the problem of maximizing expected utility in (C.18) yields the same solution we have derived before:

$$c^*_{g,t}(a,\lambda_i) = (1-\beta)\sum_{h=0}^{\infty}\beta^h E_{i,t}[y_{g,t+h}] + (1-\beta)\beta^{-1}a + \sigma\sum_{h=0}^{\infty}\beta^{h+1}r^n_{i,t+h}.$$

Note that, for any $C$, we can write the utility value as

$$v_{i,t}(c) = v(0) + \frac{1}{2}\frac{\partial^2 v}{\partial C^2}c^2 - \frac{\partial^2 v}{\partial C^2}\cdot c\cdot c^*_{g,t}(a,1) + \text{terms independent of } C,$$

where $c^*_{g,t}(a,1)$ denotes the rational expectations demand.

The realized utility cost of inattention is given by

$$v_{i,t}(c^*_{g,t}(a,1)) - v_{i,t}(c^*_{g,t}(a,\lambda_i)) = -\frac{1}{2}\frac{\partial^2 v}{\partial C^2}\left(c^*_{g,t}(a,1) - c^*_{g,t}(a,\lambda_i)\right)^2, \tag{C.19}$$

and note that

$$\left(c^*_{g,t}(a,1) - c^*_{g,t}(a,\lambda_i)\right)^2 = \left(\sum_{h=1}^{\infty}\frac{\partial C}{\partial Y_h}(1-\lambda_i)y_{g,t+h}\right)^2 = \sum_{h=1}^{\infty}\sum_{\tilde{h}=1}^{\infty}\frac{\partial C}{\partial Y_h}\frac{\partial C}{\partial Y_{\tilde{h}}}(1-\lambda_i)^2 y_{g,t+h}y_{g,t+\tilde{h}}. \tag{C.20}$$

It follows that the ex-ante utility cost of inattention is given by

$$\mathcal{C}_g(\lambda_i) = -\frac{1}{2}\frac{\partial^2 v}{\partial C^2}\sum_{h=1}^{\infty}\sum_{\tilde{h}=1}^{\infty}\frac{\partial C}{\partial Y_h}\frac{\partial C}{\partial Y_{\tilde{h}}}(1-\lambda_i)^2 \cdot \gamma_g^2 \sigma_{h,\tilde{h}}, \tag{C.21}$$

where $\sigma_{h,\tilde{h}}$ denotes the ex-ante perceived covariance between $y_{t+h}$ and $y_{t+\tilde{h}}$ which is assumed to depend only on the horizons and not date $t$.

### C.1.5 Horizon-varying attention

We may think about situations where individuals have different attentions based on the forecast horizon. So, suppose that attention varies with the horizon, then we replace the structural relation (3.16) with

$$\overline{E}_{g,t}[y_{t+h}] = \lambda_{g,h} y_{t+h}. \tag{C.22}$$

In this case,

$$\overline{E}_t y_{t+h} = \overline{\lambda}_h y_{t+h} \quad \text{and} \quad \mathrm{CD}_h \equiv \mathbb{C}\mathrm{ov}\left(\gamma_g, \lambda_{g,h}/\overline{\lambda}_h\right),$$

where $\overline{\lambda}_h \equiv \sum_g \pi_g \lambda_{g,h}$.

Equilibrium output now satisfies

$$y_t = (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\cdot(1+\mathrm{CD}_h)\cdot\overline{\lambda}_h y_{t+h} + \sigma\sum_{h=0}^{\infty}\beta^h r_{t+h}^n. \tag{C.23}$$

It follows that the results in Proposition 9 still hold under the caveat that we must require the properties to be met for all horizons.

Defining amplification in the same way, we find that

$$\mathcal{A}_t = \sum_{h=1}^{\infty}\{\mathrm{CD}_h + (1+\mathrm{CD}_h)\,\mathcal{A}_{t+h}\}\cdot\frac{(1-\beta)\cdot\beta^{h-1}\cdot\overline{\lambda}_h y_{t+h}}{y_t^{\mathrm{RA}}} \tag{C.24}$$

It follows that amplification is positive if $\mathrm{CD}_h > 0$ for all $h$ and negative if $\mathrm{CD}_h < 0$ for all $h$. However, exactly how amplification depends on persistence, and the discount factor becomes less clear. This expression, however, suggests two facts: (1) the longer the effects on output, the more significant is the impact of correlated disagreement, and (2) the more important the general equilibrium channel, the more significant the impact of correlated disagreement. The first can be achieved via a persistent shock, and the second with a higher marginal propensity to consume.

To endogenize attention, I allow individuals to optimize their level of attention for each horizon $\lambda_{i,h}$. It turns out that the utility costs of inattention can be written analogously to what we have found before

$$\mathcal{C}_g(\boldsymbol{\lambda}_i) = -\frac{1}{2}\frac{\partial^2 v}{\partial C^2}\sum_{h=1}^{\infty}\sum_{\tilde{h}=1}^{\infty}\frac{\partial C}{\partial Y_h}\frac{\partial C}{\partial Y_{\tilde{h}}}(1-\lambda_{i,h})(1-\lambda_{i,\tilde{h}})\cdot\gamma_g^2\sigma_{h,\tilde{h}}. \tag{C.25}$$

Albeit not essential, I assume, as in Gabaix (2014), that people perceive no correlation across the variables. Optimal attention thus solves

$$\min_{\boldsymbol{\lambda}_I} -\frac{1}{2}\frac{\partial^2 v}{\partial C^2}\sum_{h=1}^{\infty}\left(\frac{\partial C}{\partial Y_h}\right)^2(1-\lambda_{i,h})^2\cdot\gamma_g^2\sigma^2 + \kappa\sum_h \lambda_{i,h}. \tag{C.26}$$

**Proposition 14.** *Optimal attention to horizon $h$ is given by*

$$\lambda_{i,h} = \lambda_{g,h} \equiv \max\left\{0, 1-\frac{\kappa}{\Lambda_h\gamma_g^2}\right\},$$

*where $\Lambda_h \equiv -\frac{1}{2}\frac{\partial^2 v}{\partial C^2}\left(\frac{\partial C}{\partial Y_h}\right)^2\sigma^2$. It follows that:*

1. *Attention $\lambda_{g,h}$ is increasing in $\gamma_g$.*

2. *Attention is decreasing in horizon $h$. There exists $H > 0$ such that $\lambda_{g,h} = 0$ for all $h > H$.*

This extended model still holds the central result that attention is increasing in the income cyclicality. Furthermore, we also find that attention is decreasing in the forecast horizon. The reason for this implication is as follows. The present the value of a change in income at date $h$ is given by $\beta^h$, which means that the longer the horizon, the lower the impact that those changes in income have on contemporaneous consumption. So, people choose to devote more attention to incomes that are relatively close in time than to incomes that are further away in the future.

Note also that for sufficiently far-off events, individuals become fully inattentive. Intuitively, for enough distant events, their impact on current consumption would be so small that it does not pay off to exert the cognitive effort of trying to forecast them. Interestingly, this model generates an endogenous "finite planning horizon", a behavioral feature analyzed in Woodford (2018) and Woodford and Xie (2019, 2022).

## C.2 Appendix to section 3.3

In this appendix, I describe the data used in section 3.3. The data comes from the Survey of Consumer Expectations (SCE) and the Current Population Survey (CPS).

**Survey of Consumer Expectations** The SCE is a monthly internet rotating panel survey of one thousand and three hundred (1,300) households that started in June 2013. New respondents are drawn to match demographic factors from the American Community Survey, ensuring population representativeness, and stay on the panel for up to twelve months. To increase data availability, I aggregate individual responses to the quarterly level by averaging within that time frame.

In this paper, I use the responses to the following question: "*Suppose again that, 12 months from now, you are working in the exact same/main job at the same place you cur-*

*rently work, and working the exact same number of hours. In your view, what would you say is the percent chance that 12 months from now your earnings on this job, before taxes and deductions, will have...*" Respondents are asked to assign probabilities to ten different bins: higher than 12%, between 8% and 12%, between 4% and 8%, between 2% and 4%, between 0% and 2%, between -2% and 0%, between -4% and -2%, between -8% and -4%, between -12% and -8%, and lower than -12%.

The SCE estimates a density distribution for household forecasts using the approach in Engelberg et al. (2009). I assume that this estimated mean captures $E_{i,t}[\Delta y_{i,t+h}]$ where the horizon $h = 4$ quarters or 1 year.

**Current Population Survey** The CPS is a monthly survey of around sixty thousand U.S. households (60,000) conducted by the BLS, starting from 1940. This survey contains detailed microdata on employment and income characteristics of members within a household.

To estimate the income cyclicality parameters, I use yearly data from the ASEC March Supplement of the CPS from 2000 to 2019. I remove the Covid-19 recession from this estimation due to its unusual features in terms of labor market incidence. Using the monthly responses for 2012 to 2021, I also compute state-level average income growth $\Delta \overline{y}_{S,t}$ for 2013-21.

### C.2.1 Forecast error

In the CPS data, I consider individuals who are in the labor force aged 20 to 64, who are active in the labor force, and who are not in the military. As in Acemoglu and Autor (2011), I multiply top-coded weekly earnings and hourly wages by 1.5. When not available, I compute weekly earnings using the information on hourly wages and weekly hours of work. I deflate these weekly earnings by the CPI to measure real earnings. I then use weekly earnings to compute average income growth at the state level using the sample earnings weights. I also aggregate the SCE responses to obtain a state-level average forecast using the sample weights.

Using this data, the state-level average forecast error is defined as

$$\overline{\mathrm{FE}}_{S,t} \equiv \Delta y_{i,t+h} - \overline{E}_{S,t}[\Delta y_{i,t+h}]. \tag{C.27}$$

### C.2.2 Income cyclicality

To estimate $\gamma_g$, I use March Supplement CPS data. I restrict the analysis to households aged 20 to 64 active in the labor force and not in the military. I focus on the set of 14 census industries by matching the 1990 industry information to their corresponding industry. The precise matching can be found in table C.1.

Table C.1: Census industry and 1990 Industrial Class. System

| | Industry | Start | End | | Industry | Start | End |
|---|---|---|---|---|---|---|---|
| 1 | Agriculture, Forestry, Fishery | 10 | 32 | 8 | Non-durable Man. | 100 | 229 |
| 2 | Public Administration | 900 | 932 | 9 | Durable Man. | 230 | 392 |
| 3 | Bus. and Repair Services | 721 | 760 | 10 | Retail Trade | 580 | 691 |
| 4 | Prof. and Related Serv. | 812 | 893 | 11 | Wholesale Trade | 500 | 571 |
| 5 | Mining | 40 | 50 | 12 | Personal Services | 761 | 791 |
| 6 | Transp., Commun., Public Util. | 400 | 472 | 13 | Finance, Insur., Real Est. | 700 | 712 |
| 7 | Construction | 60 | 60 | 14 | Ent. and Recr. Serv. | 800 | 810 |

As in Acemoglu and Autor (2011), I multiply top-coded weekly earnings and hourly wages by 1.5. When not available, I compute weekly earnings using the information on hourly wages and weekly hours of work. I deflate these weakly earnings by the CPI to measure real earnings. I use the unique individual identifier to match individuals across consecutive years.[1] I compute income growth for each individual and calculate nationwide aggregate income growth using individual earnings weights.

At the industry level, I regress individual income growth on aggregate income growth and recover the estimated parameter $\tilde{\gamma}_g$. I include a vector of controls for a cubic polynomial of age, sex, race, state, and level of education. In practice, I find that the conclusions do not change if we exclude the vector of controls. I renormalize $\gamma_g = \tilde{\gamma}_g/(\sum_g \pi_g \tilde{\gamma}_g)$ using the industry shares in 2018.

### C.2.3 Robustness

Table C.2: Robustness exercises 2

| | Magnitude of Forecast Errors | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\overline{\gamma}_S$ | $-1.02$ | $-1.73^{**}$ | $-1.62^{**}$ | $-1.56^{**}$ | $-1.76^{**}$ | $-1.64^{**}$ | $-1.63^{**}$ | $-1.19^{*}$ |
| *Quarter F.E.* | ✓ | | | | | | | ✓ |
| *High-skill share* | | ✓ | | | | | | ✓ |
| *Numeracy share* | | | ✓ | | | | | ✓ |
| *Avg. Tenure* | | | | ✓ | | | | ✓ |
| *Avg. Age* | | | | | ✓ | | | ✓ |
| *Med. Income Share* | | | | | | ✓ | | ✓ |
| *High Income Share* | | | | | | | ✓ | ✓ |

[1] I use only matches for which race and sex coincide and for which age is consistent across the two observations.

**Corrected forecast errors**

In the baseline empirical exercise, I use the state's average forecast error as obtained in the data. However, note that in that baseline analysis, the unpredictable component of forecast errors is also increasing in income cyclicality. So, the expected magnitude of forecast errors can decrease in cyclicality only if attention increases sufficiently fast with income cyclicality.

In this appendix, in order to remove the dependence on the state's business cycle exposure, I divide the state's forecast error by its average exposure, $\overline{\gamma}_S$ and looking at the following regression:

$$(\overline{\mathrm{FE}}_{S,t}/\overline{\gamma}_{S,t})^2 = \alpha + \beta\overline{\gamma}_{S,t} + \varepsilon_{S,t}. \tag{C.28}$$

The main result can be found in Figure C.1 and the robustness results taking into account a variety of relevant control variables in Table C.3.

Figure C.1: Income cyclicality and the magnitude of forecast errors

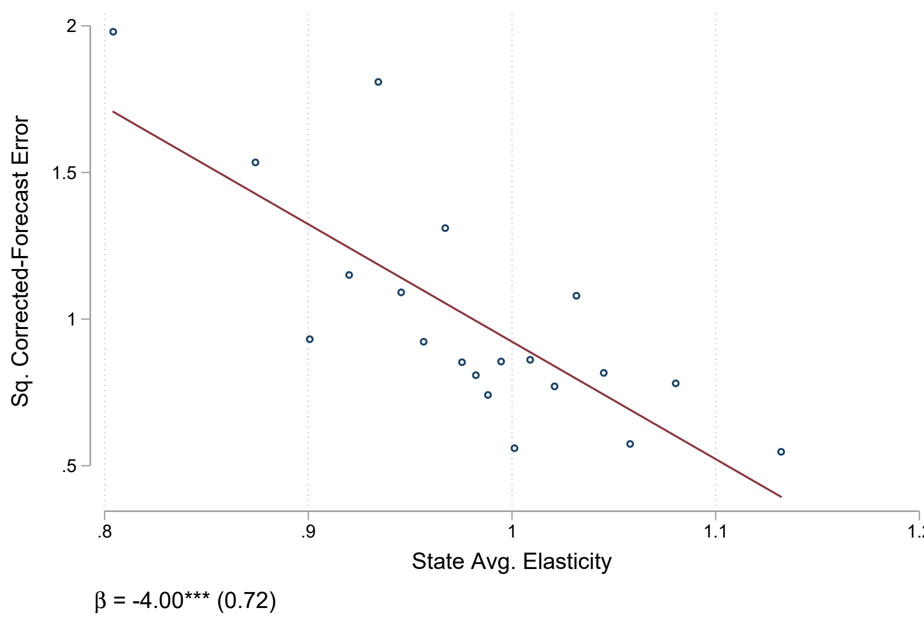


Table C.3: Robustness exercises

| | Magnitude of Forecast Errors | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\overline{\gamma}_S$ | $-3.37^{***}$ | $-4.10^{***}$ | $-4.00^{***}$ | $-3.94^{***}$ | $-4.15^{***}$ | $-4.02^{***}$ | $-4.02^{***}$ | $-3.55^{***}$ |
| *Time F.E.* | ✓ | | | | | | | ✓ |
| *High-skill share* | | ✓ | | | | | | ✓ |
| *Num. share* | | | ✓ | | | | | ✓ |
| *Avg. Tenure* | | | | ✓ | | | | ✓ |
| *Avg. Age* | | | | | ✓ | | | ✓ |
| *Med. Inc. share* | | | | | | ✓ | | ✓ |
| *High Inc. share* | | | | | | | ✓ | ✓ |

## C.3 Appendix to section 3.4

### C.3.1 Unions and labor supply

In this section, I derive the wage Phillips curve. At time $t$, union $u$ sets the wage to maximize

$$\sum_{h\geq 0}\beta^h\left[u'\left(C_{t+h}\right)\left(1-\tau_{t+h}\right)\frac{W_{u,t+h}N_{u,t+h}}{P_{t+h}}-v'\left(N_{t+h}\right)N_{u,t+h}-\frac{1}{2\tilde{\kappa}_w}\left(\frac{W_{u,t+h}}{W_{u,t+h-1}}-1\right)^2\right].$$

I assume that the union works to maximize a utility valuation of the income derived from the union labor supply and the utility cost of labor supply, subject to quadratic wage adjustment costs. Note that, to measure the utility valuation, I use the aggregates for consumption and labor supply, which implies that the union ignores the distributional consequences of its decisions. Alternatively, we could assume that the union maximizes an average utility valuation considering these distributional consequences, as in Auclert et al. (2018). In practice, this would make little quantitative difference but would have the computational cost of computing the average marginal utility of consumption and labor at each point. For these reasons, I focus on this more straightforward representation of the Phillips curve.

