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Ambiguity, Investor Disagreement, and Expected Stock Returns

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ABSTRACT

I set up a disagreement model where traders not only have different interpretations of a public signal that conveys information of a stock, but are also uncertain about the information quality of others' interpretations. The model along with traders being ambiguity-averse predicts a positive relation between investor disagreement (ID) and expected stock return. Consistent with the model's prediction, I find that stocks in the highest ID decile outperform stocks in the lowest ID decile by 9.2 percent annually, adjusted for exposures to the market return as well as size, value, momentum, and liquidity factors. In addition, stocks with higher ID prior to earnings announcements earn significantly higher earnings announcement returns. Furthermore, investor disagreement also increases following firm-specific news events.

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CHAPTER 1

Investor Disagreement in Asset Pricing

People disagree. In financial markets, investors have different interpretations of public information. Many theoretical models in the economics and finance literature also assume that investors can differ in how they interpret information.[1] In my dissertation, I ask the following questions: how is investor disagreement related to expected stock return? How do we measure investor disagreement? In particular, I incorporate traders' ambiguity aversion and their ambiguity about others' interpretations into a disagreement model and show that there exists a positive relation between investor disagreement and expected stock return. Consistent with the model's prediction, I find that stocks with higher investor disagreement experience higher future returns, both in the monthly setting and in the earnings announcement setting.

The literature still disagrees on how investor disagreement should be related to expected stock returns. The two competing views are represented by Miller [1977] and Merton [1987]. First, Miller [1977] posits that in the presence of short-sales constraints, stock prices are biased upward (lower future returns) when disagreement among investors is high. This is because when pessimists can't freely trade on the negative information as a result of short-sales constraints, asset prices are mainly set by optimists. Morris [1996],

[1] See, for example, Harrison and Kreps [1978], Harris and Raviv [1993], Scheinkman and Xiong [2003], Cao and Ou-Yang [2008], and Banerjee and Kremer [2010].

Viswanathan [2001], and Chen et al. [2002] also suggest that prices typically reflect a more optimistic valuation due to high short-sale costs.

In a traditional CAPM world, idiosyncratic risk is not priced since investors can hold efficiently diversified portfolios. Merton [1987], however, argues that investors tend to hold stocks they are familiar with and thus hold under-diversified portfolios. Naturally, they demand compensation to hold low visibility stocks with idiosyncratic risk. Since high disagreement indicates higher variation in earning streams, stocks with high divergence of opinion should earn higher future returns. However, as Merton [1987]'s hypothesis only applies to stocks with low visibility, it is difficult to explain the relation between disagreement and expected stock returns for high visibility stocks.[2]

On the other hand, some models consider investor disagreement as a source of "speculation risk" and also predict a positive relation between disagreement and future stock returns. For instance, David [2008] constructs a general equilibrium model in which two types of agents have heterogeneous beliefs about future fundamental growth. Agents face the risk that market prices move more in line with the trading models of competing agents than with their own, and thus speculate with each other. Gao et al. [2019] also argue that when investors agree to disagree, they both expect to profit at the expense of their trading counterparties. One assumption of David [2008] is that each trader is absolutely convinced that his own belief is correct.[3] In other words, traders agree to disagree all the time and never use others' beliefs to update their priors. This assumption seems too strong as in reality traders often take into account others' beliefs after observing them.

[2]Boehme et al. [2009] provide a detailed summary and empirically show that idiosyncratic risk is positively related to future stock returns only within stocks with low visibility.

[3]This assumption is similar to that of the overconfidence model of Daniel et al. [2005], in which each investor assigns an excessively large weight on his own model.

In my dissertation, I set up a disagreement model based on Kandel and Pearson [1995]'s market-trading model with disagreement. To begin with, there is a public signal which is equal to the sum of the true value of a stock and some noise. Two types of traders know the true variance of the noise but have different prior beliefs on the mean of the noise. For simplicity, I use the word "interpretations" to describe traders' prior prior mean beliefs of the noise, and investor disagreement is defined as the absolute value of the difference in two types of interpretations. By setup, higher interpretation of the signal corresponds to a more pessimistic valuation.

I make two assumptions. First, I relax the strong assumption in David [2008] that traders agree to disagree and ignore other traders' interpretations. In contrast, I assume that when traders observe the other type's interpretation, they believe that it can range from being less accurate to being more accurate than their own. This assumption stems from the fact that most traders in the market lack the ability, time, and information to accurately evaluate others' interpretations. Formally, as traders are ambiguous about the information quality of the other type's interpretation, they assign a range of information precision to it. In addition, in most ambiguity models, it is the variance of the signal that is difficult to judge.[4] In this model, however, the public signal is not ambiguous as its variance is known. In other words, ambiguity lies in the other type's interpretation of the signal rather than the signal itself.

[4] See, for example, Epstein and Schneider [2007], Epstein and Schneider [2008], Easley and O'Hara [2010], and Illeditsch [2011].

Second, the essential behavioral assumption is that traders are ambiguity-averse.[5] Several papers have provided evidence that many people exhibit ambiguity aversion. For example, in an experiment involving 104 individuals who are asked to choose between an ambiguous urn and a risky urn, Halevy [2007] finds that 61% are ambiguity-averse, 22% are ambiguity neutral, and 17% prefer the ambiguous urn. Using a Unicredit sample of 1,686 retail investors, Butler et al. [2014] find that 52% are ambiguity-averse and 25% are ambiguity-neutral. Dimmock et al. [2016] find that out of $3,258$ respondents in the American Life Panel (ALP), 52% are ambiguity averse, 10% are ambiguity-neutral, and 38% are ambiguity-seeking. In order to model ambiguity aversion, traders' preferences will be represented using the maxmin expected utility model of Gilboa and Schmeidler [1989].[6] Under the maxmin expected utility, agents have a set of probability measures and evaluate any action using the probability that minimizes the expected utility of that action.

I show that the expected stock return is increasing in investor disagreement, holding fixed the average interpretation of the signal. The intuition is that ambiguity aversion motivates traders to take into account the other type's interpretation asymmetrically, i.e., traders give more (less) weight to the other type's interpretation if it is higher (lower) than their own interpretation. In other words, when updating their beliefs, optimistic traders assign relatively more weight to the pessimistic view while pessimistic traders

[5]The concept of ambiguity aversion in economics can be traced back to at least the Ellsberg Paradox (Ellsberg [1961]), which suggests that individuals are averse to vague probabilities and may not act as if they have a single prior.

[6]There are different forms of preferences in the literature that reflect ambiguity aversion, including the smooth ambiguity model by Klibanoff et al. [2005] and Klibanoff et al. [2009], "α-maxmin" model of Ghirardato et al. [2004], and "robust control" by Hansen and Sargent [2007] and Hansen and Sargent [2011]. Though this paper adopts the maxmin expected utility formulation, a brief discussion of the extent to which alternative models would push on the results can be found in Section 1.4.

assign relatively less weight to the optimistic view. Hence, when investor disagreement is higher, stock price reflects a more pessimistic valuation (higher expected return).

In order to test the model's prediction, we need an investor disagreement (ID) measure. Past studies typically measure investor disagreement by analyst forecast dispersion, trading volume or volatility.[7] There are, however, some concerns with theses approaches. For instance, analysts receive similar training and interact with other analysts frequently, so the views of analyst forecasts may under-represent the views among investors, as proposed by Daniel et al. [2002], Anderson et al. [2005], and Erturk [2006]. Furthermore, analysts are biased in their forecasts due to agency problems, and thus tend to make overly optimistic forecasts, incorporate negative news into their forecasts sluggishly, and follow trends.[8] On the other hand, standard turnover or volatility variation can be more consistent with classical asset pricing stories that do not feature investor disagreement at all, making it difficult to attribute those variations to dispersion in beliefs.

Consistent with an implication of Kandel and Pearson [1995], the model also jointly pins down volume and price change. In particular, I use simulation to show that the correlation coefficient between volume and absolute price change is smaller when ID is higher. The intuition is that, if investors actively trade in the exact opposite directions (high investor disagreement), large trading volume can be accompanied by a small price

[7]For analyst forecast dispersion, see, e.g., Diether et al. [2002], Doukas et al. [2006], Sadka and Scherbina [2007], and Barinov [2013]. For trading volume, see, e.g., Garfinkel and Sokobin [2006], Garfinkel [2009], and Berkman et al. [2009]. For volatility, see, e.g., Boehme et al. [2006] and Chatterjee et al. [2012].
[8]For analyst optimism, see, e.g., De Bondt and Thaler [1985], La Porta [1996], Dechow and Sloan [1997], and Brown [2001]. For slow incorporation of negative information by analysts, see, e.g., Chan et al. [1996], Easterwood and Nutt [1999], Lim [2001], and Conrad et al. [2006]. For analyst herding, see, e.g., Graham [1999], Welch [2000], Lamont [2002], and Hong and Kubik [2003].

change (correlation between volume and absolute price change is low).[9] In other words, the correlation coefficient of daily trading volume and absolute price change serves as a negative indicator for ID. Empirically, ID for a stock at the end of a given month is computed as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past two months, multiplied by -1.[10]

I proceed to examine the relation between ID and expected stock returns in the cross section. At the end of each month, I sort stocks into ten decile portfolios based on ID. Then, I examine the returns for each ID decile in the subsequent month. I find that when moving from the lowest ID decile to the highest ID decile, mean returns increase almost monotonically. In particular, stocks in the highest ID decile significantly outperform stocks in the lowest ID decile by 65 basis points per month with a Newey and West [1987] t-statistic of 3.91 from January 1983 to December 2019. The corresponding monthly differences in CAPM, three-, four-, and five-factor alphas are 0.87% (t-statistic = 5.64), 0.71% (t-statistic = 5.47), 0.75% (t-statistic = 5.94), and 0.77% (t-statistic = 6.28), respectively.[11] Consistent with the model's prediction, the univariate portfolio results indicate a strongly positive relation between ID and expected stock returns.