When setting the wage $W_{u,t}$, the union behaves monopolistically, taking into account the response of demand which is given by

$$N_{u,t} = \left(\frac{W_{u,t}}{W_t}\right)^{-\mu_w} N_t,$$

Taking first-order conditions, we obtain the following non-linear Phillips curve

$$\left(e^{\pi_t^w} - 1\right) e^{\pi_t^w} = \tilde{\kappa}_w \left(\mu_w - 1\right) \left[-u'\left(C_t\right)\left(1-\tau_t\right)Y_t + \frac{\mu_w}{\mu_w - 1} v'\left(N_t\right) N_t\right] + \beta_t \left(e^{\pi_{t+1}^w} - 1\right) e^{\pi_{t+1}^w}.$$

Linearizing this equation, we obtain

$$\pi_t^w = \kappa_w \left[\sigma^{-1} c_t + \psi^{-1} n_t - \left(y_t - \hat{\tau}_t - n_t\right)\right] + \beta \pi_{t+1}^w, \tag{C.29}$$

where $\kappa_w \equiv \tilde{\kappa}_w \mu_w v'\left(N\right) N$.

### C.3.2 Jacobians without FIRE in Section 3.4.6

In this appendix, I provide the central sketch for the result in Section 3.4.6. Following Auclert et al. (2021) and Auclert et al. (2020), I consider a generic representation of a heterogeneous-agent problem as a mapping from some inputs $\mathbf{X}_t$ to a time-path of aggregates $\mathbf{C}_t$. In the model of this paper, the heterogeneous-agent blocks are each group of households, the aggregates are the group's average consumption and savings, and the inputs are their incomes, taxes, and interest rates. To simplify, I will work with a representation with a single input and output, but the analysis can be easily extended to multiple inputs and outputs, see Auclert et al. (2021). Furthermore, I also assume that all individuals in a single group share the same beliefs. This can be easily extended.

Let $v_t$ denote the marginal utility of consumption The generic problem is

$$\mathbf{v}_t = u\left(\mathbf{v}_{t+1}^{e,t}, X_t\right), \quad \text{for } t \geq 0 \tag{C.30}$$

$$\mathbf{v}_s^{e,t} = u\left(\mathbf{v}_{s+1}^{e,t}, X_s^{e,t}\right), \quad \text{for } t \geq 0, s \geq t+1 \tag{C.31}$$

$$\mathbf{D}_{t+1} = \Lambda\left(\mathbf{v}_{t+1}^{e,t}, X_t\right)' \mathbf{D}_t, \quad \text{for } t \geq 0 \tag{C.32}$$

$$C_t = c\left(\mathbf{v}_{t+1}^{e,t}, X_t\right)' \mathbf{D}_t. \tag{C.33}$$

Here $\mathbf{v}_t$ is the marginal utility of consumption which is related to the future expected future marginal utility of consumption $\mathbf{v}_{t+1}^{e,t}$ and the input today $X_t$. The problem is discretized to $n_g$ grid points, so $\mathbf{v}_t$ is $n_g \times 1$. The distribution over these grid points is given by $\mathrm{D}_{t+1}$ and the individual consumption choices are given by $c\left(\mathbf{v}_{t+1}^{e,t}, X_t\right)$. Equation (C.30) thus represents the Euler equation, and (C.31) defines the predicted future Euler equations. Equation (C.32) determines how the distribution is updated given some transition matrix $\Lambda$. Finally, equation (C.33) determines how individual choices are aggregated.

We are interested in the consumption response to an increase in the actual variable $X_t$ and the expectations expectation $X_s^{e,t}$. As it turns out, we can write the response to an unanticipated $\partial X_t$ as follows

$$\frac{\partial C_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s, \\ \mathcal{J}_{t-s,0} & \text{if } t \geq s, \end{cases}$$

where $\mathcal{J}$ denotes the FIRE Jacobian. The effects of a shock to beliefs can be written as

$$\frac{\partial C_t}{\partial X_s^{e,m}} = \begin{cases} 0 & \text{if } t < m \text{ or } s \leq m, \\ \frac{\partial C_{t-m}}{\partial X_{s-m}^{e,0}} = \mathcal{J}_{t-m,s-m} - \mathcal{J}_{t-m-1,s-m-1} & \text{if } t > m \text{ and } s > m, \\ \mathcal{J}_{0,s-m} & \text{if } t = m \text{ and } s > m. \end{cases}$$

It follows that

$$\begin{aligned} dC_t &= \sum_{s=0}^{t} \mathcal{J}_{t-s,0} dX_s + \sum_{m=0}^{t-1} \sum_{s=m+1}^{\infty} \left(\mathcal{J}_{t-m,s-m} - \mathcal{J}_{t-m-1,s-m-1}\right) dX_s^{e,m} \\ &\quad + \sum_{s=t+1}^{\infty} \mathcal{J}_{0,s-t} dX_s^{e,t} \\ \Leftrightarrow dC_t &= \sum_{s=0}^{t} \mathcal{J}_{t-s,0} \left(dX_s - dX_s^{e,s-1}\right) + \sum_{m=1}^{t} \sum_{s=m+1}^{\infty} \mathcal{J}_{t-m,s-m} \left(dX_s^{e,m} - dX_s^{e,m-1}\right) \\ &\quad + \sum_{s=0}^{\infty} \mathcal{J}_{t,s} dX_s^{e,0}. \end{aligned}$$

Using the defition that $X_s^{e,t} = X_s$ if $s \leq t$, we can write the above expression in vector form:

$$d\mathbf{C} = \mathcal{J} \cdot \underbrace{d\mathbf{X}^{e,0}}_{\text{Initial belief}} + \sum_{t \geq 1} \mathcal{R}_t \cdot \underbrace{\left(d\mathbf{X}^{e,t} - d\mathbf{X}^{e,t-1}\right)}_{\text{Forecast Revision at time } t}, \tag{C.34}$$

where $\mathcal{J} \equiv [\mathcal{J}_{t,s}]$ and

$$\mathcal{R}_t \equiv \begin{bmatrix} 0 & \mathbf{0}_t' \\ \mathbf{0}_t' & \mathcal{J} \end{bmatrix}.$$

### C.3.3 Utility cost of inattention

For every $(a, z)$ such that the policy function implies $c_{g,t}^*(a, z) < (1-\tau_t)y(z_{i,t}, Y_{g,t}) + (1-r_t)a$, we can proceed in the same way as before. I define the value function of an individual with full attention as follows:

$$V_{i,t}(a,z) = \max_{c}\{u(c) + \beta_{i,t}\mathbb{E}_t[V_{i,t+1}((1-\tau_t)y(z_{i,t},Y_{g,t}) + (1-r_t)a - c, z')]\} \tag{C.35}$$

and the objective function in this problem is

$$v_{i,t}(c;a,z) \equiv u(c) + \beta_{i,t}\mathbb{E}_t[V_{i,t+1}((1-\tau_t)y(z_{i,t},Y_{g,t}) + (1-r_t)a - c, z'). \tag{C.36}$$

Instead, the problem of an inattentive individual is given by

$$u(c) + \beta_{i,t}E_{i,t}[V_{i,t+1}((1-\tau_t)y(z_{i,t},Y_{g,t}) + (1-r_t)a - c, z')]. \tag{C.37}$$

Following the same steps, we find that the realized utility cost of inattention is given by

$$v_{i,t}(c^*_{g,t}(a,1);a,z) - v_{i,t}(c^*_{g,t}(a,\lambda_i);a,z) = -\frac{1}{2}\frac{\partial^2 v(c(a,z);a,z)}{\partial c^2}\left(c^*_{g,t}(a,z,1) - c^*_{g,t}(a,z,\boldsymbol{\lambda}_i)\right)^2$$
$$= -\frac{1}{2}\frac{\partial^2 v(c(a,z);a,z)}{\partial c^2}\sum_{X,\tilde{X},h,\tilde{h}}\frac{\partial c(a,z)}{\partial X_h}\frac{\partial c(a,z)}{\partial \tilde{X}_{\tilde{h}}}(1-\lambda^X_{i,h})(1-\lambda^{\tilde{X}}_{i,\tilde{h}})X_{t+h}X_{t+\tilde{h}}.$$

It follows that the ex-ante utility cost of inattention is given by

$$\mathcal{C}_g(a,z)(\boldsymbol{\lambda}_i) = -\frac{1}{2}\frac{\partial^2 v(c(a,z);a,z)}{\partial c^2}\sum_{X,\tilde{X},h,\tilde{h}}\frac{\partial (a,z)}{\partial X_h}\frac{\partial c(a,z)}{\partial \tilde{X}_{\tilde{h}}}\left(1-\lambda^X_{i,h}\right)\left(1-\lambda^{\tilde{X}}_{i,\tilde{h}}\right)\sigma_{X_h,\tilde{X}_{\tilde{h}}}. \tag{C.38}$$

where $\sigma_{X_h,\tilde{X}_{\tilde{h}}}$ denotes the ex-ante perceived covariance between $X_{t+h}$ and $\tilde{X}_{t+\tilde{h}}$.

If $c^*_{g,t}(a,z) = (1-\tau_t)y(z_{i,t},Y_{g,t}) + (1-r_t)a$ then the individual is at the borrowing constraint. Note that if an individual is borrowing constrained, then their consumption is not changing given changes in future variables, i.e.,

$$\frac{\partial c(a,z)}{\partial X_h} = 0, \tag{C.39}$$

for $h \geq 1$. This implies that the misoptimization costs of inattention to future variables are exactly zero, i.e., $\mathcal{C}_g(a,z)(\boldsymbol{\lambda}_i) = 0$. These two facts put together allow us to write

$$\mathcal{C}_g(a,z,\boldsymbol{\lambda}_i) = -\frac{1}{2}\frac{\partial^2 v(c(a,z);a,z)}{\partial c^2} \sum_{X,\tilde{X},h,\tilde{h}} \frac{\partial c(a,z)}{\partial X_h}\frac{\partial c(a,z)}{\partial \tilde{X}_{\tilde{h}}}\left(1-\lambda_{i,h}^{X}\right)\left(1-\lambda_{i,\tilde{h}}^{\tilde{X}}\right)\sigma_{X_h,\tilde{X}_{\tilde{h}}}, \tag{C.40}$$

which equals zero since the partial derivatives of the consumption function are equal to zero.

# C.4 Appendix to section 3.5

## C.4.1 Individual demand

The household enters time $t$ with assets $A_{i,t}$ and chooses consumption and savings to solve

$$\max E_{i,t} \sum_{h=0}^{\infty}\prod_{s=0}^{h-1}\left(\beta_{i,t+s}\right)\left[u(C_{i,t}) - v\left(N_{i,t}\right)\right], \quad \text{subject to}$$

$$C_{i,t+h} + A_{i,t+h+1} = (1-\tau_{t+h})Y_{g,t+h} + (1+r)A_{i,t+h}.$$

The Euler equation is still given by (C.1) and its linearized form (C.2).

Linearizing the budget constraint, we obtain

$$\begin{aligned} c_{i,t+h} + a_{i,t+h+1} &= (1-\tau)\cdot y_{g,t+h} - d\tau_{t+h}\cdot Y + \beta^{-1}a_{i,t+h} \\ \Leftrightarrow c_{i,t+h} + a_{i,t+h+1} &= y_{g,t+h} - \tau_{t+h} + \beta^{-1}a_{i,t+h} \end{aligned}$$

since $\tau = 0$ and $Y = 1$ in steady state. We can again aggregate flow-of-funds constraints and obtain

$$\sum_{h=0}^{\infty}\beta^h c_{i,t+h} = \sum_{h=0}^{\infty}\beta^h[y_{g,t+h} - \tau_{t+h}] + \beta^{-1}a_{i,t}. \tag{C.41}$$

Proceeding as before finally shows that

$$c_{i,t} = (1-\beta)\sum_{h=0}^{\infty}\beta^h E_{i,t}[y_{g,t+h} - \tau_{t+h}] + (1-\beta)\beta^{-1}a_{i,t} + \sum_{h=0}^{\infty}\beta^{h+1}r^n_{i,t+h}.$$

## C.4.2 Proof of proposition 12

The government-spending multiplier satisfies

$$\frac{dy_t}{dG_t} = 1 + (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\left[(1+\text{CD})\cdot\overline{\lambda}^Y\frac{dy_{t+h}}{dG_{t+h}} - \overline{\lambda}^\tau\right]\rho_G^h. \tag{C.42}$$

I guess and verify that the multiplier is constant over time $dy_t/dG_t = \Omega$:

$$\begin{aligned}\Omega &= 1 + \varrho_G\left[(1+\text{CD})\cdot\overline{\lambda}^Y\Omega - \overline{\lambda}^\tau\right]\\ \Leftrightarrow \Omega &= \frac{1-\varrho_G\cdot\overline{\lambda}^\tau}{1-\varrho_G\cdot(1+\text{CD})\cdot\overline{\lambda}^Y} > 0,\end{aligned}$$

where $\varrho_G \equiv (1-\beta)\rho_G/(1-\beta\rho_G) \in (0,1)$.

Note that

$$\frac{d\Omega}{d\text{CD}} = \Omega\cdot\frac{\varrho_G\cdot\overline{\lambda}^Y}{1-\varrho_G\cdot(1+\text{CD})\cdot\overline{\lambda}^Y} > 0 \tag{C.43}$$

and

$$\Omega \geq 1 \Leftrightarrow \frac{1-\varrho_G\cdot\overline{\lambda}^\tau}{1-\varrho_G\cdot(1+\text{CD})\cdot\overline{\lambda}^Y} \geq 1 \Leftrightarrow (1+\text{CD})\cdot\overline{\lambda}^Y \geq \overline{\lambda}^\tau. \tag{C.44}$$

## C.4.3 Proof of proposition 13

The government-spending multiplier satisfies

$$\frac{dy_t}{dG_t} = 1 + (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\left[(1+\text{CD})\cdot\overline{\lambda}^Y\frac{dy_{t+h}}{dG_{t+h}} + \text{TC}\cdot\overline{\lambda}^Y - \overline{\lambda}^\tau\right]\rho_G^h. \tag{C.45}$$

I guess and verify that the multiplier is constant over time $dy_t/dG_t = \Omega$:

$$
\begin{aligned}
\Omega &= 1 + \varrho_G \left[(1+\text{CD}) \cdot \overline{\lambda}^Y \Omega - \overline{\lambda}^\tau\right] \\
\Leftrightarrow \Omega &= \frac{1 - \varrho_G \cdot (\overline{\lambda}^\tau + \text{TC} \cdot \overline{\lambda}^Y)}{1 - \varrho_G \cdot (1+\text{CD}) \cdot \overline{\lambda}^Y} \\
\Leftrightarrow \Omega &= \frac{dy_t^0}{dG_t} + \frac{\varrho_G \cdot \text{TC} \cdot \overline{\lambda}^Y}{1 - \varrho_G \cdot (1+\text{CD}) \cdot \overline{\lambda}^Y}
\end{aligned}
$$

where $\varrho_G \equiv (1-\beta)\rho_G/(1-\beta\rho_G) \in (0,1)$, and $dy_t^0/dG_t \equiv \frac{1-\varrho_G \cdot \overline{\lambda}^\tau}{1-\varrho_G \cdot (1+\text{CD}) \cdot \overline{\lambda}^Y}$.

With homogeneous beliefs $\lambda_g^Y = \overline{\lambda}^Y$ (of which FIRE is a special case with $\lambda_g = 1$), we find that

$$
TC = \mathbb{C}\text{ov}\left(\omega_g, \overline{\lambda}^Y / \overline{\lambda}^Y\right) = 0.
$$

This means that

$$
dy_t/dG_t = \frac{1 - \varrho_G \cdot \overline{\lambda}^\tau}{1 - \varrho_G \cdot (1+\text{CD}) \cdot \overline{\lambda}^Y},
$$

and so targeting does not affect the spending multiplier.

Instead, suppose that attention is heterogeneous. Then,

$$
\frac{d\Omega}{d\text{TC}} = \frac{\varrho_G \cdot \overline{\lambda}^Y}{1 - \varrho_G \cdot (1+\text{CD}) \cdot \overline{\lambda}^Y} > 0, \tag{C.46}
$$

which implies that the spending multiplier increases if the covariance between $\omega_g$ and $\lambda_g$ is higher.

## C.5 Extensions

### C.5.1 Horizon-independent attention

The baseline quantitative model allows attention to vary with horizon. In this appendix, I assess the robustness of the quantitative results to assuming that horizon must be constant over time. The interest for this analysis stems from the fact that this independence on horizon is closest to what would be obtained in a model with rational expectations in which individuals receive noisy signals of the fundamental shocks, as in Angeletos and Huo (2021), or in models of sticky information, as in Mankiw and Reis (2002).

Formally, I now assume that beliefs are given by

$$E_{i,t}[dX_{t+h}] = \lambda_i^X \cdot \mathbb{E}_t[dX_{t+h}] + (1 - \lambda_i^X) \cdot E_{i,t-1}[dX_{t+h}], \tag{C.47}$$

and proceed as before to optimize for $\lambda_i^X$ for each variable $X$.

$$\lambda_g^Y = \max\left\{0, 1 - \frac{\kappa^Y}{\sum_z \int \frac{\partial^2 v(a,z)}{\partial c^2} \sum_{h=1}^{\infty} \left(\frac{\partial c(a,z)}{\partial Y_{g,h}}\right)^2 D(da, z) \cdot \gamma_g^2 \sigma_Y^2}\right\} \tag{C.48}$$

and

$$\lambda_g^X = \max\left\{0, 1 - \frac{\kappa^X}{\sum_z \int \frac{\partial^2 v(a,z)}{\partial c^2} \sum_{h=1}^{\infty} \left(\frac{\partial c(a,z)}{\partial X_h}\right)^2 D(da, z) \cdot \sigma_X^2}\right\} \tag{C.49}$$

for $X = \tau, r$.

The quantitative results can be found below. In sum, the results emphasized in the main text also hold in this case.