In addition to univariate portfolio analysis, I perform bivariate portfolio analysis to ensure that the significantly positive return differences between high and low ID decile

[9]Kim and Verrecchia [1991] and Harris and Raviv [1993] indicate that when there's no investor disagreement, volume should be perfectly proportional to absolute price change.
[10]The number of trading days is around 44 in two months. Using only one trading month to compute a correlation coefficient may be subject to lack of statistical power. The asset pricing implications of ID are the same if I instead use turnover ratio to substitute for trading volume, or use squared return to substitute for absolute price change.
[11]The returns reported here are equal-weighted as in Table 2.1. I also present value-weighted returns in Table 2.2. For robustness checks, I also use DGTW-adjusted returns following Daniel et al. [1997] and Wermers [2003], and the results remain the same. The results are available upon request.

are not driven by well-known stock characteristics or risk factors. In particular, I control one at a time for 12 return predictors, including firm size (SIZE), book-to-market (BM) ratio, the cumulative return over the 11 months prior to the portfolio formation month (MOM), short-term reversal (REV), average turnover ratio (TURN), idiosyncratic volatility (IVOL) as defined in Ang et al. [2006], Amihud [2002] illiquidity ratio (ILLIQ), demand for lottery stocks with extreme positive returns (MAX) as defined in Bali et al. [2011], institutional ownership ratio (IOR), the stock beta (BETA), co-skewness (COSKEW) as defined in Harvey and Siddique [2000], and analyst forecast dispersion (DISP) as defined in Diether et al. [2002]. After controlling for each of the above variables, the return differences between high ID and low ID decile portfolios are in the range of 0.34% and 0.68% per month with Newey and West [1987] t-statistics ranging from 3.41 to 6.78. The corresponding 5-factor alpha differences are in the range of 0.45% to 0.72% and are all highly significant.

It is, however, possible that some information is lost via portfolio aggregation. Hence, in order to control for multiple variables simultaneously, I implement Fama and MacBeth [1973] regressions to examine the cross-sectional relation at the stock level. The results suggest that the relation between ID and future stock returns remains positive and highly statistically significant when a large set of control variables is included. I also perform a battery of robustness checks. I find that the significantly positive relation between ID and future stock returns persists in high and low sentiment periods (Baker and Wurgler [2006]), NBER recessions and expansions, and high and low economic uncertainty periods (Jurado et al. [2015], Ludvigson et al. [2015], and Baker et al. [2016]). All these results provide strong evidence for a positive relation between ID and expected stock returns.

Next, I provide solid evidence that investor disagreement (ID) is persistent. First, I run ID on lagged ID with a large set of control variables. The coefficient on lagged ID is 0.468 with an adjusted R-squared of 34.54%, which implies that ID is highly persistent. In addition, I examine the average month-to-month ID transition matrix and find that all diagonal probabilities exceed 10%. In particular, the diagonal probabilities are 43.03% and 38.06% for the lowest and highest ID decile, respectively. I further vary both the number of months in the formation of ID and the portfolio holding periods and find that the long-short ID strategy is robust to those variations.

I also examine whether the positive relation between ID and future stock returns holds in the earnings announcement setting. As firms typically use earnings announcements to communicate relevant information to the market, there exists a sudden increase of information prior to the earnings announcement for investors to disagree on.[12] Using portfolio sorts and stock level cross-sectional regressions, I find that stocks with high ID prior to the earnings announcement experience significantly higher cumulative abnormal returns around the earnings announcement period compared to stocks with low ID. In particular, stocks in the highest ID decile prior to earnings announcements outperform stocks in the lowest ID decile by 65 basis points in the 3-day window around earnings announcements with a Newey and West [1987] t-statistic of 4.77. The positive relation is also robust to variations in different windows to compute ID prior to earnings announcements or different earnings announcement windows.

In addition, I examine whether earnings announcements resolve disagreement among investors. I find that ID on average increases after earnings announcements. In particular,

[12] Ball and Brown [1968], Krinsky and Lee [1996], Back et al. [2018], and Yang et al. [2020] argue that the leaking of information is pervasive prior to earnings announcements.

compared to good earnings news, bad earnings news trigger a larger increase in ID. I also obtain firm-specific public news stories from RavenPack and classify them into six different news categories (Financial, Legal, M&A, Operational, Ratings, and Others). It turns out that ID also increases after these firm-specific news stories. In contrast, ID before and after macroeconomic announcements like FOMC meetings remain virtually the same.

This paper mainly contributes to the disagreement literature, both theoretically and empirically. To the best of my knowledge, this is the first paper to incorporate ambiguity aversion into a disagreement model to study the relation between investor disagreement and expected stock returns. In addition, to the extent that investors experience ambiguity, it seems especially plausible that they do so when thinking about the interpretations belonging to other investors. This observation plus the experimental evidence on ambiguity aversion motivates the way it is used in our model. Furthermore, the model's prediction applies to all stocks, whereas Merton [1987]'s hypothesis applies only to stock with low visibility and Miller [1977]'s hypothesis requires the presence of short-sales constraints. Consistent with the model's prediction, I find that stocks with higher investor disagreement experience higher future returns.

The paper also sheds light on the earnings announcement and disclosure literature. As Berkman et al. [2009] point out, it has been difficult to isolate the effect of disagreement from other effects such as momentum or post-earnings announcement drift in the traditional monthly returns setting. Hence, examining the relation between disagreement and expected stock returns in the earnings announcement setting is important. In addition, how uncertainty evolves following disclosure events has long received considerable attention.[13] This paper complements the literature by providing the evolution of investor disagreement following both firm-specific and macroeconomic news.

1.1. A disagreement model with ambiguity aversion

In this section I introduce the model. The basic setup follows Kandel and Pearson [1995]. There are three time periods ($t = 0$, $t = 1$, and $t = 2$) and two assets in a competitive market: a risk-free asset with a zero rate of return and a stock in zero net supply with an uncertain payoff X that is realized at $t = 2$. Figure 1.1 presents the timeline.

(1) A public signal S arrives.
(2) Traders observe S and form interpretations.
(3) Traders observe the other type's interpretation.

$t = 0$ $t = 1$ $t = 2$

X is realized.

Figure 1.1. **Model Timeline.** There are two types of traders in the market with equal mass indexed by $i = 1, 2$. X is the true value of the stock and is realized at $t = 2$. At $t = 0$, type i traders' prior on $X \sim N(X_i, Z_i^{-1})$. At $t = 1$, a public signal $S = X + \eta$ arrives, where $\eta \perp X$ and $\eta \sim N(\mu_\eta, \sigma_\eta^2)$. Everything about S is common knowledge except for the mean μ_η. Type i traders believe that $\mu_\eta \sim N(\mu_i, \sigma^2)$, where μ_i denotes type i traders' interpretation of S. Let μ_{-i} denote the other type's interpretation of S from type i traders' perspective. Type i traders observe μ_{-i} and believe that $\mu_{-i} = \mu_\eta + \epsilon$, $\mu_\eta \perp \epsilon$, $\epsilon \sim N(0, \sigma_\epsilon^2)$, and $\sigma_\epsilon^2 \in [\underline{\sigma_\epsilon^2}, \overline{\sigma_\epsilon^2}]$, where $0 < \underline{\sigma_\epsilon^2} < \sigma^2 < \overline{\sigma_\epsilon^2} < \infty$.

[13] For instance, Patell and Wolfson [1979] have documented immediate decline in volatility after earnings announcements, which reflects the resolution of uncertainty. In addition, Billings et al. [2015] find that implied volatility decreases after guidance announcements, while Rogers et al. [2009] find that earnings announcements increase short-term volatility.

There is a continuum of type 1 traders and a continuum of type 2 traders in the market, with each type constituting half of the total traders. Traders strive to maximize their final wealth W at $t = 2$, and are endowed with negative exponential utility functions $U(W) = -e^{-\lambda W}$, where λ is the coefficient of absolute risk aversion.

At $t = 0$, traders have different prior beliefs on X. In particular, type i traders' prior beliefs of X are given by normal distributions of mean X_i and precision Z_i, where $i \in \{1, 2\}$. In addition, traders don't know others' beliefs and likelihood functions.

At $t = 1$, a public signal S arrives and traders observe S. The informative while noisy signal is given by $S = X + \eta$, where η is independent of X, $\eta \sim N(\mu_\eta, \sigma_\eta^2)$, and $0 < \sigma_\eta^2 < \infty$. Everything about S is common knowledge except for the mean μ_η. In particular, type i traders believe that

$$\mu_\eta \sim N(\mu_i, \sigma^2), \tag{1.1}$$

where μ_i denotes type i traders' interpretation of S and $0 < \sigma^2 < \infty$. In particular, type i traders think that S is higher than X if $\mu_i > 0$, and higher μ_i implies a more negative view of the same signal. In addition, I refer to type 1 traders as pessimistic traders and type 2 traders as optimistic traders if $\mu_1 > \mu_2$, and vice versa.

At $t = 1$, traders also observe the other type's interpretation of S, which means that type 1 traders observe μ_2 and type 2 traders observe μ_1. To simplify notations, let μ_{-i} denote the other type's interpretation from type i traders' point of view. In other words, type i traders observe μ_{-i}.

As most traders in reality don't have enough information or skills to correctly evaluate the information quality of the other type's interpretation, I make the following assumption:

Assumption 1. *When traders observe the other type's interpretation, they believe that it can range from being less precise to being more precise than their own.*

Assumption 1 indicates that traders take into account the other type's interpretation impartially: they think that it can range from being less accurate to being more accurate than their own. Formally, type i traders think that

$$\mu_{-i} = \mu_\eta + \epsilon, \quad \mu_\eta \perp \epsilon, \quad \epsilon \sim N(0, \sigma_\epsilon^2), \quad \sigma_\epsilon^2 \in [\underline{\sigma_\epsilon^2}, \overline{\sigma_\epsilon^2}], \tag{1.2}$$

where $0 < \underline{\sigma_\epsilon^2} < \sigma^2 < \overline{\sigma_\epsilon^2} < \infty$ and $i \in \{1, 2\}$. When σ_ϵ^2 is higher (lower) than σ^2, type i traders believe that compared to their own interpretation μ_i, the other type's interpretation μ_{-i} is less (more) precise.[14] The information quality of μ_{-i} is thus captured by the range of precisions $[1/\overline{\sigma_\epsilon^2}, 1/\underline{\sigma_\epsilon^2}]$.

In order to update their priors on μ_η, type i traders apply Bayes's rule to obtain a family of posteriors:

$$\mu_\eta \sim N(\mu_i + \frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2}(\mu_{-i} - \mu_i), \frac{\sigma^2 \sigma_\epsilon^2}{\sigma^2 + \sigma_\epsilon^2}), \quad \sigma_\epsilon^2 \in [\underline{\sigma_\epsilon^2}, \overline{\sigma_\epsilon^2}]. \tag{1.3}$$

For tractability, let $\underline{\sigma_\epsilon^2} = \sigma^2(1 - \alpha)$, $\overline{\sigma_\epsilon^2} = \sigma^2(1 + \alpha)$, and $0 < \alpha < 1$. Next, motivated by several experimental studies in the literature, I assume that traders exhibit ambiguity aversion.[15]

Assumption 2. *Traders are ambiguity-averse.*

[14] Since $\mu_{-i} = \mu_\eta + \epsilon$, $\mu_\eta = \mu_{-i} - \epsilon$. Hence, $\mu_\eta \sim N(\mu_{-i}, \sigma_\epsilon^2)$. From (1) we know that $\mu_\eta \sim N(\mu_i, \sigma^2)$.
[15] See, for example, Halevy [2007], Butler et al. [2014], and Dimmock et al. [2016].

Traders' preferences of ambiguity aversion will be represented by the maxmin expected utility of Gilboa and Schmeidler [1989]. Under ambiguity aversion, traders have a set of probability measures and evaluate any action using the probability that maximizes the expected utility of that action.

Next, I introduce traders' problem at $t = 0$ and $t = 1$. Following the setup in Kandel and Pearson [1995], traders at $t = 0$ don't take into account the fact that prices will be "wrong" at $t = 1$ because others are potentially using different likelihood functions to update their beliefs and trade.

1.2. Trade at $t = 0$

At $t = 0$, each type i trader solves the following problem:

$$\max_{m_{i,0}} E_{i,0} \; - e^{-\lambda m_{i,0}(X-P_0)}, \tag{1.4}$$

where $E_{i,0}$ denotes expectation with respect to X of type i traders at $t = 0$ and $m_{i,0}$ denotes the position held by each of them at $t = 0$. The resulting demand is

$$m_{i,0}(P_0) = (X_i - P_0)\frac{Z_i}{\lambda} \tag{1.5}$$

Using the market-clearing condition ($\frac{1}{2}m_{1,0} + \frac{1}{2}m_{2,0} = 0$), the equilibrium price is

$$P_0^* = \frac{Z_1 X_1 + Z_2 X_2}{Z_1 + Z_2}. \tag{1.6}$$

The equilibrium holdings are $m_{1,0}(P_0^*)$ and $m_{2,0}(P_0^*)$.

1.3. Trade at $t = 1$

At $t = 1$, each type i trader solves the following problem:

$$\max_{m_{i,1}} \min_{\sigma_\epsilon^2 \in [\underline{\sigma_\epsilon^2}, \overline{\sigma_\epsilon^2}]} E_{i,1} \; - e^{-\lambda m_{i,1}(X - P_1)}, \tag{1.7}$$

where $E_{i,1}$ denotes expectation with respect to X of type i traders at $t = 1$ and $m_{i,1}$ denotes the position held by each of them at $t = 1$. Note that traders' posterior mean on μ_η is negatively related to the expected utility before maximization. Hence, ambiguity-averse traders select an information quality $\sigma_\epsilon^2 \in [\underline{\sigma_\epsilon^2}, \overline{\sigma_\epsilon^2}]$ that generates the highest posterior mean on μ_η. That is, if the other type's interpretation is higher ($\mu_{-i} - \mu_i > 0$), type i traders act as if μ_{-i} is precise ($\sigma_\epsilon^2 = \underline{\sigma_\epsilon^2}$). In contrast, if the other type's interpretation is lower ($\mu_{-i} - \mu_i < 0$), type i traders act as if μ_{-i} is imprecise ($\sigma_\epsilon^2 = \overline{\sigma_\epsilon^2}$). Formally, at $t = 1$, type i traders' posterior belief on μ_η is given by

$$\begin{cases} \mu_\eta \sim N(\mu_i + \frac{\sigma^2}{\sigma^2 + \underline{\sigma_\epsilon^2}}(\mu_{-i} - \mu_i), \frac{\sigma^2 \underline{\sigma_\epsilon^2}}{\sigma^2 + \underline{\sigma_\epsilon^2}}), & \text{if } \mu_{-i} - \mu_i > 0 \\ \mu_\eta \sim N(\mu_i + \frac{\sigma^2}{\sigma^2 + \overline{\sigma_\epsilon^2}}(\mu_{-i} - \mu_i), \frac{\sigma^2 \overline{\sigma_\epsilon^2}}{\sigma^2 + \overline{\sigma_\epsilon^2}}), & \text{if } \mu_{-i} - \mu_i < 0. \end{cases} \tag{1.8}$$

Next, plug in $\underline{\sigma_\epsilon^2} = \sigma^2(1 - \alpha)$ and $\overline{\sigma_\epsilon^2} = \sigma^2(1 + \alpha)$. Type i traders' posterior belief on μ_η is given by

$$\begin{cases} \mu_\eta \sim N(\frac{(1-\alpha)\mu_i + \mu_{-i}}{2-\alpha}, \frac{1-\alpha}{2-\alpha}\sigma^2), & \text{if } \mu_{-i} - \mu_i > 0 \\ \mu_\eta \sim N(\frac{(1+\alpha)\mu_i + \mu_{-i}}{2+\alpha}, \frac{1+\alpha}{2+\alpha}\sigma^2), & \text{if } \mu_{-i} - \mu_i < 0. \end{cases} \tag{1.9}$$

The market-clearing condition is $\frac{1}{2}m_{1,1} + \frac{1}{2}m_{2,1} = 0$.

1.4. Investor disagreement and future return

The following proposition indicates a positive relation between investor disagreement and future stock return.

Proposition 1. *Suppose* $\sigma^2 << \sigma_\eta^2$*. Let the expected stock return* $R = P_2^* - P_1^*$*. Then,*

$$R = X - \Big(\frac{Z_1X_1 + Z_2X_2 + \sigma_\eta^{-2}\{(S-\mu_1) + (S-\mu_2) - \frac{2\alpha}{(2-\alpha)(2+\alpha)}|\mu_1 - \mu_2|\}}{Z_1 + Z_2 + 2\sigma_\eta^{-2}}\Big) \tag{1.10}$$

and is increasing in investor disagreement, $|\mu_1 - \mu_2|$*, holding fixed the average interpretation of the signal,* $\frac{\mu_1+\mu_2}{2}$*.*

Proof. See Appendix A. □

The intuition is as follows. At $t = 1$, traders are uncertain about the information quality of the other type's interpretation and since they are ambiguity-averse, they assign an information quality $\sigma_\epsilon^2 \in [\underline{\sigma_\epsilon^2}, \overline{\sigma_\epsilon^2}]$ to it that generates the highest posterior mean on μ_η, which corresponds to the lowest expected utility before maximization. In other words, ambiguity-averse traders tend to place more emphasis on the larger of the two interpretations.