Figure C.2: Optimal attention

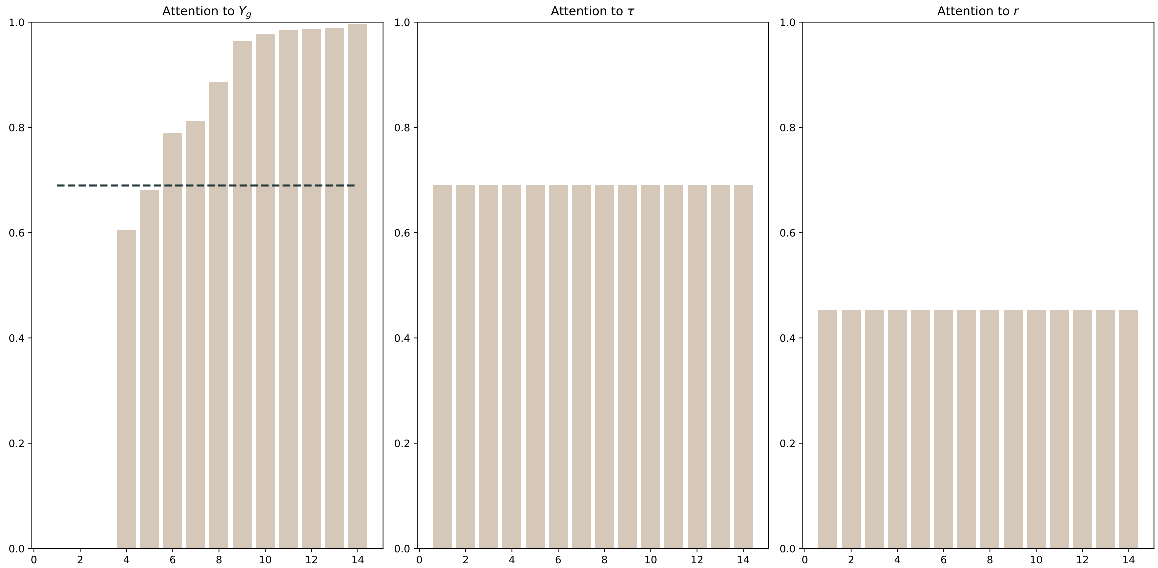



Figure C.3: Consumption response

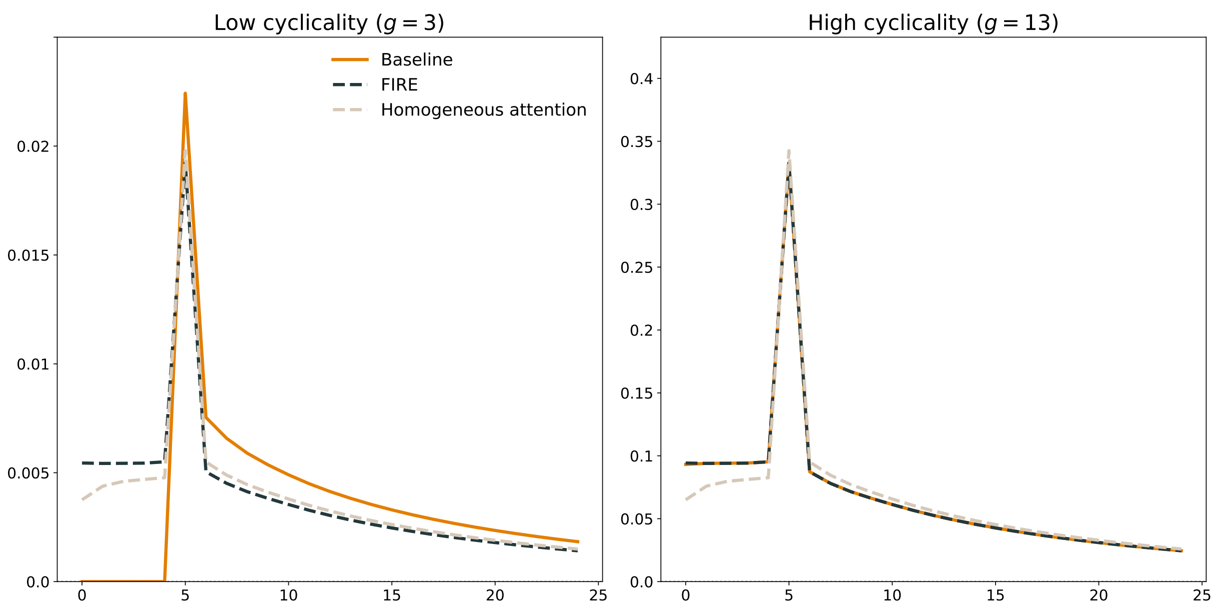


Figure C.4: Consumption response for all groups

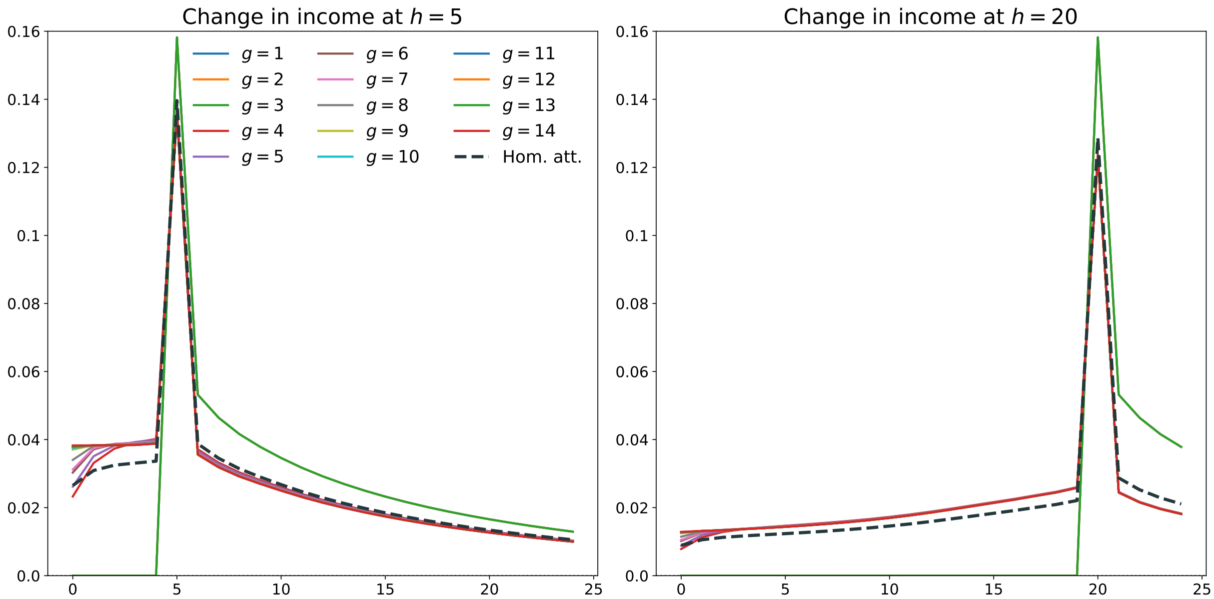



Figure C.5: Business-cycle amplification

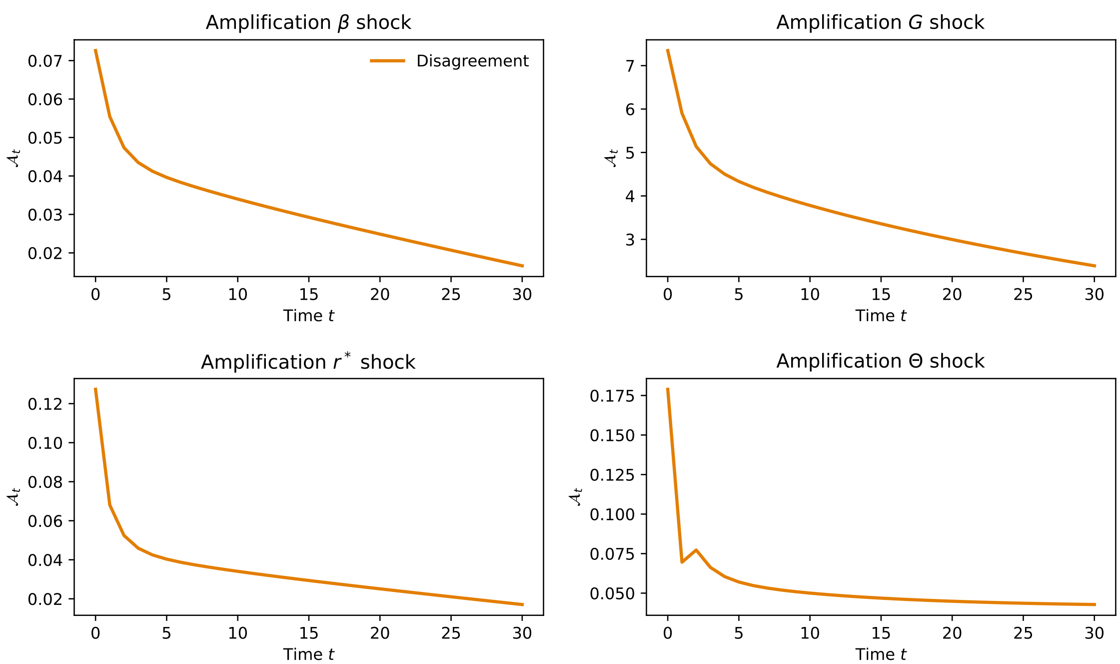


Figure C.6: Business-cycle amplification: The role of persistence

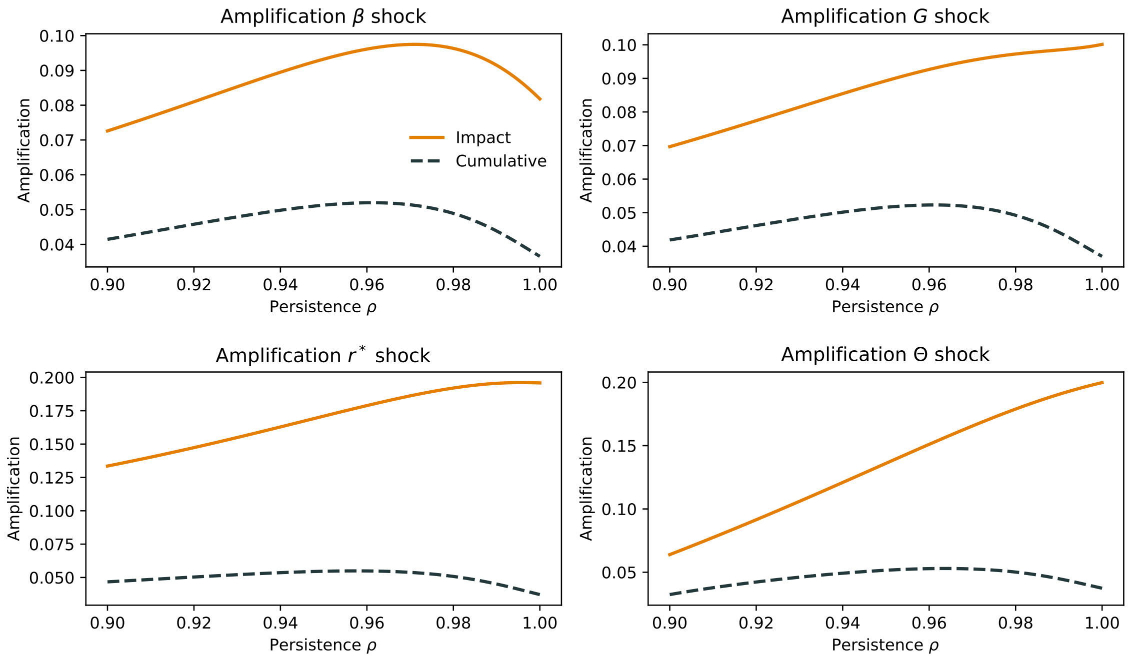



Figure C.7: Business-cycle amplification: The role of monetary policy

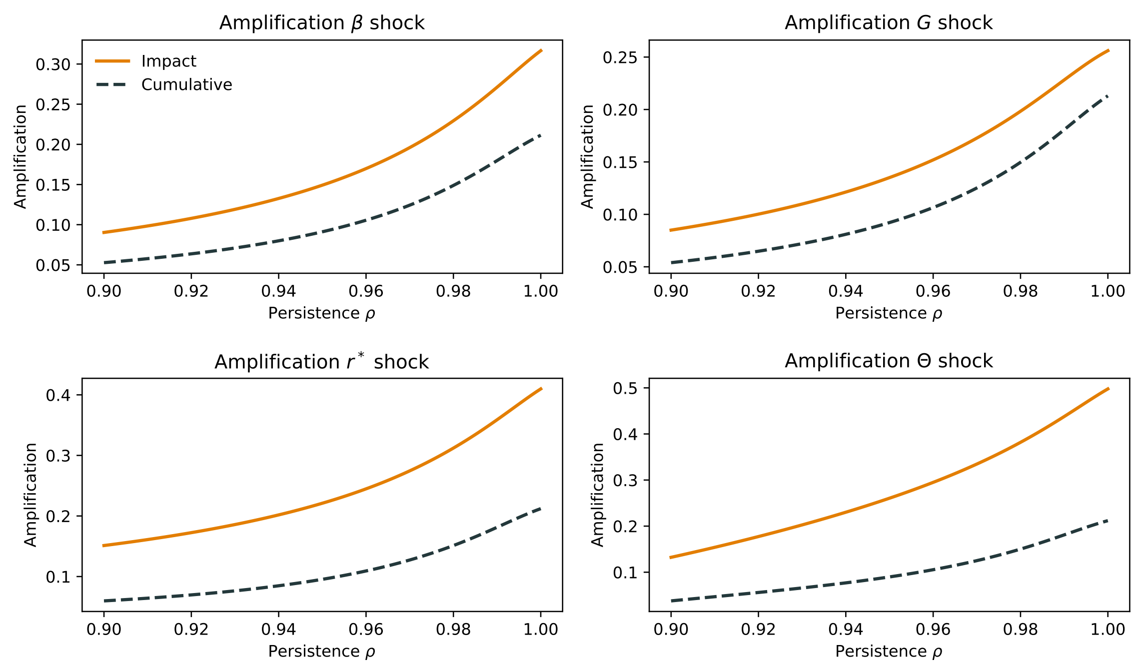


Figure C.8: Targeted spending multipliers

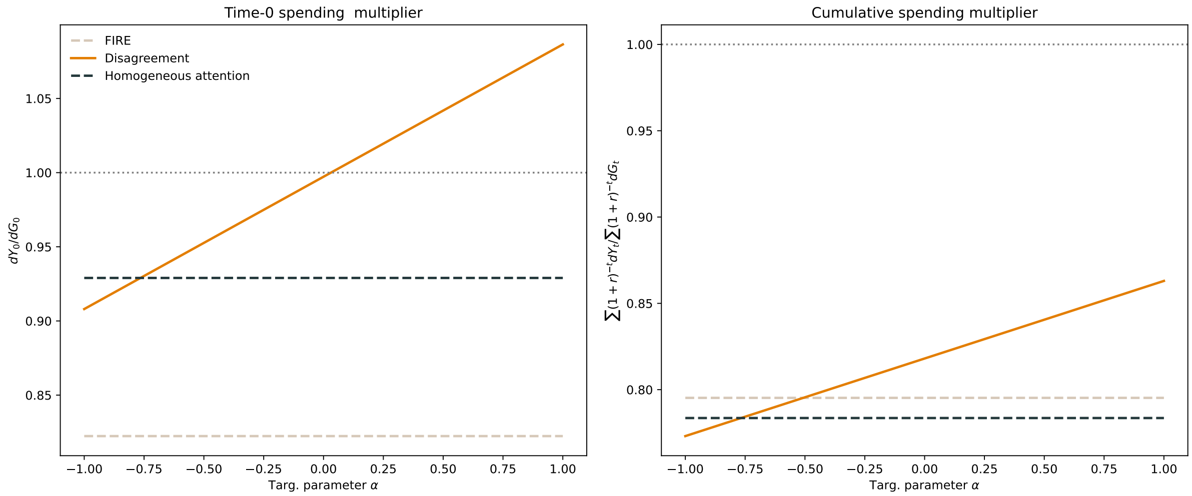
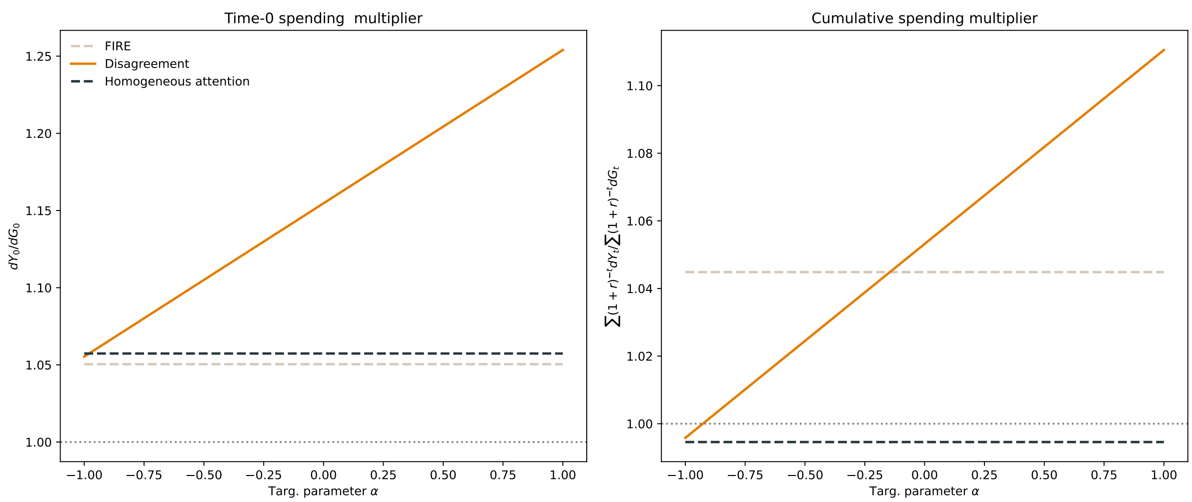



Figure C.9: Targeted spending multipliers: The role of monetary policy




## C.5.2 Allowing for perceived correlations

Following Gabaix (2014), in the baseline quantitative model, I assume that individuals perceive all variables to be uncorrelated. In this appenix, I show that the quantitative results are robust to allowing for perceptions in correlations. Specifically, I assume that individuals perceive each variable to evolve as an AR(1):

$$dX_{t+1} = \rho_X dX_t + \eta_{t+1}^X, \tag{C.50}$$

where $\rho_X$ denotes the persistence of variable $X$ and $\mathbb{V}\text{ar}(dX_t) = \sigma_X^2$ and $\mathbb{C}\text{orr}(dX_t, d\tilde{X}_t) = \rho_{X,\tilde{X}}$. I assume that peoples perceived correlations are equal to their empirical counterparts and use U.S. data to estimate these correlations. The empirical standard deviations and correlations can be found in table C.4.

Table C.4: Empirical covariances

| Param. | Description | Value | Param. | Description | Value |
|---|---|---|---|---|---|
| $\sigma_y$ | St. Dev. GDP | 0.03 | $\rho_r$ | Persistence real rate | 0.59 |
| $\sigma_r$ | St. Dev. real rate | 0.13 | $\rho_{y,\text{tax}}$ | Correl. GDP-taxes | 0.51 |
| $\sigma_{\text{tax}}$ | St. Dev. taxes | 0.01 | $\rho_{y,r}$ | Correl. GDP-real rate | 0.12 |
| $\rho_y$ | Persistence GDP | 0.87 | $\rho_{\text{tax},r}$ | Correl. taxes-real rate | $-0.09$ |
| $\rho_{\text{tax}}$ | Persistence Taxes | 0.89 | | | |

Solving the optimal attention problem becomes more difficult, since the problem is no longer separable across forecasting variables. However, it is still possible to find a closed form solution to this problem which takes the form:

$$\boldsymbol{\lambda}_g = 1 - \boldsymbol{\Psi}^{-1}\boldsymbol{\kappa}$$

where

$$\boldsymbol{\lambda}_g = \begin{bmatrix} \lambda_1^Y \\ \lambda_2^Y \\ \ldots \\ \lambda_1^\tau \\ \ldots \\ \lambda_1^r \\ \ldots \end{bmatrix}, \quad \boldsymbol{\Psi} \equiv \begin{bmatrix} \Lambda_{Y_0,Y_0}\sigma_{Y_0,Y_0} & \Lambda_{Y_0,Y_1}\sigma_{Y_0,Y_2} & \ldots & \Lambda_{Y_0,\tau_0}\sigma_{Y_0,\tau_0} & \ldots \\ \Lambda_{Y_1,Y_0}\sigma_{Y_1,Y_0} & \Lambda_{Y_1,Y_1}\sigma_{Y_1,Y_2} & \ldots & \Lambda_{Y_1,\tau_0}\sigma_{Y_1,\tau_0} & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \Lambda_{\tau_0,Y_0}\sigma_{\tau_0,Y_0} & \Lambda_{\tau_0,Y_1}\sigma_{\tau_0,Y_2} & \ldots & \Lambda_{\tau_0,\tau_0}\sigma_{\tau_0,\tau_0} & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \Lambda_{r_0,Y_0}\sigma_{r,Y_0} & \Lambda_{\tau_0,Y_1}\sigma_{r_0,Y_2} & \ldots & \Lambda_{r_0,\tau_0}\sigma_{r_0,\tau_0} & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}, \quad \boldsymbol{\kappa} \equiv \begin{bmatrix} \kappa_Y \\ \kappa_Y \\ \ldots \\ \kappa_\tau \\ \ldots \\ \kappa_r \\ \ldots \end{bmatrix}.$$

The quantitative results can be found below. In sum, the results emphasized in the main text also hold in this case.