As a result, when updating their beliefs on μ_η, optimistic traders consider the pessimistic view to be more accurate than their own, while pessimistic traders consider the optimistic view to be less accurate than their own. Hence, when holding fixed the average interpretation of the signal, the stock price reflects pessimism more as interpretations become more polarized (high investor disagreement).

Furthermore, since X is realized at $t = 2$ and is not affected by traders' interpretations, there is no speculation risk involved. The assumption of $\sigma^2 << \sigma_\eta^2$ indicates that S

is extremely imprecise. This is consistent with that fact that in reality, firms do not communicate with investors on a daily basis, so publicly available information such as news reports and analyst forecasts reveal very less about the true value of the stock.

One thing worth mentioning is that I use the maxmin expected utility to model traders' ambiguity aversion. However, the only use of ambiguity aversion is to motivate traders to give more (less) weight to the other type's interpretation when it is higher (lower) than their own. Other ambiguity aversion preferences would also generate traders' asymmetric response and thus lead to the same conclusion. The only difference is the extent of this asymmetry. As the maxmin expected utility is the most strict version of ambiguity aversion, it leads to the most asymmetric traders' response.

1.5. Measuring investor disagreement

The setup of the model implies a natural measure of investor disagreement by examining the joint behavior of volume and price change.

Proposition 2. *Let $V^*_{0,1}$ be the equilibrium trading volume from $t = 0$ to $t = 1$. It can be shown that*

$$V^*_{0,1} = |A + B\Delta P^*_{0,1}|, \tag{1.11}$$

where $\Delta P^*_{0,1} = (P^*_1 - P^*_0)$,

$$A = \frac{\sigma_\eta^{-2}}{4\lambda}\frac{\alpha^2}{(2-\alpha)(2+\alpha)}(\mu_2 - \mu_1), B = \frac{1}{4\lambda}(Z_1 - Z_2). \tag{1.12}$$

Proof. See Appendix B. □

Note that when there is no disagreement in the market, i.e., $\mu_1 = \mu_2$, equilibrium trading volume is perfectly proportional to absolute price change (Kim and Verrecchia [1991] and Harris and Raviv [1993]), and there exists no trading volume given zero price change. However, when disagreement exists so that $\mu_1 \neq \mu_2$, there can exist trading volume given zero price change. In particular, Kandel and Pearson [1995] uses (1.11) to provide explanation for the existence of large trading volume around some earnings announcements with small price changes.

One implication of the model is that, when investor disagreement, $|\mu_1 - \mu_2|$, is higher, the relation between the equilibrium trading volume ($V^*_{0,1}$) and absolute price change ($|\Delta P^*_{0,1}|$) is weaker. To illustrate this idea, Figure 1.2 plots the correlation between equilibrium trading volume and absolute price change over different values of $(\mu_1 - \mu_2)$. Without loss of generality, μ_1 is fixed to 0, so $(\mu_1 - \mu_2)$ varies under different values of μ_2. For a given value of $(\mu_1 - \mu_2)$, I draw 100,000 observations from the distribution of $\eta \sim N(\mu_\eta, \sigma^2_\eta)$ and thus acquire 100,000 observations of S since $S = X + \eta$. The equilibrium trading volume, absolute price change, and the correlation between the two can be computed accordingly.

Figure 1.2 suggests that, the correlation coefficient between equilibrium trading volume and absolute price change is decreasing in investor disagreement, $|\mu_1 - \mu_2|$. When there is no disagreement, equilibrium trading volume and absolute price change are perfectly correlated. In addition, as long as investor disagreement is not too high, the correlation coefficient of trading volume and absolute price change is positive, which is consistent with the findings in the past literature.[16]

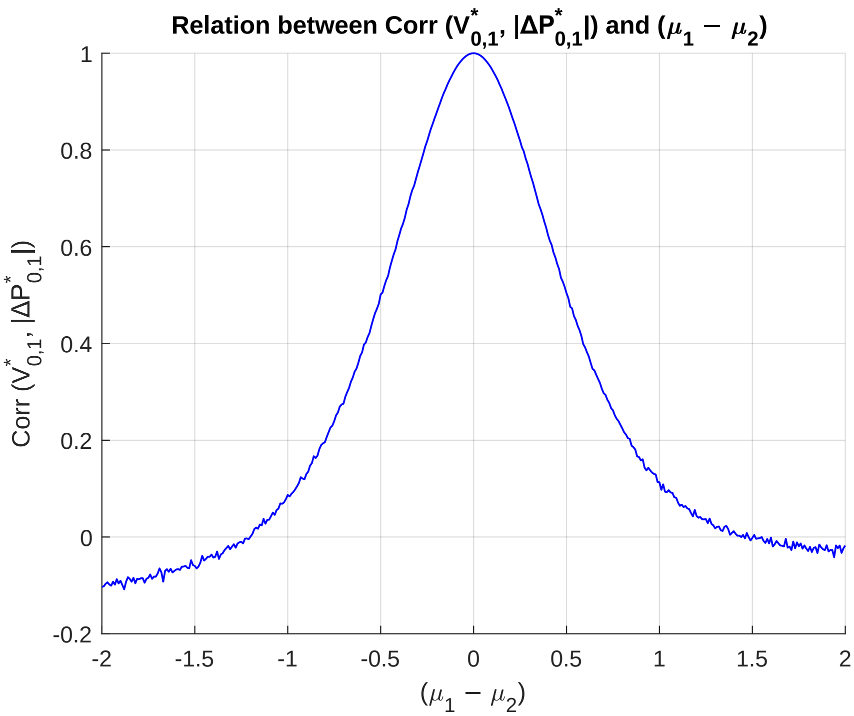


Figure 1.2. **Relation between** $Corr(V^*_{0,1}, |\Delta P^*_{0,1}|)$ **and** $(\mu_1 - \mu_2)$**.** The figure plots the correlation coefficient between equilibrium trading volume and absolute price change as a function of $(\mu_1 - \mu_2)$. Without loss of generality, μ_1 is fixed to 0, so $(\mu_1 - \mu_2)$ varies under different values of μ_2. For a given value of $(\mu_1 - \mu_2)$, I draw 100,000 observations from the distribution of $\eta \sim N(\mu_\eta = 0, \sigma^2_\eta = 400)$ and thus acquire 100,000 observations of S since $S = X + \eta$. The equilibrium trading volume, absolute price change, and the correlation between the two can be computed accordingly. Other model parameters are as follows: $X = 50$, $X_1 = 51$, $X_2 = 49$, $Z_1^{-1} = 9.98$, $Z_2^{-1} = 10.02$, $\lambda = 0.9$, $\alpha = 0.9$, and $\sigma^2 = 0.1$.

[16] Past literature has documented a positive contemporaneous relation of volume and volatility. See for example, Clark [1973], Tauchen and Pitts [1983], Karpoff [1987], Gallant et al. [1992], and Andersen [1996].

Hence, the contemporaneous correlation coefficient of daily trading volume and absolute price change serves as a negative indicator for investor disagreement (ID).[17] When the correlation coefficient between the two is smaller, it is more likely that disagreement among investors is higher.

1.6. Conclusion

This chapter studies the relation between investor disagreement and expected stock returns from a behavioral point of view. Following Kandel and Pearson [1995], I set up a disagreement model in which two types of traders have different interpretations of a public signal that conveys information of a stock. I make two assumptions. First, traders are uncertain about the information quality of the other type's interpretation. Second, traders exhibit ambiguity-aversion.

I show that there exists a positive relation between investor disagreement and future stock return. The intuition is that ambiguity aversion motivates traders to take into account the other type's interpretation asymmetrically. That is, optimistic traders attach relatively more weight to the pessimistic valuation while pessimistic traders attach relatively less weight to the optimistic view. As a result, the stock price reflects a more pessimistic valuation (higher expected return) as investor disagreement becomes higher, holding fixed the average interpretation of the signal. The model also indicates that when investor disagreement is higher, the correlation coefficient of trading volume and absolute price change is lower.

[17] In cross-section asset pricing, the implicit assumption is that firms are ex-ante identical in a way that b and $|Z_1 - Z_2|$ are virtually the same across firms.

CHAPTER 2

Empirical Analysis

In this chapter, I test the model's implication by examining the relation between ID and expected stock returns in the cross-section. First, I describe the data and sample selection. Second, I define the ID measure. Third, I perform univariate portfolio-level analysis. Fourth, I discuss average stock characteristics in each ID decile portfolio. Fifth, I perform bivariate portfolio-level analysis to examine the return-predicting power of ID after controlling for commonly known stock characteristics and risk factors. Sixth, I implement Fama and MacBeth [1973] regressions to examine the cross-sectional relation at the stock level while controlling for multiple variables simultaneously. Finally, I examine the robustness of the relation by using different samples, ID formation periods, and portfolio holding periods.

2.1. Data

The stock sample includes all common stocks (share code 10 or 11) traded on NYSE, AMEX, and Nasdaq from the Center for Research in Security Prices (CRSP) for the period from January 1983 to December 2019. The second data set is Compustat, which is used to obtain the equity book values for computing the book-to-market ratios of individual firms. Stocks are required to have non-missing firm size (SIZE), book-to-market (BM) ratio, and momentum (MOM), which are defined in detail in Appendix C.

2.2. Measuring investor disagreement

The first step involves measuring investor disagreement (ID) for each stock-month. I define ID at the end of a given month as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past two months (around 44 trading days), multiplied by -1. For example, investor disagreement of a stock at the end of October is defined as the correlation coefficient between its daily trading volume and absolute price change over September and October, multiplied by -1.

A stock trading day t is eligible if the price per share on $t-31$ is at least 5 dollars and has non-missing return and volume. This is to ensure the results are not driven by small, illiquid stocks or by bid-ask bounce. All returns are delisting-adjusted. Stocks are required to have at least 30 eligible trading days to compute ID. Figure 2.1 plots the time-series distribution of the number of all CRSP common stocks and eligible stocks to compute ID.

2.3. Univariate portfolio-level analysis

I first perform univariate portfolio-level analysis to examine the relation between ID and expected stock returns in the cross section. At the end of each month, I sort stocks into ten decile portfolios based on ID. Decile 1 (low ID) is the portfolio of stocks with the lowest investor disagreement, and decile 10 (high ID) is the portfolio of stocks with the highest investor disagreement. Stocks are held for one month after being assigned into ID decile portfolios.

Table 2.1 presents the equal-weighted monthly average returns of ID-sorted decile portfolios. When moving from the lowest to highest ID decile, the next-month average excess return increases almost monotonically from 0.18% to 0.82%. The average excess return difference between decile 10 (high ID) and decile (low ID) is 0.65% with a corresponding Newey and West [1987] t-statistic of 3.91.

In addition to the average excess returns, Table 2.1 also presents the risk-adjusted returns (alphas) from regressing monthly excess return on contemporaneous risk factors. CAPM alpha is the intercept from the regression of excess portfolio returns on a constant and excess market return (MKT). Three-factor alpha is the intercept from the regression of excess portfolio returns on a constant, the excess market return (MKT), a size factor (SMB), and a book-to-market factor (HML). Four-factor alpha is the intercept from the regression of excess portfolio returns on a constant, the excess market return (MKT), a size factor (SMB), a book-to-market factor (HML), and a momentum factor (UMD) of Carhart [1997]. Five-factor alpha is the intercept from the regression of excess portfolio returns on a constant, the excess market return (MKT), a size factor (SMB), a book-to-market factor (HML), a momentum factor (UMD), and a liquidity factor (LIQ) of Pástor and Stambaugh [2003]. If the factor model can capture the cross-sectional variation in stock returns, then the corresponding alpha should be statistically indistinguishable from zero.

As shown in the third column in Table 2.1, CAPM alpha increases from -0.63% to 0.23% per month when moving from the lowest to highest ID decile. The difference in CAPM alphas between the high and low ID portfolios is 0.87% per month with a Newey and West [1987] t-statistic of 5.64. The next three columns present similar alpha results from the three-factor, four-factor, and five-factor models. When moving from the lowest to the highest ID decile, the three-factor alpha increases from −0.57% to 0.14%, the four-factor alpha increases from −0.50% to 0.25%, and the five-factor alpha increases from −0.51% to 0.26%. The difference in alphas between the high ID and low ID portfolios is 0.71% (t-statistic=5.47), 0.75% (t-statistic=5.94), and 0.77% (t-statistic=6.28) per month for the three-factor, four-factor, and five-factor model, respectively.

In addition, I examine the source of the risk-adjusted return difference between high ID and low ID portfolios. Is it generated by outperformance of high ID stocks or underperformance of low ID stocks? The last column of Table 2.1 indicates the strongly significant five-factor alpha spread (t-statistic=6.28) is driven by both the outperformance of high ID stocks (significantly positive with a t-statistic of 2.69) and the underperformance of low ID stocks (significantly negative with a t-statistic of −6.43).

Table 2.2 presents evidence from the value-weighted decile portfolios of ID. The results are slightly weaker but in general consistent with the equal-weighted portfolio results.

Stocks in decile 1 (low ID) generate a value-weighted average excess return of 0.50% per month, while stocks in decile 10 (high ID) generate higher value-weighted average excess return of 0.91% per month. The average return differential is 0.50% per month with a Newey and West [1987] t-statistic of 2.78. The difference in alphas between the high ID and low ID portfolios is 0.52% (t-statistic=3.56), 0.40% (t-statistic=3.20), 0.37% (t-statistic=2.95), and 0.36% (t-statistic=2.84) per month for the CAPM, three-factor, four-factor, and five-factor model, respectively.

In Table 2.1 and Table 2.2, I also report betas with respect to MKT, SMB, HML, UMD, and LIQ risk factors. In both cases, MKT betas and HML betas are significantly negative and significantly positive, respectively, suggesting that compared to stocks in the lowest ID decile, stocks in the highest ID decile are less exposed to market risk and have a tilt towards value stocks. To test the hypothesis that all 10 alphas are jointly equal to zero, I implement GRS test of Gibbons et al. [1989]. For both equal-weighted and value-weighted and for all regression models, the GRS test rejects at 1% level.[1]

Overall, the univariate portfolio analysis suggests a positive relation between ID and expected stock returns.

2.4. Average stock characteristics

Next, I examine the stock composition of investor disagreement (ID) decile portfolios. In particular, Table 2.3 presents for each ID decile, the time-series average of mean values of stock characteristics, including firm size (SIZE), book-to-market (BM) ratio, the cumulative return (in percent) over the 11 months prior to the portfolio formation month (MOM), the return (in percent) in the portfolio formation month (REV), average turnover ratio (TURN), idiosyncratic volatility (IVOL) as defined in Ang et al. [2006], Amihud [2002] illiquidity ratio (ILLIQ), lottery demand (MAX) as defined in Bali et al. [2011], institutional owernship ratio (IOR) defined the ratio of shares owned by institutions as reported in 13F filings in the last quarter, the stock beta (BETA), and co-skewness (COSKEW) as defined in Harvey and Siddique [2000]. Definitions of these variables are given in Appendix C. The weights are based on the number of observations in each portfolio in each month and there is an average of 306 stocks per decile portfolio.

The first row of Table 2.3 reports that the average investor disagreement (ID) increases from -0.77 to 0.07 when moving from the lowest to highest ID decile. The average ID

[1]For brevity I didn't report the results here. However, all the test statistics are available upon request.

in the subsequent month increases monotonically from -0.58 to -0.12 from the lowest to highest ID decile, which sheds light on the persistence of investor disagreement.

Fama and French [1992] and Fama and French [1993] report that on average, small stocks earn higher future returns than large stocks. The third row of Table 2.3 indicates the average market capitalization (SIZE) slightly increases and then decreases when moving from the low ID decile to high ID decile. In fact, SIZE is relatively large around middle ID deciles. This is perhaps because large firms benefit more from their disclosure policy compared to small firms (Diamond and Verrecchia [1991]) due to lower information and proprietary costs. As large firms on average tend to be more transparent, investors disagree less.

This result provides further support for the return differences between high and low ID decile in Table 2.2 and Table 2.3 since if small stocks do earn higher subsequent returns, then low ID decile should earn higher returns than middle ID decile.

The average book-to-market (BM) ratio for each investor disagreement (ID) decile is reported in the fourth row. As ID increases across the deciles, BM increases monotonically. The concentration of high book-to-market stocks in the high ID deciles casts doubt on the positive relation between ID and expected stock returns, as Fama and French [1992] and Fama and French [1993] document that stocks with high BM ratio stocks (value stocks) earn higher subsequent returns than stocks with low BM ratio (growth stock).

Looking at the fifth and sixth row of Table 2.3, one observes that as ID increases across the deciles, both momentum (MOM) and short-term reversal (REV) decrease. The decrease in MOM is good news as Jegadeesh and Titman [1993] shows that stocks that perform the best (worst) over intermediate horizons tend to do well (poorly) in the future.

If past losers do continue to perform badly in the future, high ID stocks should experience low instead of high returns. However, the decrease in REV across ID deciles casts doubt on the significance of the long-short ID strategy, as stocks tend to exhibit return reversal due to initial price overreaction to good news and bid-ask bounce (Jegadeesh [1990] and Lehmann [1990]).

Gervais et al. [2001] find that stocks withe higher volume earn higher returns, which is known as the high volume return premium. Looking at the seventh row of Table 2.3, stock turnover ratio (TURN) decreases monotonically when ID increases. The pattern is good news for the positive relation between ID and expected stock returns, as the concentration of high trading volume stocks in low ID deciles would suggest these portfolios earn higher instead of lower returns observed in the data.