Figure C.10: Optimal attention

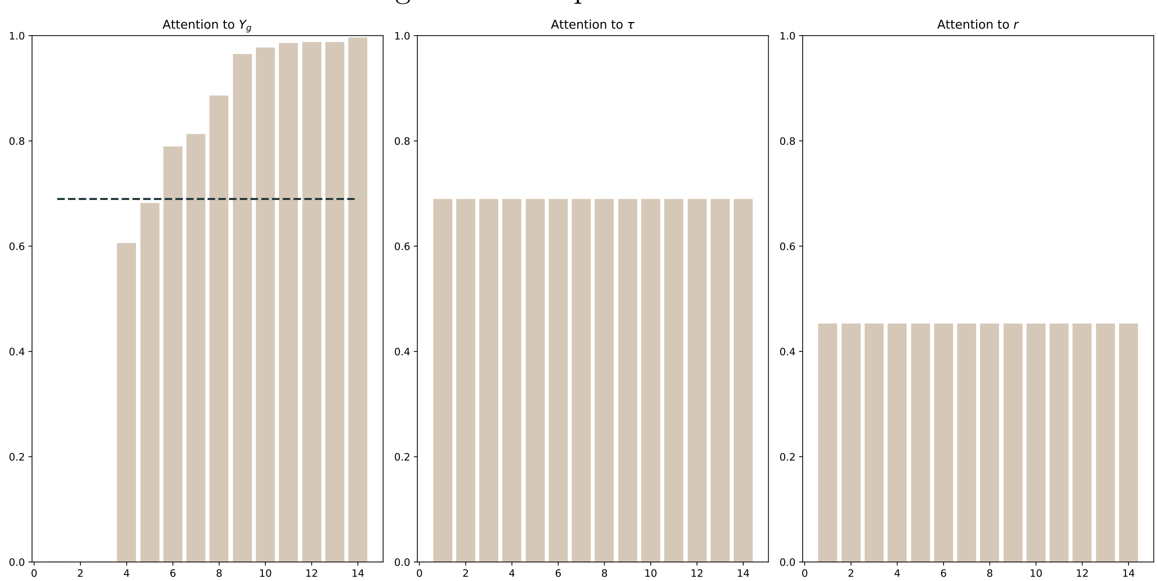


Figure C.11: Consumption response

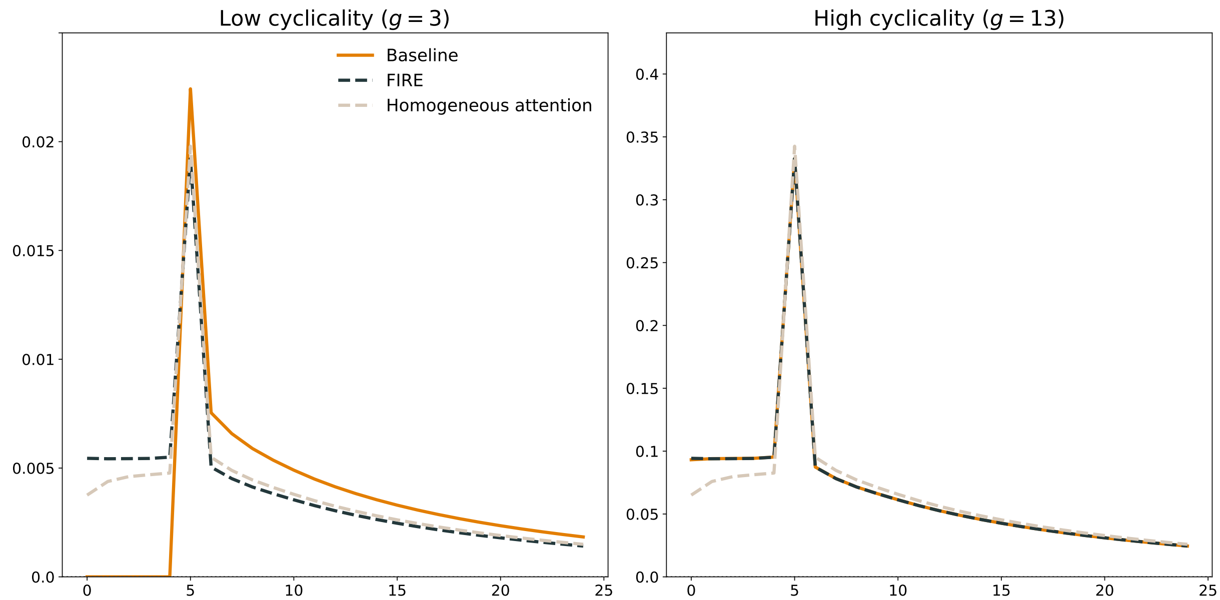



Figure C.12: Consumption response for all groups

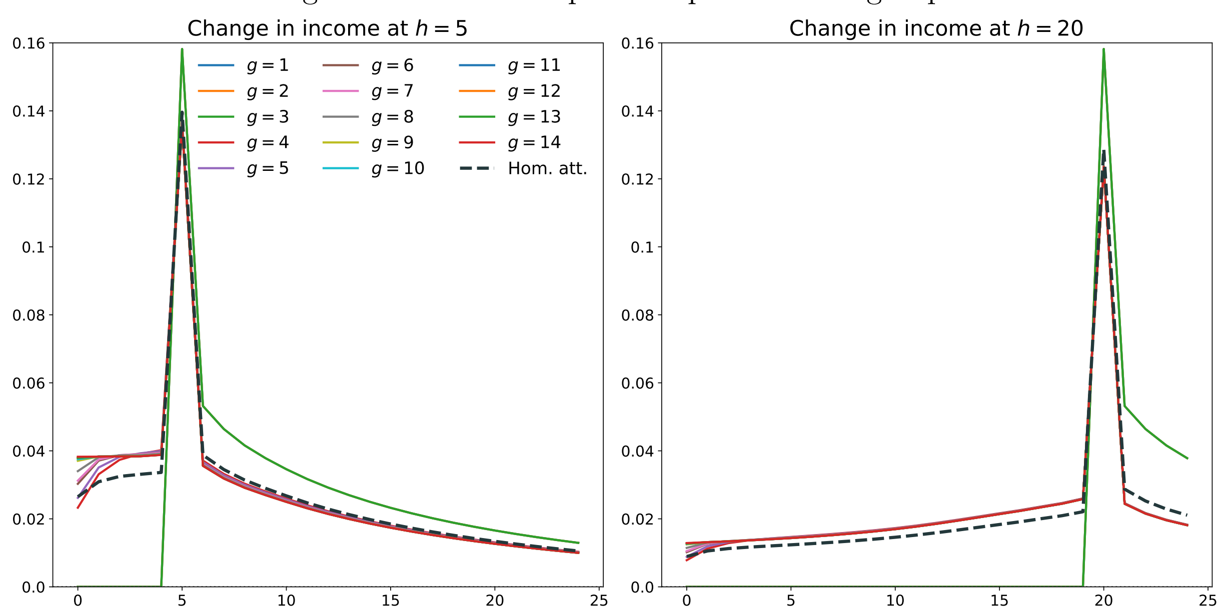


Figure C.13: Business-cycle amplification

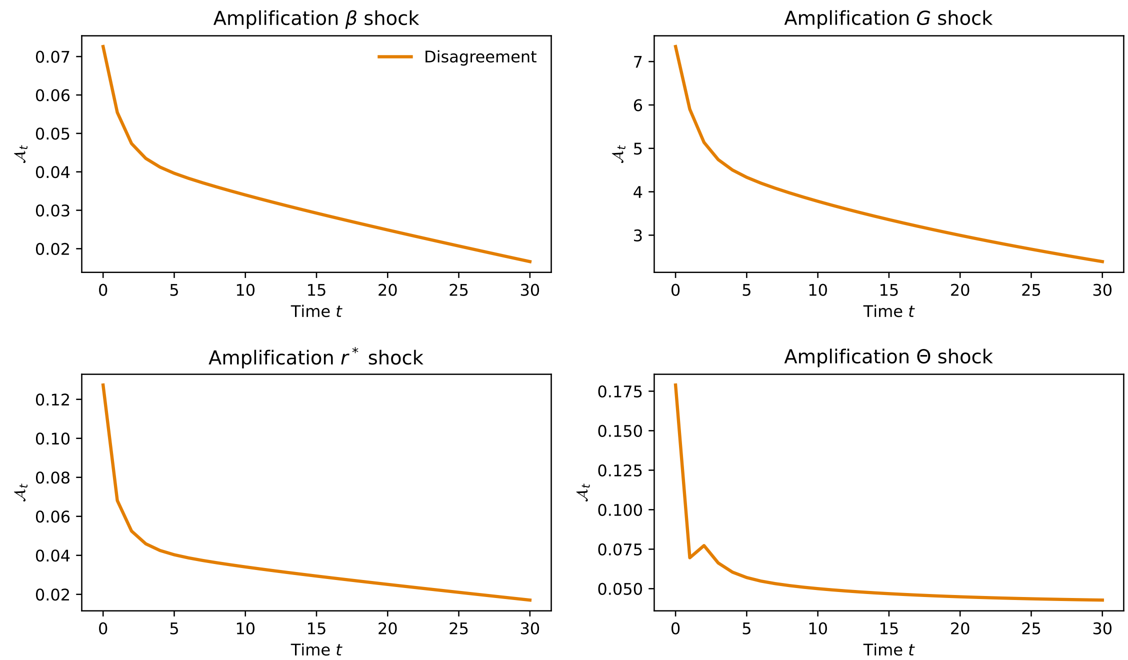



Figure C.14: Business-cycle amplification: The role of persistence

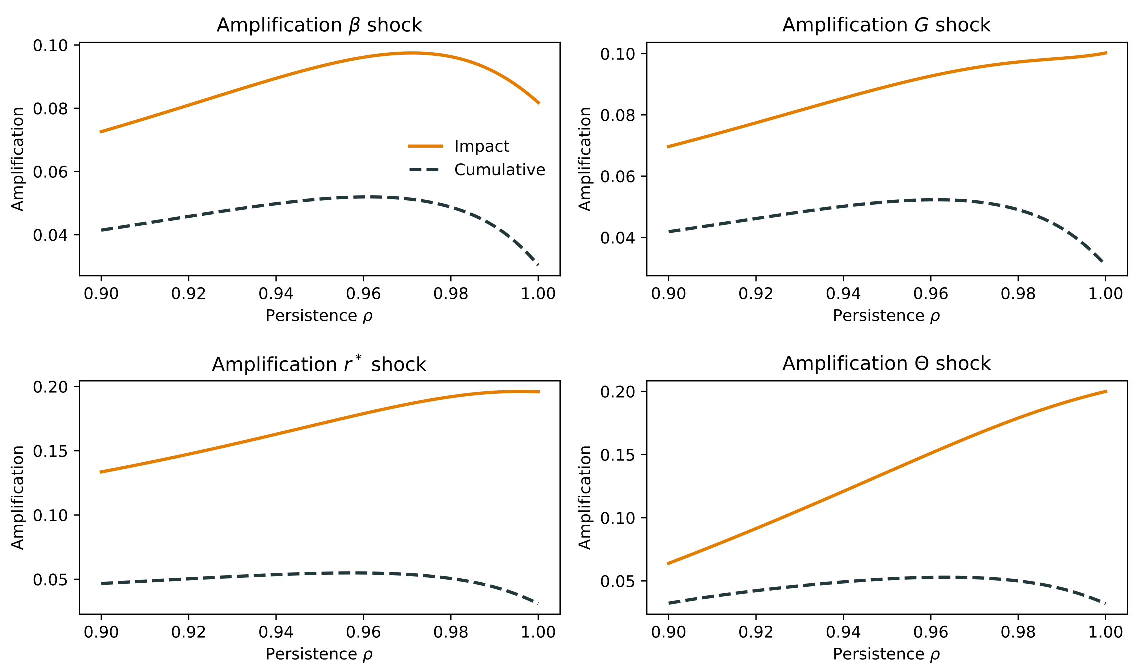


Figure C.15: Business-cycle amplification: The role of monetary policy

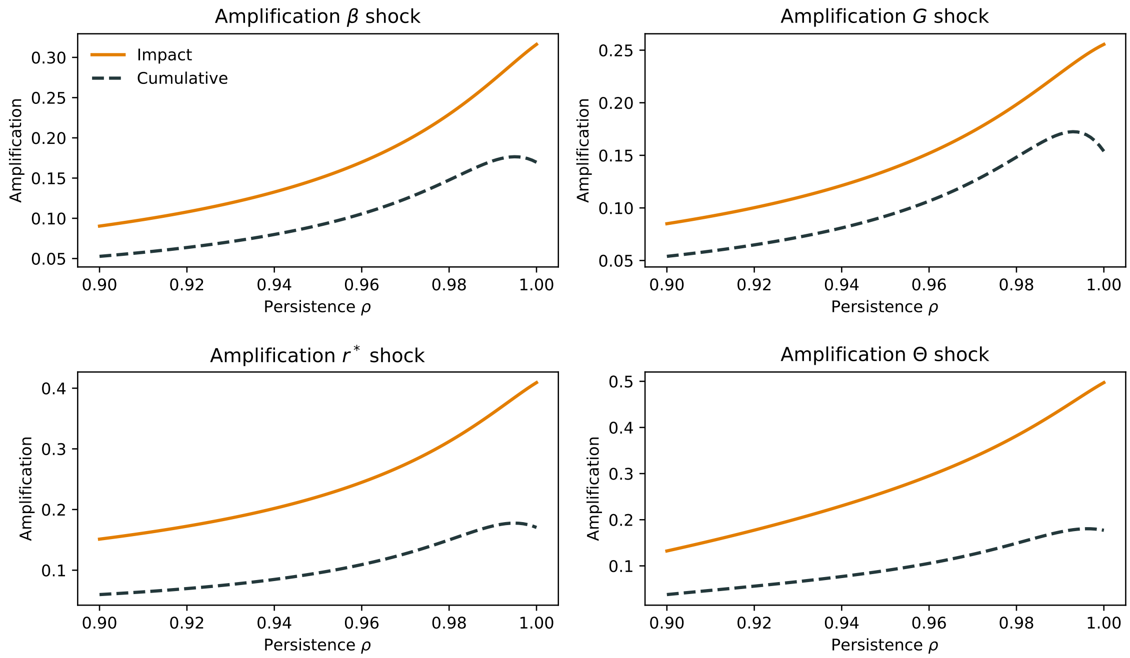



Figure C.16: Targeted spending multipliers

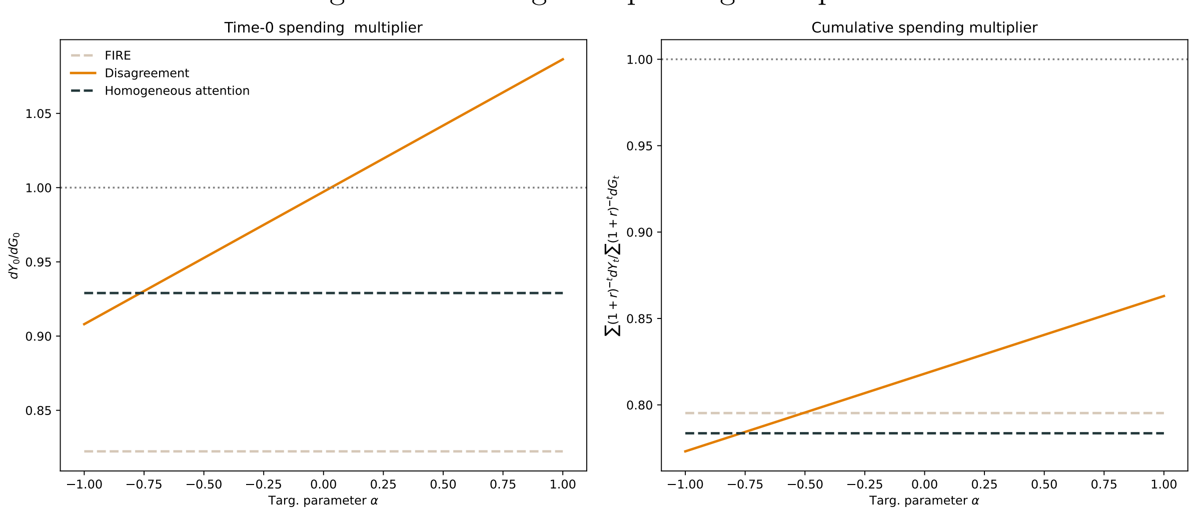
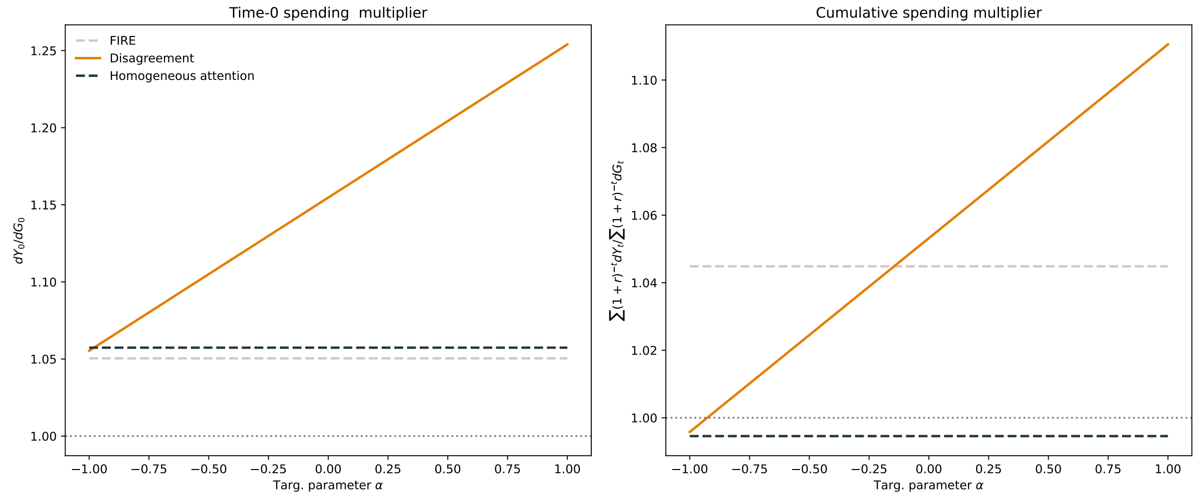


Figure C.17: Targeted spending multipliers: The role of monetary policy




## C.5.3 Budget deficits

The baseline quantitative model assumes that debt is kept constant at its steady-state level $B_t = B$. In this appendix, I allow debt to vary over time by assuming a fiscal rule for taxes as in Auclert et al. (2020). Formally, I assume that

$$\tau_t = \tau + \psi \frac{B_t - B}{Y}. \tag{C.51}$$

This expression implies that taxes are updated smoothly so that in the long-run government debt converges back to the original steady state. However, note that it also implies that upon increasing spending, the government only starts raising taxes a period later. In this sense, this extension allows for budget deficits. Following Auclert et al. (2020), I assume that the response of taxes to deviations of debt is given by $\psi = 0.1$ per annum, which is in line with the empirical results of the fiscal literature.