Next, the eight row of Table 2.3 indicates that as ID increases across deciles, average idiosyncratic volatility (IVOL) decreases. As Ang et al. [2006] present evidence that stocks with high idiosyncratic volatility generate lower future returns, the negative relation between ID and idiosyncratic volatility raises concern on the positive relation between ID and future stock returns. On the other hand, Amihud [2002] suggests that expected stock returns increase in illiquidity. Looking at the ninth row of Table 2.3, there exists no striking pattern of illiquidity across ID deciles.

As shown in the tenth row of Table 2.3, the average demand for lottery stocks with extreme positive returns (MAX) is lower for stocks in high ID deciles. Since Bali et al. [2011] and Bali et al. [2017] document that low MAX stocks earn higher expected returns than high MAX stocks, the negative relation between ID and MAX casts doubt on the positive relation between ID and future stock returns.

Looking at the eleventh row of Table 2.3, institutional ownership ratio (IOR) decreases as ID increases. This negative relation between ID and IOR provides support of the positive relation between ID and expected stock returns, as Asquith et al. [2005] find that short-sale constrained stocks with low institutional ownership significantly underperform than high institutional ownership stocks.

Next, the twelfth row of Table 2.3 indicates that when ID increases across deciles, average stock beta (BETA) decreases monotonically. This pattern suggests that high ID stocks are less exposed to market risk. If stocks are compensated more for bearing more exposure to market risk, stocks with higher ID should instead earn lower future returns. Hence, the negative relation between ID and BETA is good news for the return differences between the high and low ID decile as reported in Table 2.1 and Table 2.2.

On the other hand, in the thirteenth row of Table 2.3, average co-skewness (COSKEW) first increases then decreases when moving from the lowest to the highest ID decile. Compared to low ID deciles, high ID deciles on average have lower co-skewness, which further provides support for the positive relation between ID and future stock returns, since Harvey and Siddique [2000] report that stocks with high co-skewness generate lower one-month-ahead returns. In the fourteenth row of Table 2.3, average analyst forecast dispersion (DISP) appears to be stable across ID deciles, which indicates that ID is not picking up or related to dispersion in beliefs among analysts.

In sum, Table 2.3 indicates that compared to low ID stocks, high ID stocks on average have high book-to-market (BM) ratio, low intermediate-horizon momentum (MOM), low short-term reversal (REV), low turnover ratio (TURN), low idiosyncratic volatility (IVOL), low demand for lottery stocks (MAX), and low exposure to market risk (BETA).

In particular, the fact that high ID stocks having high BM, low REV, low IVOL, and low MAX seems to dampen the validity of the positive relation between ID and expected stock returns. In the next section, I use bivariate portfolio sorts to show that the positive relation between ID and expected stock returns is not driven by the above return predictors.

2.5. Bivariate portfolio-level analysis

The section studies whether the relation between investor disagreement (ID) and expected stocks returns still holds after controlling for the well-known cross-sectional return predictors: market capitalization (SIZE), book-to-market (BM) ratio, momentum (MOM), short-term reversal (REV), turnover ratio (TURN), idiosyncratic volatility (IVOL), illiquidity (ILILQ), demand for lottery stocks with extreme positive returns (MAX), institutional ownership ratio (IOR), the stock beta (BETA), co-skewness (COSKEW), and analyst forecast dispersion (DISP).

I first examine whether the results in Table 2.1 and Table 2.2 are simply capturing a size effect. Each month, I assign stocks to one of five quintiles based on firm size (SIZE).[2] Within each size quintile, stocks are further sorted into deciles based on ID in the previous month. I then examine the next month returns in each portfolio. Table 2.4 shows that the return differential is positive and highly significant in all size quintiles. In addition, the average equal-weighted monthly return differential between high ID and low ID stocks decreases when moving from the smallest to the largest size quintile (except when going from the second to the third size quintile).

[2] As a robustness check, I also form portfolios using NYSE-based market capitalization. The results are similar and are available upon request.

In particular, the long-short ID strategy for the smallest and the largest size quintile on average generates a return of 1.07% and 0.28% per month, with a Newey and West [1987] t-statistic of 3.75 and 2.14, respectively. In addition, the corresponding CAPM, three-factor, four-factor, and five-factor alphas are all significantly positive. Specifically, the five-factor alpha differences are in the range of 0.29% to 1.32% per month with t-statistics ranging from 2.29 to 4.57. The above results indicate that the strongly positive relation between ID and expected stock returns is not driven by size effect.

Table 2.5 presents the results of two-way cuts on book-to-market (BM) ratio and ID. The return differential and corresponding CAPM, three-factor, four-factor, and five-factor alphas between low and high ID stocks are highly significant in all book-to-market quintiles, indicating that the positive relation between ID and expected stock returns is not simply capturing a book-to-market effect. In addition, compared to other BM quintiles, the long-short ID strategy in the lowest BM quintile generates the highest return of 0.97% per month with a Newey and West [1987] t-statistic of 5.33.

Table 2.6 presents the double sorts results on momentum (MOM) and ID. Again, the return differential and CAPM, three-factor, four-factor, and five-factor alphas between high and low ID stocks remain highly significant across all momentum quintiles. In particular, the return differential between high and low ID stocks is the highest in the stocks that are past losers. In particular, the long-short ID strategy generates a five-factor alpha of 1.54% with a Newey and West [1987] t-statistic of 6.57 in the lowest momentum quintile.

Overall, Table 2.5, Table 2.6, and Table 2.7 indicate that the significantly positive relation between ID and future stock returns cannot be explained by the well-known size, value, or momentum effect. In addition, the return differential between high and low ID stocks is most pronounced in small stocks, growth stocks, and stocks that perform poorly over the past year.

I proceed to control for other commonly used return-predicting stock characteristics. In each month, stocks are first sorted into deciles based on a control variable and then, within each decile I sort stocks into deciles based on ID. Stocks are held for month and portfolio returns are equal-weighted. For brevity, I do not report returns for all 100 (10×10) portfolios. Instead, the ten investor disagreement decile portfolios are averaged over each of the ten control variable decile portfolios. Table 2.8 reports for each control variable the time-series average of excess returns, high-minus-low excess returns, and corresponding five-factor alphas, together with Newey and West [1987] t-statistics to examine their statistical significance.

Table 2.8 shows that after controlling for many cross-sectional return predictors, the return differences between high ID and low ID decile portfolios are in the range of 0.34% and 0.68% per month with Newey and West [1987] t-statistics ranging from 3.41 to 6.78. The corresponding five-factor alpha differences are in the range of 0.45% to 0.72% and are all highly significant. In particular, when controlling for TURN, IVOL, and DISP, the five-factor alpha difference is 0.65% (t-statistic=6.50), 0.45% (t-statistic=5.29), and 0.50% (t-statistic=5.03), respectively, which provides evidence that the positive relation between ID and expected stock returns is not simply picking up existing disagreement measures. Overall, the results in this section indicate that well-known firm characteristics or risk factors cannot explain the positive relation between ID and expected stock returns.

2.6. Firm-level cross-sectional regressions

So far, the significance of investor disagreement (ID) as a determinant of the cross-section of expected returns has been examined at the portfolio level (both univariate and bivariate). The portfolio-level analysis is non-parametric since no functional form on the relation between the ID and the future returns is imposed. In addition, it is possible that some information is lost via portfolio aggregation and it is difficult to control for multiple variables simultaneously via portfolio analysis. Moreover, the Gibbons et al. [1989] tests seldom come close to rejecting the hypothesis that the three-factor, four-factor, or five-factor model explains average returns. Hence, I examine the cross-sectional relation between ID and expected returns at the stock level using Fama and MacBeth [1973] regressions. The incremental predictive power of ID can be examined relative to other control variables known to explain the cross-section of returns.

Table 2.9 reports the time-series averages of the slope coefficients from the regressions of one-month-ahead stock returns on ID with and without control variables. The average slopes provide standard Fama and MacBeth [1973] tests for determining which explanatory variables on average have nonzero premiums. Specifically, I run the following monthly cross-sectional regressions at a monthly frequency from January 1983 to December 2019:

$$R_{i,m+1} = \alpha_m + \beta_m ID_{i,m} + \lambda_m X_{i,m} + \epsilon_{i,m+1}, \tag{2.1}$$

where $R_{i,m+1}$ is the realized excess return on stock i in month $m+1$, ID is the investor disagreement of stock i at the end of month m, and $X_{i,m}$ is the same set of stock-specific control variables at time m for stock i, including firm size (SIZE), book-to-market (BM) ratio, momentum (MOM), short-term reversal (REV), turnover ratio (TURN), idiosyncratic volatility (IVOL), illiquidity (ILILQ), demand for lottery stocks with extreme positive returns (MAX), institutional ownership ratio (IOR), the stock beta (BETA), co-skewness (COSKEW), and analyst forecast dispersion (DISP).

Table 2.9 reports the time-series averages of the slope coefficients with corresponding Newey and West [1987] t-statistics in parentheses. In the first column, the average slope coefficient from regressing realized returns on ID alone is 0.780 and highly significant (t-statistic = 3.85), indicating a strongly positive relation between ID and expected stock returns.