Figure C.18: Optimal attention

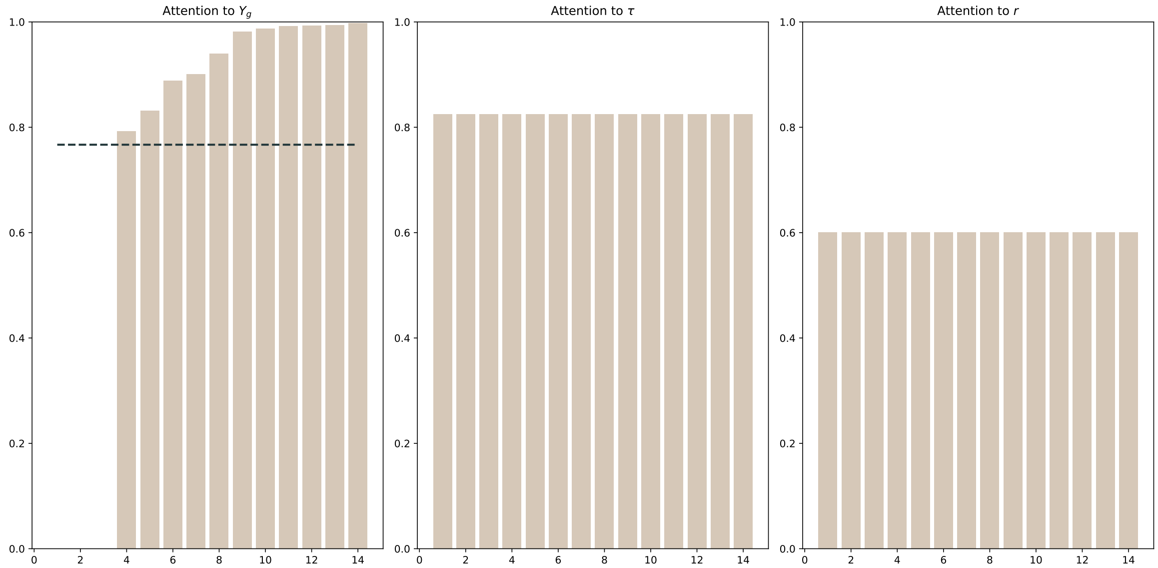



Figure C.19: Consumption response

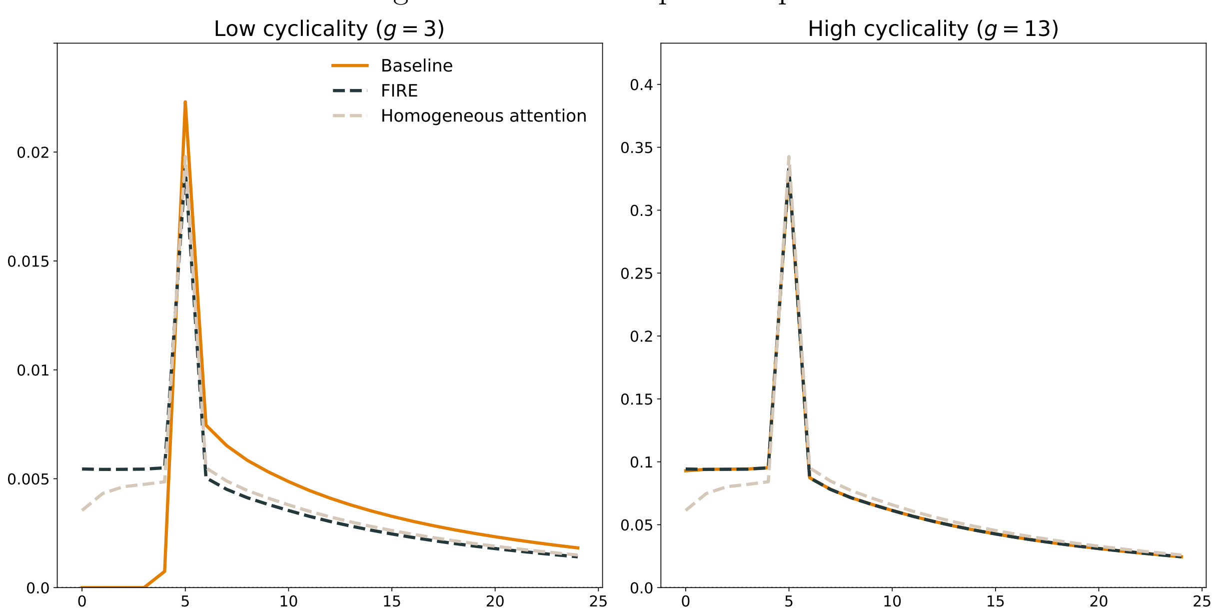


Figure C.20: Consumption response for all groups

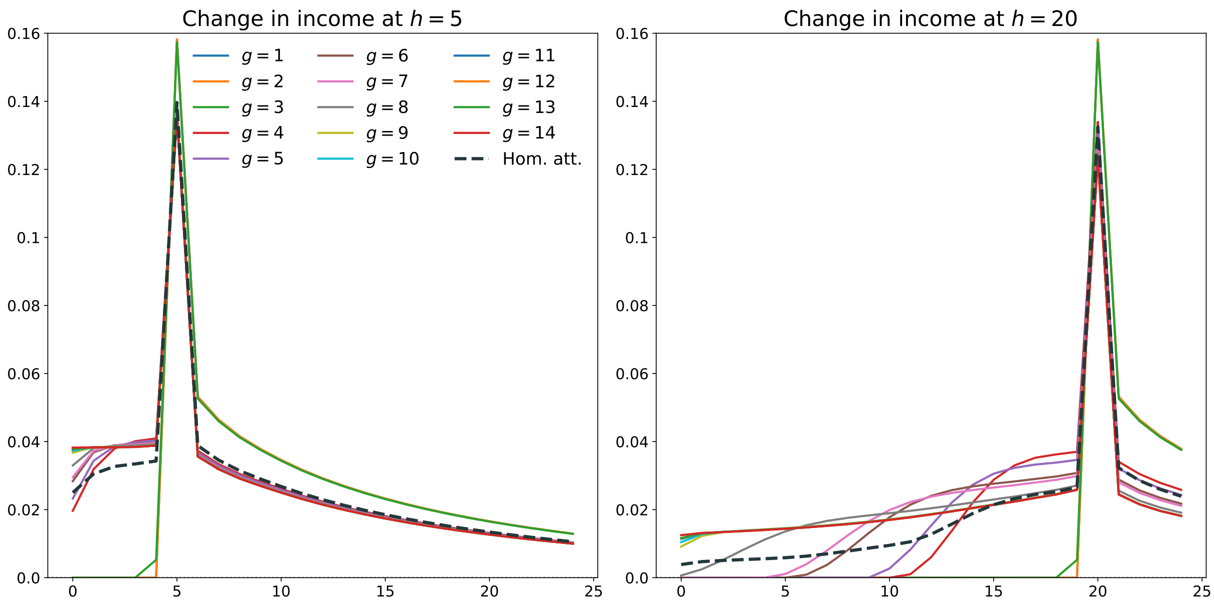



Figure C.21: Business-cycle amplification

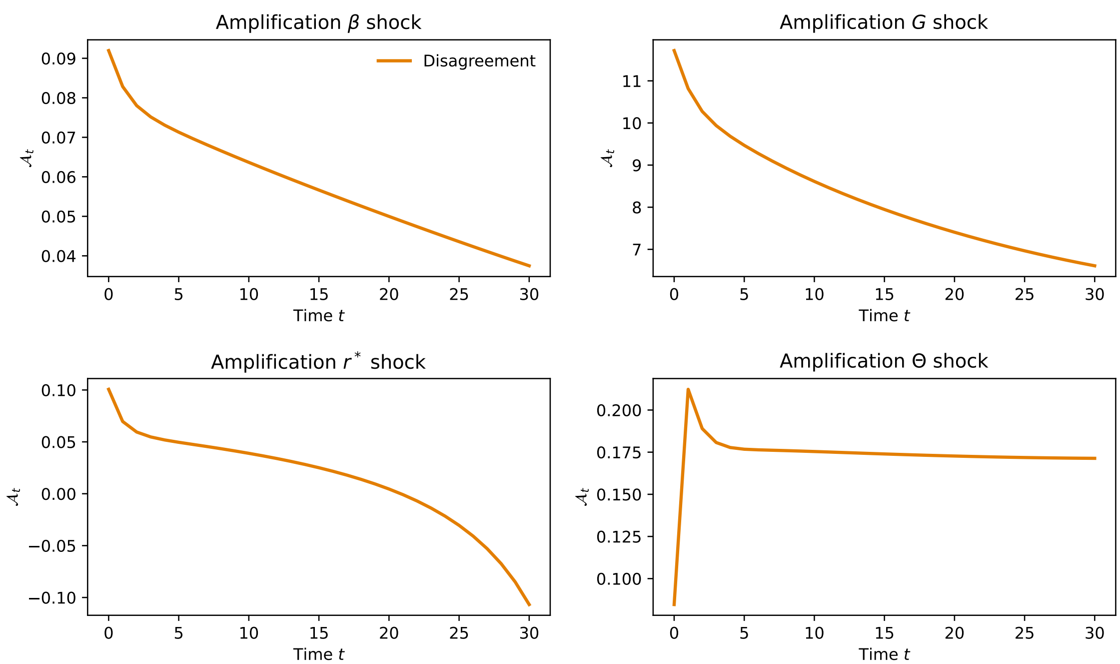


Figure C.22: Business-cycle amplification: The role of persistence

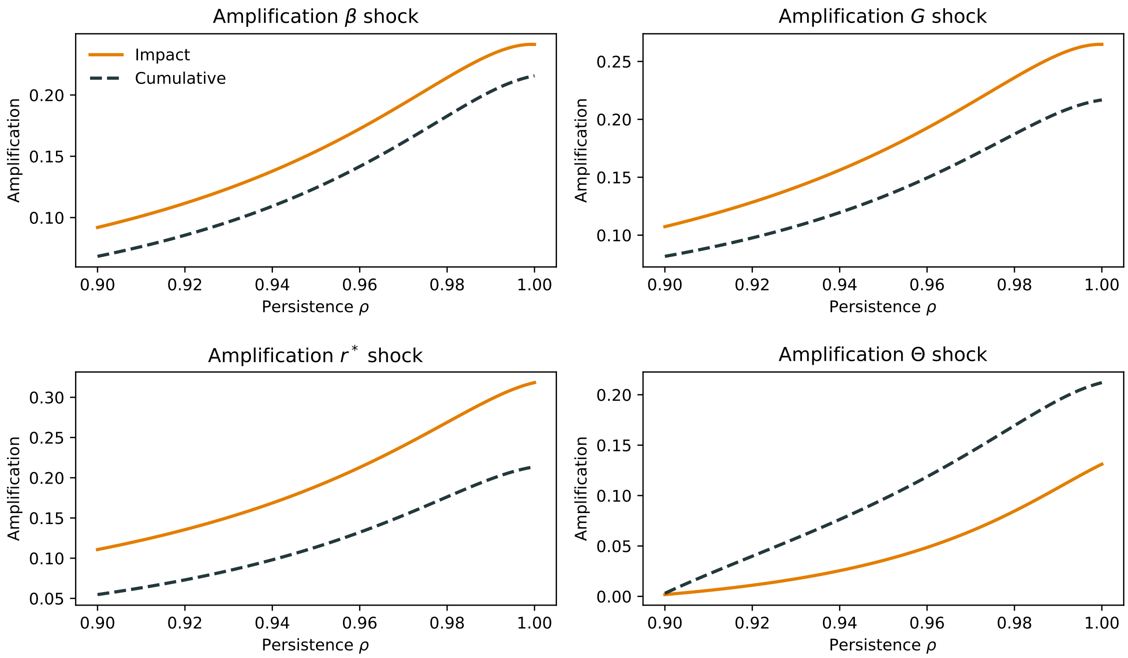



Figure C.23: Business-cycle amplification: The role of monetary policy

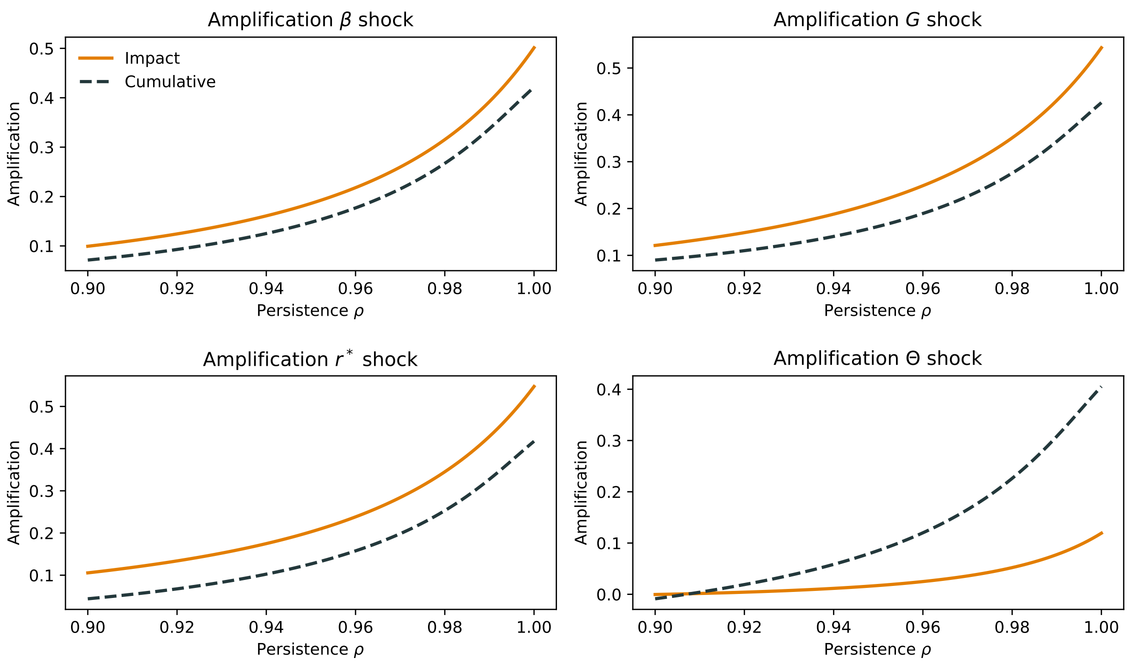


Figure C.24: Targeted spending multipliers

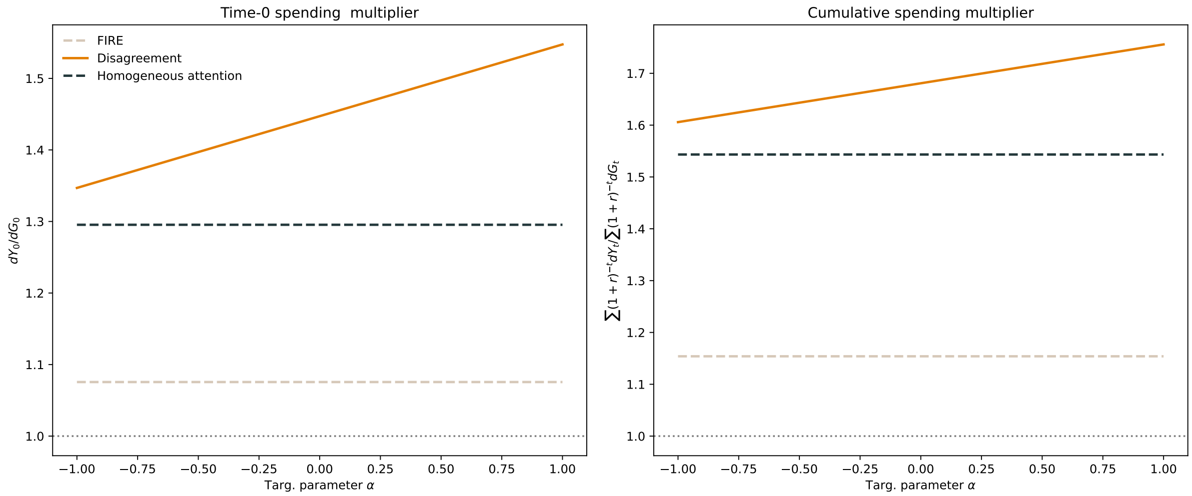
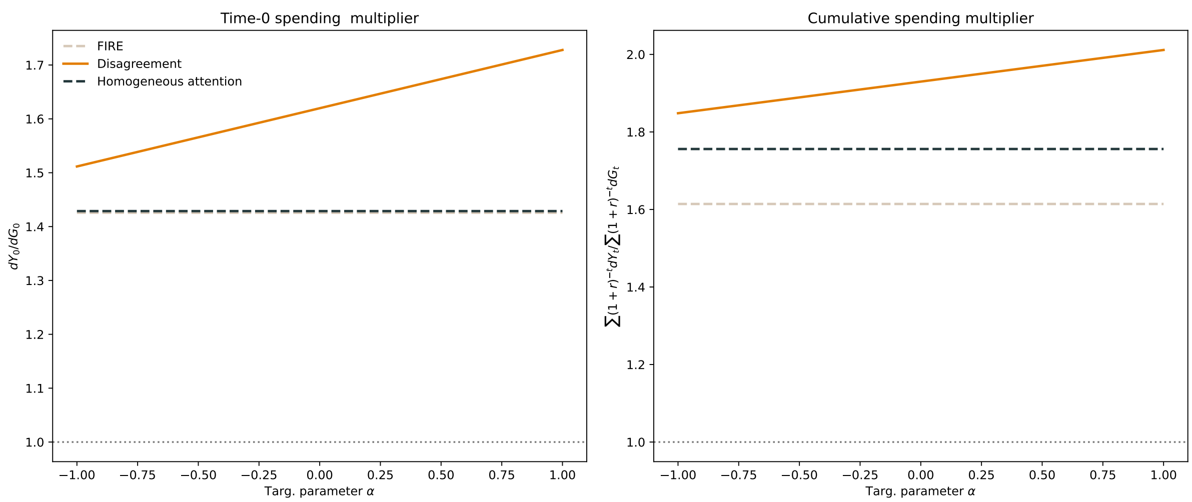



Figure C.25: Targeted spending multipliers: The role of monetary policy




### C.5.4 Progressive taxation

The baseline quantitative model assumes that labor income taxation is constant. In this appendix, I allow taxes to be progressive by assuming the constant-elasticity retention function in Heathcote et al. (2017). Formally, I assume that if an individual's pretax labor income is given by $y_{i,t}$ then their after tax labor income is given by $\tau_t y_{i,t}^{1-p}$, where $\tau_t$ and $p$ control the average level of taxation and progressivity, respectively. The modified government budget constraint is given by

$$G_t + (1 + r_t)B_t = \int_0^1 \left(y_{i,t} - \tau_t y_{i,t}^{1-p}\right) di + B_{t+1}. \tag{C.52}$$

Following Heathcote et al. (2017), I calibrate progressivity to $p = 0.181$. In my quantitative exercises, I fix progressivity and let the level of taxation $\tau_t$ vary so as to clear the government budget constraint.

Figure C.26: Optimal attention

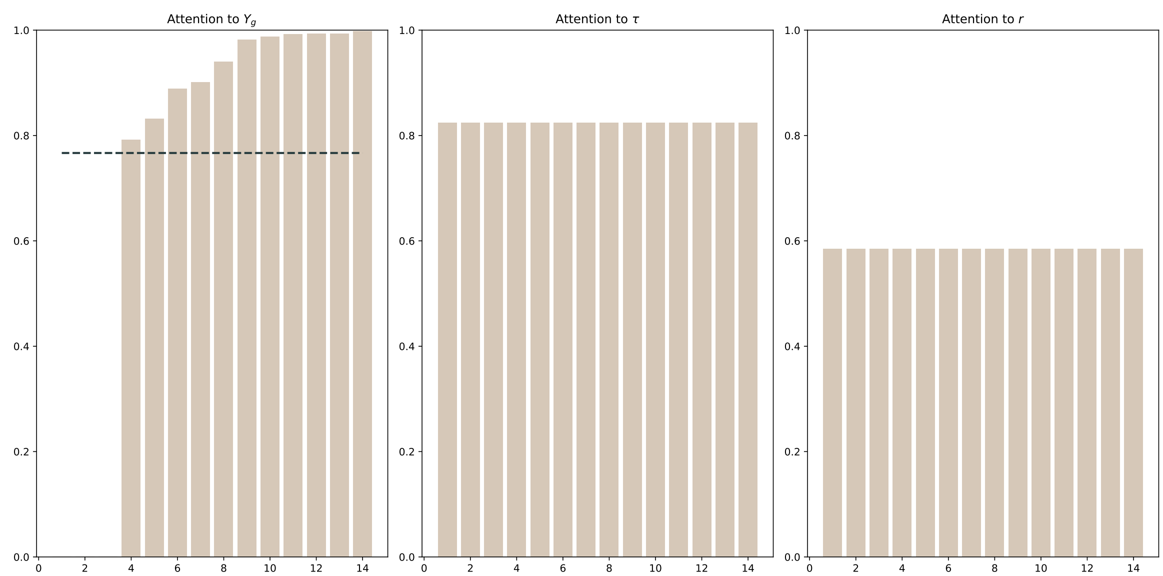


Figure C.27: Consumption response

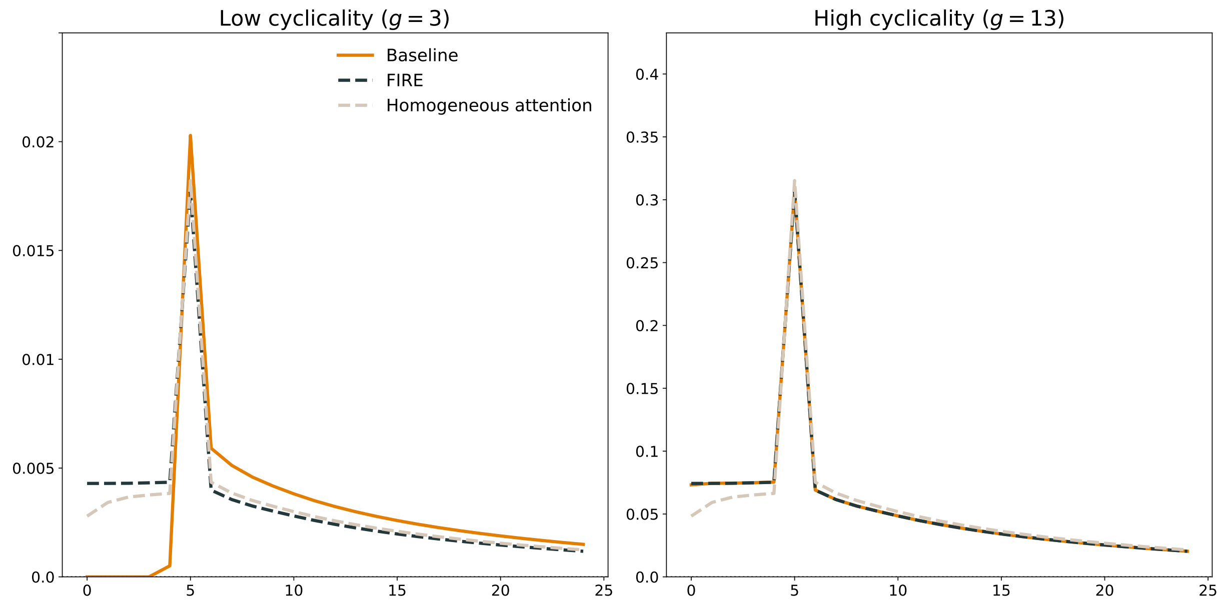



Figure C.28: Consumption response for all groups

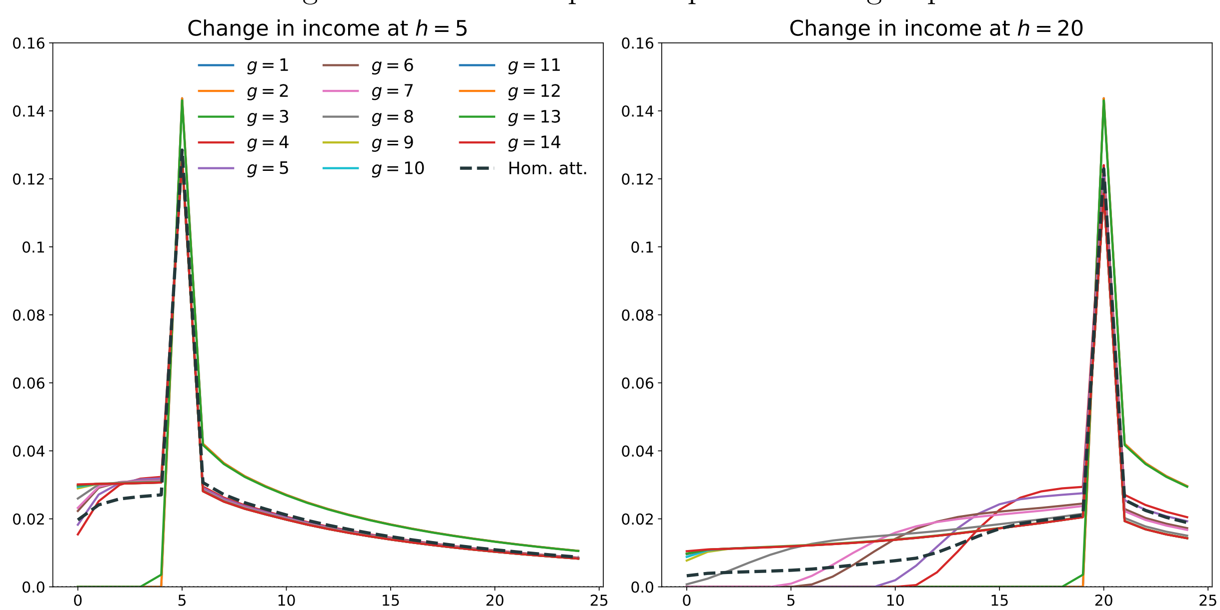


Figure C.29: Business-cycle amplification

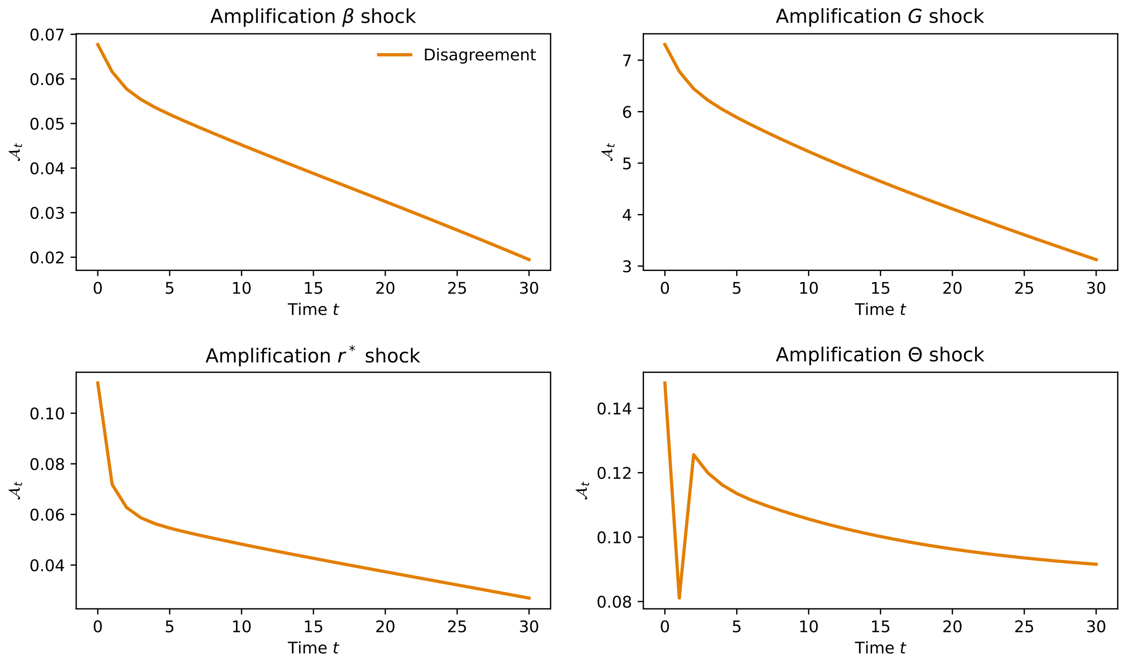



Figure C.30: Business-cycle amplification: The role of persistence

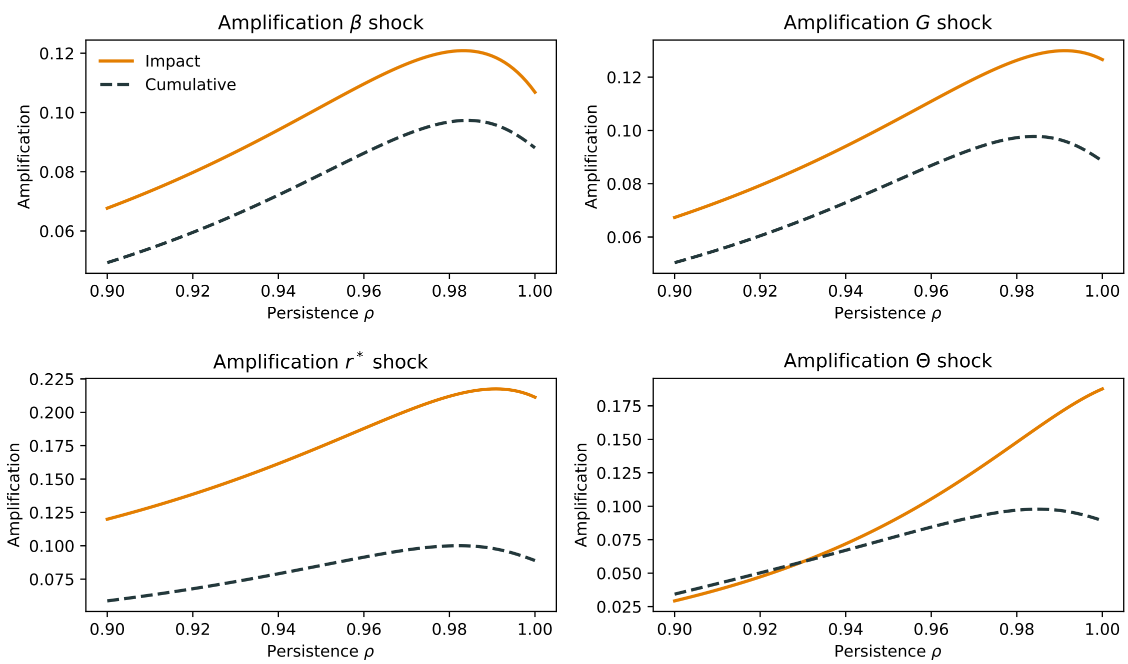


Figure C.31: Business-cycle amplification: The role of monetary policy

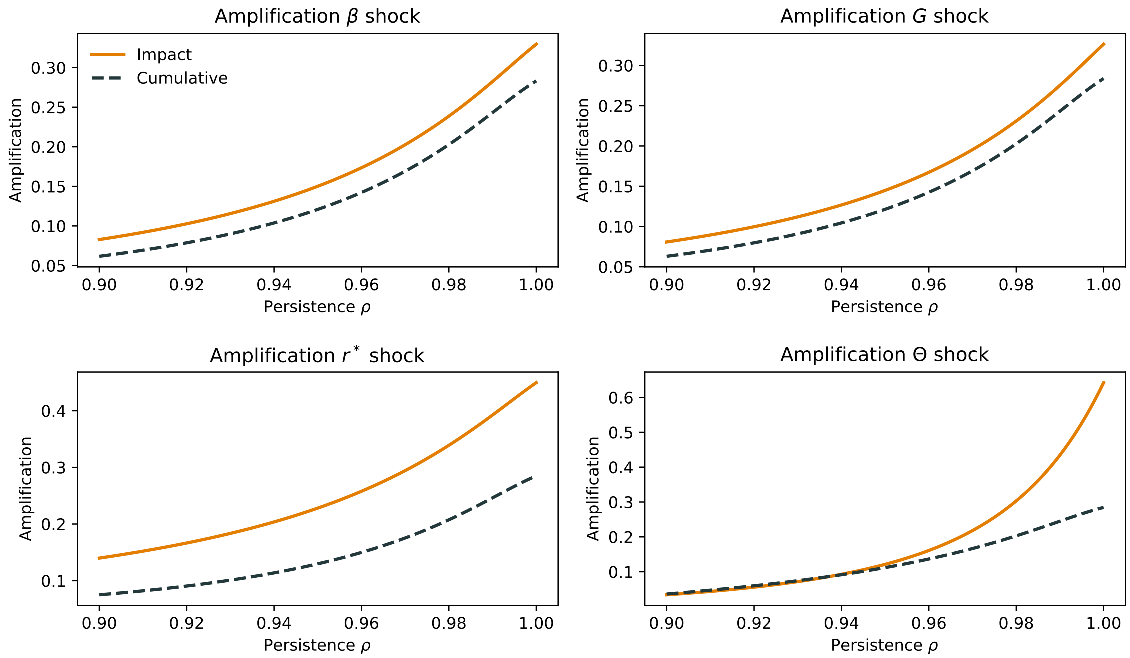



Figure C.32: Targeted spending multipliers

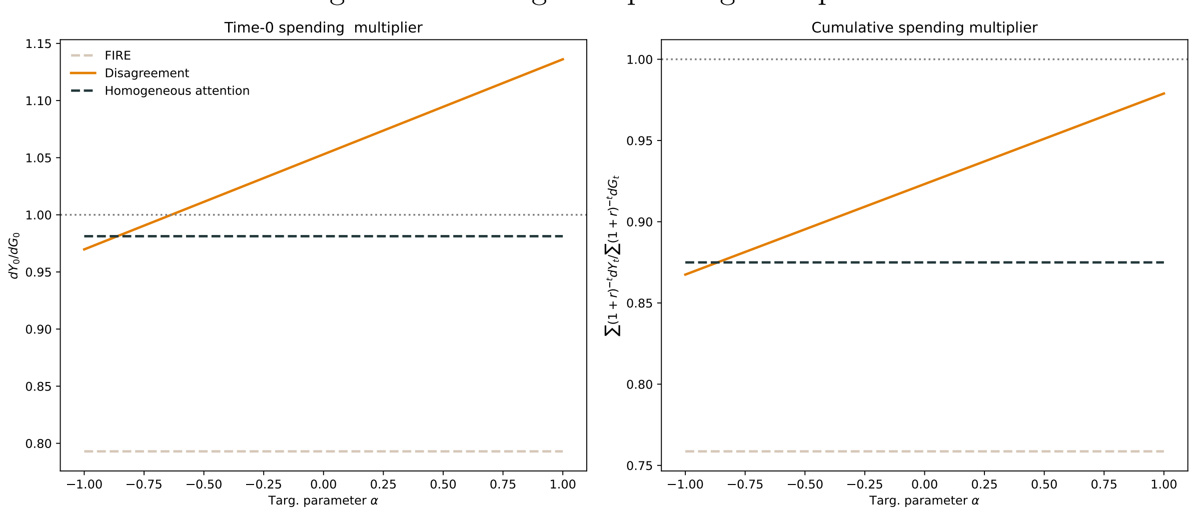
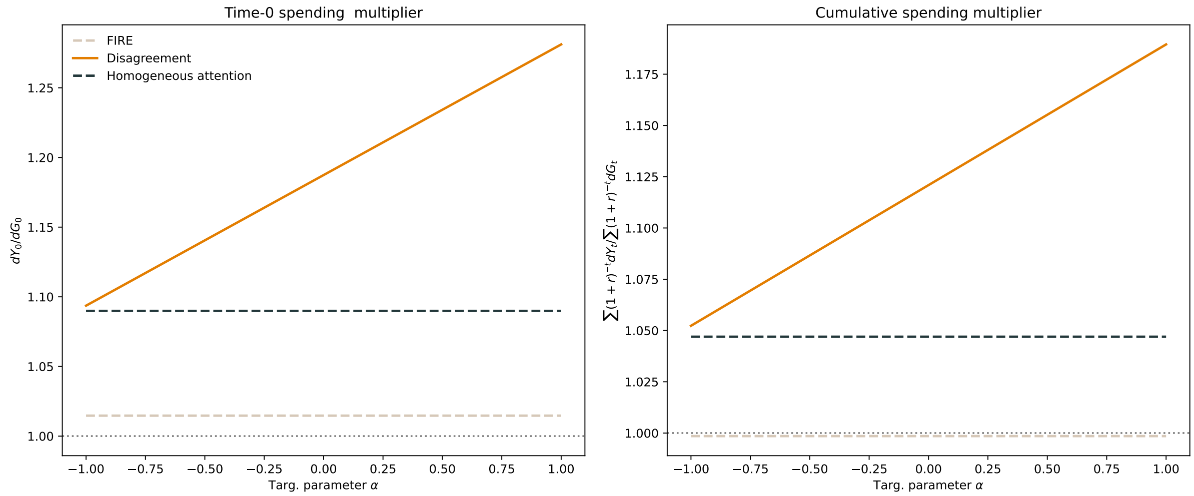


Figure C.33: Targeted spending multipliers: The role of monetary policy




## C.5.5 Acyclical Income Risk

In the baseline quantitative model, I assume that income risk is countercyclical by calibrating $\zeta = -0.5$. In order to assess the robustness of the results in this paper to this assumption, this appendix assumes that $\zeta = 0$ and recomputes the main quantitative results.