Column 2 of Table 2.9 controls for firm size (SIZE), book-to-market (BM) ratio, and momentum (MOM), and the coefficient on ID remains economically and statistically significant. Column 3 further controls for the short-term reversal (REV) and turnover ratio (TURN). Still, the average slope on ID is positive and highly significant. Column 4 of Table 2.9 shows that after including idiosyncratic volatility (IVOL), illiquidity (ILLIQ), demand for lottery sock with extreme positive returns (MAX), and institutional ownership ratio, the average slope on ID becomes 0.367 with a highly significant Newey and West [1987] t-statistic of 3.73. Column 5 further includes market beta (BETA) and co-skewness (COSKEW), and the coefficient on ID is still significantly positive. Finally, Column 6 incorporates analyst forecast dispersion and the coefficient on ID shrinks to 0.251 with a Newey and West [1987] t-statistic of 2.86.

The coefficients on most control variables are consistent with evidence in the literature. Stocks exhibit strong intermediate-horizon momentum and short-term reversals. The average slopes are significantly negative for idiosyncratic volatility, institutional ownership ratio, and analyst forecast dispersion, which is consistent with the evidence in Ang et al. [2006], Asquith et al. [2005], and Diether et al. [2002].

Overall, the multivariate Fama-MacBeth regression results in Table 2.9 indicate that when simultaneously controlling for various stock characteristics and risk factors, the average slopes on ID remain positive and highly significant, indicating a strongly positive relation between ID and the cross-section of expected stock returns.

Next, I provide a variety of robustness checks to examine whether the positive relation between investor disagreement (ID) and future stocks returns is nonlinear and thus changes over time. I also examine the persistence of ID.

2.7. Business cycles, investor sentiment, and economic uncertainty

I first examine whether the long-short ID strategy is sensitive to business cycles and investor sentiment in Table 2.10. In the second and the third column, the five-factor alphas and corresponding Newey and West [1987] t-statistics of each ID decile and the long-short ID strategy are reported under economic expansions and recessions. The expansions and recessions months are issued by the National Bureau of Economic Research's (NBER) Business Cycle Dating Committee.[3] Specifically, a recession is the period between a peak of economic activity and its subsequent trough. Between trough and peak, the economy is in an expansion. There are 410 expansions and 34 recessions from January 1983 to December 2019.

The equal-weighted five-factor alpha increases from -0.49% to 0.22% and from -0.55% to 0.69% per month for expansions and recessions, respectively. In particular, the difference in alphas is 0.71% (t-statistic=5.38) for expansions and 1.23% (t-statistic=2.04) for

[3] https://www.nber.org/research/business-cycle-dating

recessions. The results provide strong evidence that the significantly positive relation between ID and future stock returns is robust to different business cycles.

In addition, it is possible that the positive relation between ID and future stock returns is concentrated in certain investor sentiment periods. To mitigate this concern, I first classify each month as following either a high-sentiment month or a low-sentiment month. A high-sentiment (low-sentiment) month is one in which the value of the BW (Baker and Wurgler [2006]) sentiment index in the previous month is above (below) the median value for the sample period.[4] The fourth and the fifth column show that long-short ID strategy generates a five-factor alpha of 0.65% (t-statistic=5.25) and 0.92% (t-statistic=4.53) per month for low sentiment and high sentiment periods, respectively. The results indicate that the significantly positive relation between ID and expected stock returns is robust to investor sentiment.

Another robustness check is to examine whether macroeconomic uncertainty affects the positive relation between ID and expected stock returns. I use four economic uncertainty measures (macro, real, financial, and policy-related economic uncertainty) in the literature to classify each month as either a high-uncertainty month or a low-sentiment month. A high-sentiment month is one in which the value of the economic uncertainty index is above the median value for the sample period, and the low-sentiment months are those with below-median values.

Jurado et al. [2015] and Ludvigson et al. [2015] introduce time series measures of macroeconomic, real, and financial uncertainty.[5] In the two papers, real activity shocks

[4]The latest investor sentiment data is available till year 2018 and can be obtained from Professor Jeffrey Wurgler's website.
[5]The data is obtained from Professor Ludvigson's website.

are originated from technology, monetary policy, preferences, or government expenditure innovations, financial uncertainty arises because of expected volatility in financial markets, and macro uncertainty arises because of expected volatility in the macro economy, such as an expectation of greater difficulty in predicting future productivity, future monetary policy or future fiscal policy. Baker et al. [2016] constructs policy-related economic uncertainty index by combining newspaper coverage of policy-related economic uncertainty, the number of federal tax code provisions set to expire in future years, and disagreement among economic forecasters.[6]

Table 2.11 reports the five-factor alphas and corresponding Newey and West [1987] t-statistics of each ID decile portfolio and the long-short ID strategy. In all columns, the five-factor alphas increase when moving from the lowest ID to the highest ID decile. In addition, the five-factor alphas of the long-short ID strategy are in the range of 0.72% to 0.92% per month, with t-statistics between 3.95 and 6.68. The results indicate that the long-short ID strategy prevails in either high- or low- macro, financial, real, and policy-related economic uncertainty periods.

2.8. Persistence of ID and the long-short ID strategy performance

First, I examine whether investor disagreement (ID) is persistent. To address this question, I examine the persistence of ID by running firm-level cross-sectional regressions of ID on lagged ID and 12 lagged cross-sectional predictors including firm size (SIZE), book-to-market (BM) ratio, momentum (MOM), short-term reversal (REV), turnover ratio (TURN), idiosyncratic volatility (IVOL), illiquidity (ILILQ), demand for lottery

[6]The data is obtained from https://www.policyuncertainty.com/index.html.

stocks with extreme positive returns (MAX), institutional ownership ratio (IOR), the stock beta (BETA), co-skewness (COSKEW), and analyst forecast dispersion (DISP).

Panel A in Table 2.11 reports the average cross-sectional coefficients on ID from the univariate and multivariate cross-sectional regressions. The coefficients on ID are 0.552 and 0.468 for univariate and multivariate cross-sectional regressions, respectively, and are both extremely significant. The adjusted R-squared in both regressions are above 30%, indicating substantial cross-sectional explanatory power. The regression results suggest that stocks with high ID in one month on average tend to be of high ID in the subsequent month.

Another way to examine the persistence of ID is to compute the average month-to-month decile portfolio transition matrix. Panel B in Table 2.11 reports the results, where column (i, j) is the average probability that a stock in ID decile i in month will be in ID decile j in the following month. If ID is completely random, then all the diagonal probabilities should be approximately 10%. First, all the diagonal elements of the transition matrix exceeds 10%, indicating that ID is indeed persistent. In particular, the persistence is especially strong within the extreme deciles. Stocks in decile 10 (high ID) have a 38.06% chance of remaining in the same decile in the subsequent month, and stocks in decile 1 (low ID) have a 42.87% chance of appearing in the same decile in the following month.

In addition, I vary the number of months in the formation of ID and examine the significance and magnitude and the corresponding long-short ID strategy. In particular, ID at the end of a given month is computed as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past T months, multiplied by -1. For different formation periods ranging from 3 to 12 months, Table 2.12 reports the next-month equal-weighted excess returns, CAPM alpha, three-factor alpha, four-factor alpha, and five-factor alpha between the highest and the lowest ID decile.

The excess return of the long-ID strategy ranges from 0.46% to 0.58% per month, with Newey and West [1987] t-statistics between 2.20 and 3.44. The corresponding risk-adjusted returns are all positive and highly significant, indicating that the positive relation between ID and expected stock returns is robust to different formation months of ID.

Next, I examine the long-short ID strategy under different holding periods to ensure that the high returns generated by the long-short ID strategy are not caused by a statistical fluke. In particular, I vary the number of months one holds each ID portfolio after it has been formed following Jegadeesh and Titman [1993]. For example, when the holding period equals to 3 months, the portfolio return in month t is the average return of the decile portfolios formed in $t-1$, $t-2$, and $t-3$. Hence, each decile portfolio changes one-third of its composition each month.

Table 2.13 reports the equal-weighted excess returns, CAPM alpha, three-factor alpha, four-factor alpha, and five-factor alpha between the highest and the lowest ID decile for different holding months. Both excess returns and risk-adjusted returns remain significantly positive under all holding periods up to 12 months. In addition, the five-factor alpha decreases from 0.68% (t-statistic=6.57) to 0.37% (t-statistic=3.68) per month as the number of holding month increases. The results suggest that the positive relation between ID and future stock returns is most significant for short to intermediate horizons.

2.9. Earnings announcement setting

Next, I study investor disagreement in the earnings announcement setting. First, if there indeed exists a positive relation between ID and expected stocks returns, then it should also be the case that stocks with high ID prior to the earnings announcement significantly outperform those with low ID around the earnings announcement.