Figure C.34: Optimal attention

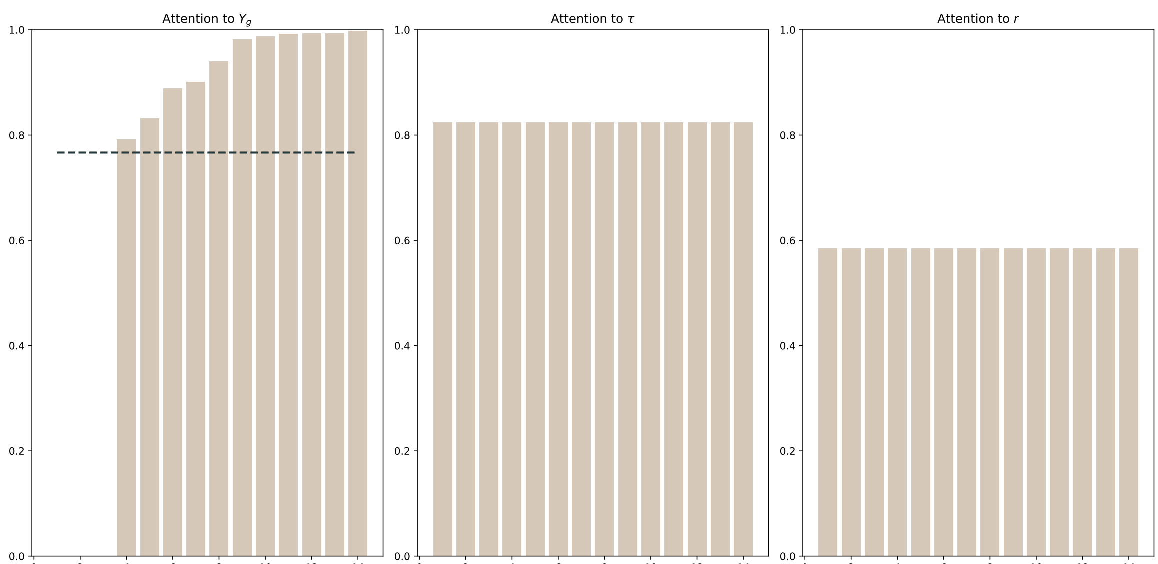


Figure C.35: Consumption response

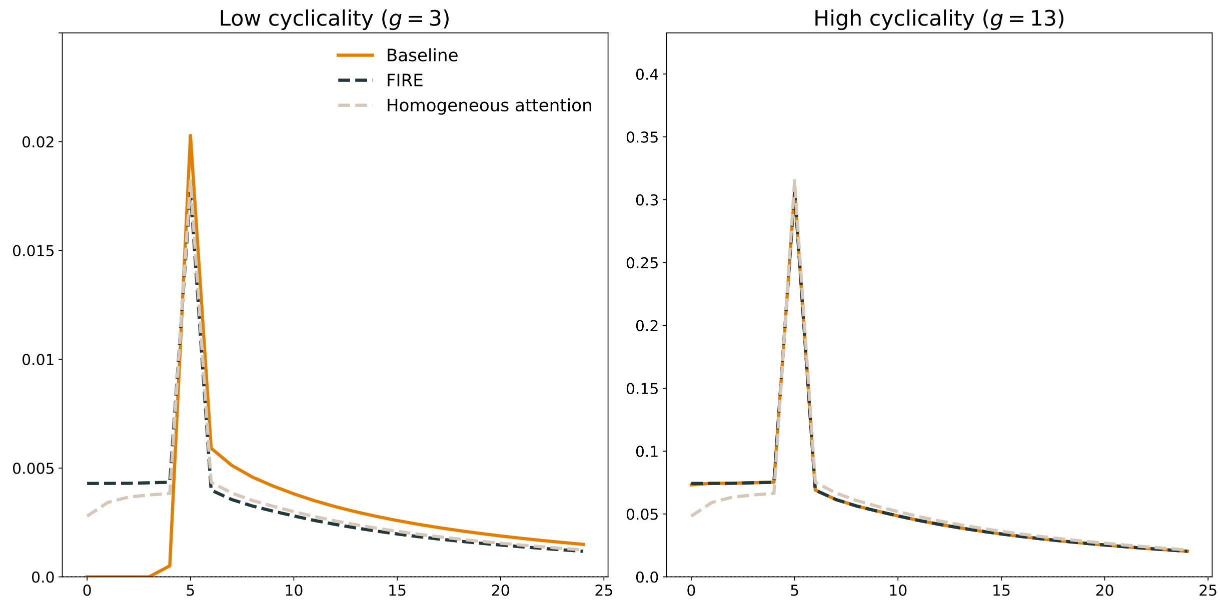



Figure C.36: Consumption response for all groups

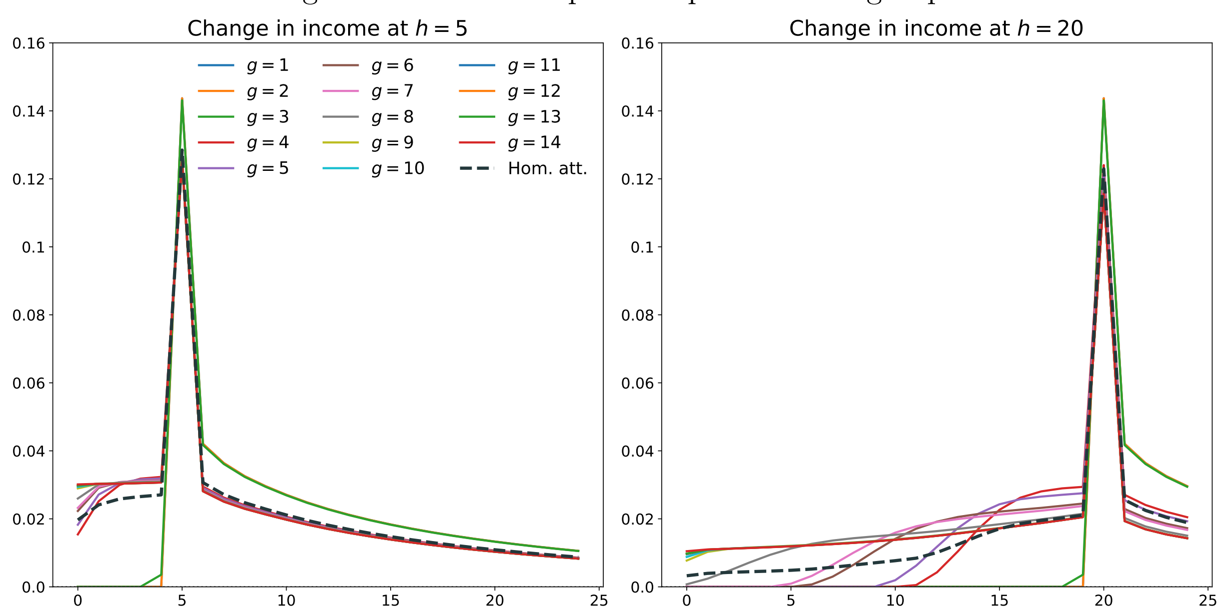


Figure C.37: Business-cycle amplification

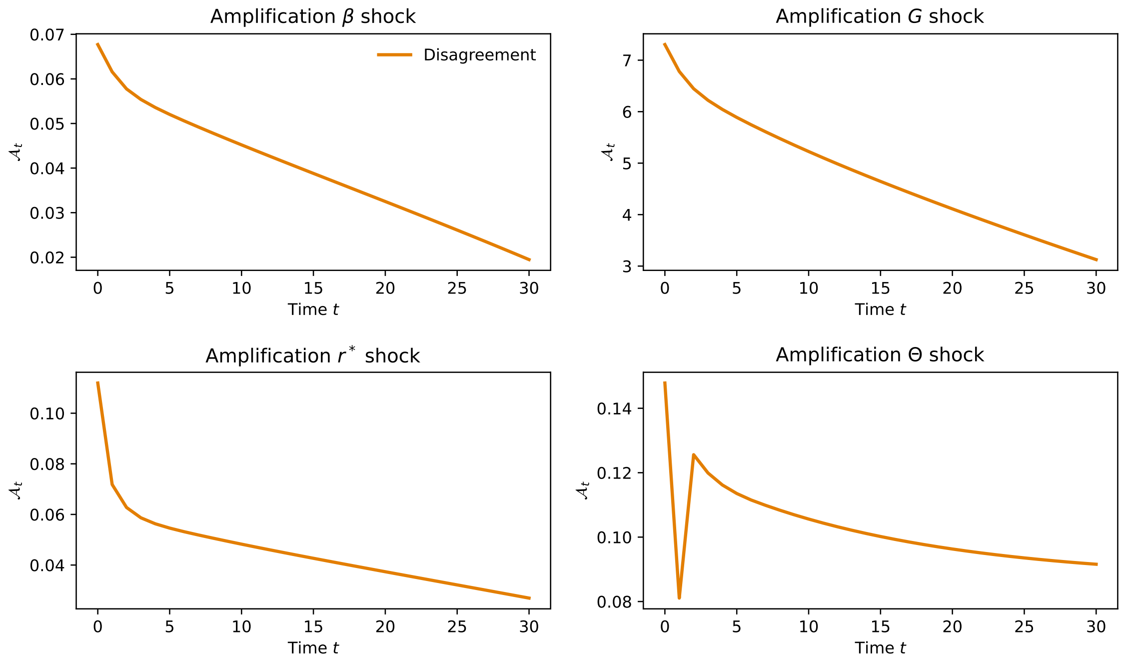



Figure C.38: Business-cycle amplification: The role of persistence

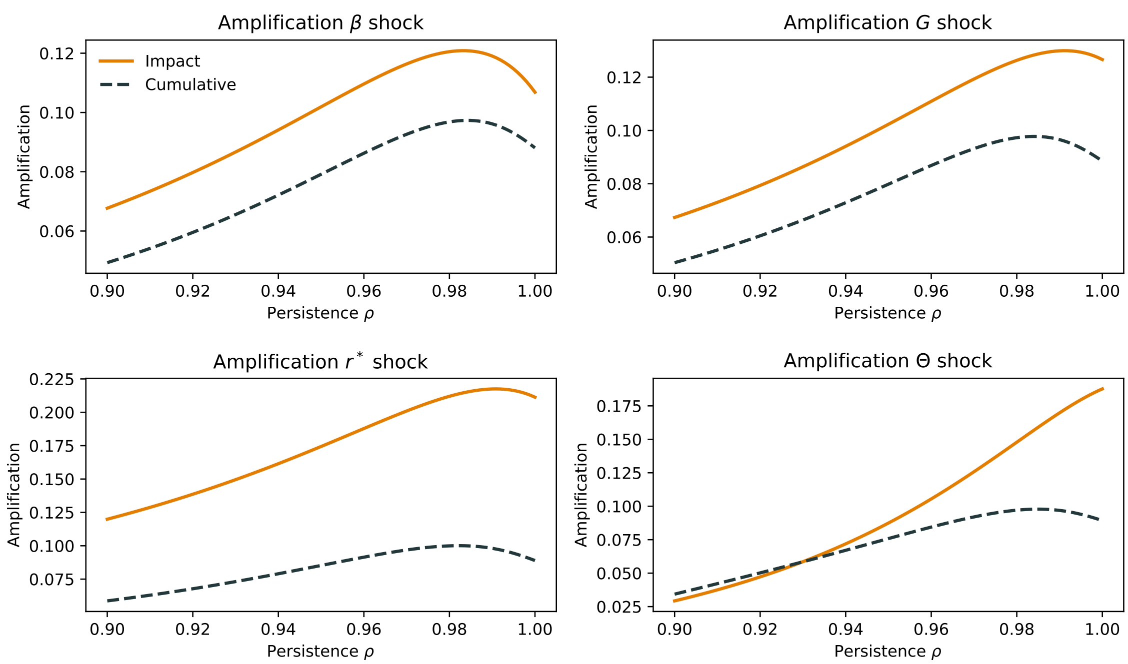


Figure C.39: Business-cycle amplification: The role of monetary policy

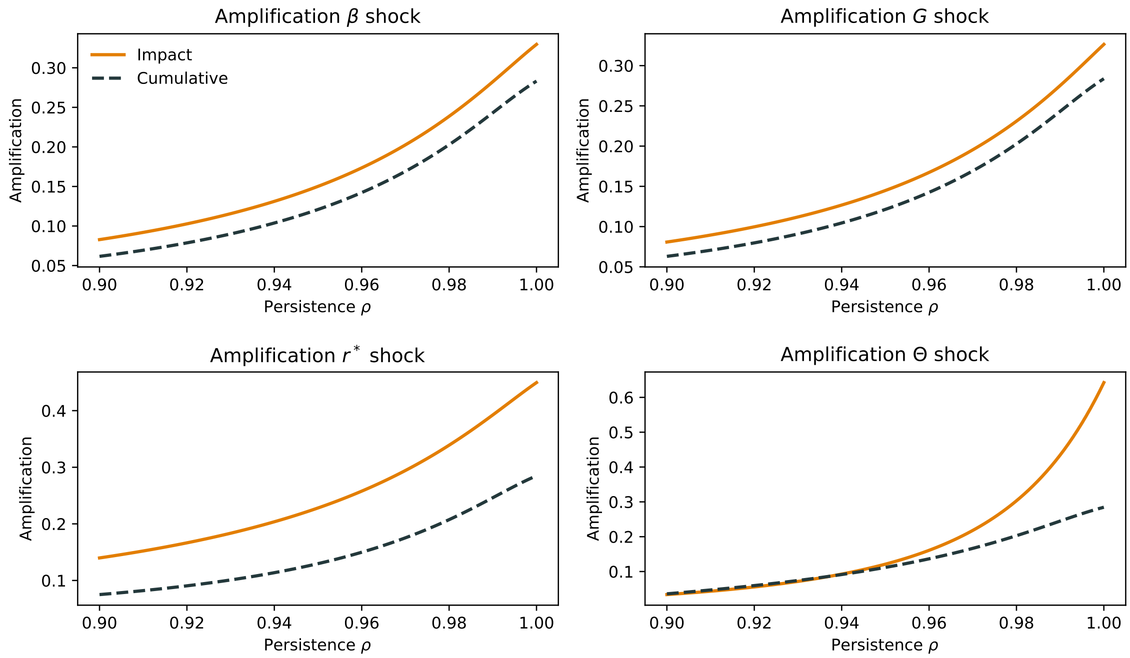



Figure C.40: Targeted spending multipliers

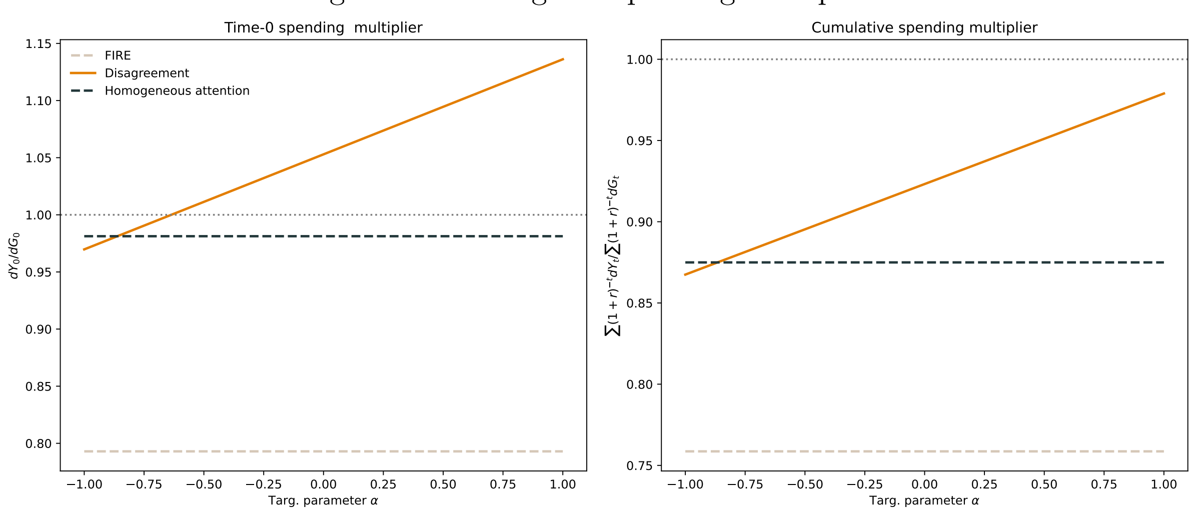


Figure C.41: Targeted spending multipliers: The role of monetary policy