2.9.1. Data and variable definitions

To test this hypothesis, I first identify earnings announcement dates of firms with common stocks traded on NYSE, NASDAQ, and AMEX from Compustat, which according to WRDS are more reliable compared to announcement dates from IBES.[7] Next, I define reference and earnings announcement period as the 44-day window $[-45, -2]$ and 3-day window $[-1, 1]$, respectively, where $t = 0$ is the earnings announcement date. As a robustness check, I also use four other variations of reference and earnings announcement period in all my following tests.[8]

ID is defined as the contemporaneous correlation coefficient of daily trading volume and absolute price change in the reference period, multiplied by -1. Again, a stock trading day t is eligible if the price per share on $t - 31$ is at least 5 dollars and has non-missing return and volume. Stocks are required to have at least 30 eligible trading days in the reference period to compute ID. Figure 2.2 plots the number of eligible stocks issuing earnings announcement in each calendar quarter from the first quarter of 1983 to the fourth quarter of 2019.

[7] https://wrds-www.wharton.upenn.edu/pages/support/support-articles/ibes/

[8] REF period $[-48, -5]$ with EAR period $[-4, 4]$, REF period $[-47, -4]$ with EAR period $[-3, 3]$, REF period $[-46, -3]$ with EAR period $[-2, 2]$, and REF period $[-45, -2]$ with EAR period $[-1, 1]$.

Following most literature studying earnings announcements, stock performance around an earnings announcement is defined as the stock's cumulative abnormal return (CAR), which is the difference between the compounded stock return and value-weighted market return (in percent) over the earnings announcement period.

Other control variables are defined similarly as in Appendix C. SIZE is the log of market capitalization in millions of dollars and BM is book-to-market ratio. RET is the return (in percent) compounded over the reference period. TURN and IVOL are the average turnover ratio and idiosyncratic volatility in the reference period. IOR is the ratio of shares owned by institutions as reported in 13F filings in the last quarter.[9] NUMEST is the number of unique analysts that have eligible fiscal year one earnings estimates on IBES in the reference period.[10]

2.9.2. Portfolio analysis

I start my analysis by examining the relation between investor disagreement (ID) in the reference period and cumulative abnormal returns (CAR) around earnings announcements. First, every calendar quarters are classified into deciles based on their ID in the reference period. Then, I compute the cross-sectional mean CAR around earnings announcements for each ID decile. Then, I compute the time-series (weighted) averages of these cross-sectional means across all quarters. The weights are based on the number of observations in each ID decile each quarter .

[9]Nagel [2005] emphasizes the relation between short-sales constraints and divergence of opinion when examining stock returns.

[10]See Israelsen [2016], Lee and So [2017], and Ali and Hirshleifer [2020] for evidence of the relation between analyst coverage and stock returns.

Table 2.14 presents time-series average of quarterly mean values of CAR around earnings announcement period within ID deciles. Looking at the second column, the average $\text{CAR}_{-1,1}$ increases from 0.12% to 0.76% when moving from the lowest to the highest ID decile. The difference in CARs is 0.65% with a significant Newey and West [1987] t-statistic of 4.77. As a robustness check, in the third to sixth column, I also examine other variations of reference and earnings announcement periods, and the results are similar. For example, in the last column the average $\text{CAR}_{-5,5}$ increases from -0.51% to 0.22% when moving from the lowest to the highest ID decile, and the return differential is 0.73% with a Newey and West [1987] t-statistic of 7.10.

Overall, the results in Table 2.14 suggest that stocks with high ID prior to the earnings announcement experience significantly higher cumulative abnormal returns in the earnings announcement period.

2.9.3. Regression analysis

Next, I perform a cross-sectional regression analysis that controls for various stock characteristics that may potentially affect the relation between investor disagreement (ID) in the reference period and cumulative abnormal returns (CAR) in the earnings announcement period. I implement Fama and MacBeth [1973] regressions in which the dependent variable is CAR in the earnings announcement period. In particular, I run the following cross-sectional regression every quarter:

$$CAR_{i,q} = \alpha_q + \beta_q ID_{i,q} + \lambda_q X_{i,q} + \epsilon_{i,q}, \tag{2.2}$$

where i refers to the stock, q refers to the calendar quarter, $\text{ID}_{i,q}$ is investor disagreement in the reference period with respect to quarter q for stock i, and $X_{i,q}$ is the set of stock-specific control variables for stock i in quarter q, and $\text{CAR}_{i,q}$ is cumulative abnormal return in the earnings announcement period for firm i in quarter q. Then, I average (weighted) the cross-sectional coefficients across all quarters, where the weights correspond to the number of observations in each quarterly cross-sectional regression. The choice of quarterly frequency is consistent with other papers in the earnings announcement literature (e.g., Garfinkel and Sokobin [2006], Johnson and So [2012], and Akbas [2016]). The coefficient of interest is ID in the reference period. If there indeed exists a positive relation between ID in the reference period and earnings announcement premium, β_q should be significantly positive.

Table 2.15 presents the results. The coefficients on ID are positive and highly significant across five different reference and earnings announcement periods. For example, looking at the second column, when the reference period is $[-45, -2]$ and the earnings announcement period is $[-1, 1]$, the coefficient on ID is 0.337 with a Newey and West [1987] t-statistic of 3.96. In other words, stocks with high ID prior to earnings announcements on average experience significantly higher cumulative abnormal returns around earnings announcements.

The coefficients on control variables are mostly consistent with the literature. The coefficients on RET are significantly negative, which is consistent with the well-known reversal effect. The coefficients on BM are significantly positive, which implies that value stocks in the reference periods tend to perform better around earnings announcements periods (Porta et al. [1997]). The coefficients on IVOL are significantly negative, which is consistent with Ang et al. [2006]. The coefficients of SIZE, however, are positive, while Chari et al. [1988] and Ball and Kothari [1991] suggest that earnings announcement returns are larger for smaller firms.

Consistent with the model's prediction, the results in this section provide evidence in support of a strongly positive relation between ID and expected stock returns, not only in the traditional monthly setting, but also in the earnings announcement setting.

2.10. Evolution of ID: earnings announcements

Next, I compute ID before and after the earnings announcement. ID before the earnings announcement date (day 0) is defined as the correlation coefficient between daily trading volume and absolute price change over the 44-day window $[-45, -2]$, multiplied by -1. ID after the earnings announcement is defined similarly over the 44-day window $[2, 45]$. Then, ΔID is defined as $\Delta \mathrm{ID} = \mathrm{ID}_{\mathrm{after}} - \mathrm{ID}_{\mathrm{before}}$.

Figure 2.3 plots the time-series average of mean values of ID before and after the earnings announcement. First, ID after earnings announcements seems to be higher than ID before earnings announcements, although the difference in magnitude is small. This is consistent with findings in Table 2.11 that ID is highly persistent.

In particular, the mean ΔID is 0.0084 with a significant Newey and West [1987] t-statistic of 4.39. In addition, I compute the mean ΔID for the sample period before and after the implementation of Regulation Fair Disclosure (Reg FD), which prevents firms from doing selective disclosure. Specifically, the pre-Reg-FD and post-Reg-FD mean ΔID are 0.0089 (t-statistic=5.67) and 0.0062 (t-statistic=2.16), respectively. The slight decrease in ID following the implementation of Reg FD could be a result of more transparent and valid firm disclosures. Overall, Figure 2.3 suggests that on average, ID increases after earnings announcements.

2.11. Evolution of ID: firm-specific public news stories

I proceed to examine whether ID also increases following firm-specific public news stories. I first obtain public news data from RavenPack News Analytics on WRDS. I select news events with a relevance score equal to 100, which according to RavenPack, means that the entity plays a key role in the news story and is considered highly relevant.

I further classify news events into six categories; Financial, Legal, M&A, Operational, Ratings, and Others. The news date is defined as the date when the first news story reporting an event about one or more entities is announced. To avoid double counting issue, subsequent news stories reporting the the same news events are not included. ID before and after news events are computed in the same fashion treating the news date as day 0.

Figure 2.4 plots the time-series average of mean values of ID before and after six types of news stories. Again, ID before and after news stories behave very similarly, reassuring the persistence of ID. In particular, the mean ΔID is 0.0177 (t-statistic=5.66) for financial news, 0.0058 (t-statistic=1.78) for legal news, 0.0089 (t-statistic=2.66) for M&A news, 0.0066 (t-statistic=2.09) for operational news, 0.0074 (t-statistic=2.21) for ratings news, and 0.0156 (t-statistic=2.66) for other news. The results indicate that other than earnings announcements, ID also increases after different types of firm-specific public news events.

2.12. Evolution of ID: FOMC meetings

Next, since investor disagreement proxies for security-level behavioral bias (Harvey et al. [2016]), it should not respond to macroeconomic events. If ID significantly increases after macroeconomic events, then it is possible that ID measure in this paper simply captures economic uncertainty instead of investor disagreement.

To mitigate this concern, I examine ID before and after Federal Open Market Committee (FOMC) meetings. There are eight regularly scheduled FOMC meetings each year and meeting minutes are made public following the meetings. Prior work (See, for example, Cieslak et al. [2019], Lucca and Moench [2015], and Bernanke and Kuttner [2005], etc.) study stock market's reaction in the form of realized stock returns to FOMC announcements. In our context, however, the hypothesis is that mean ΔID should be insignificantly different from 0.

I obtain FOMC scheduled meetings from 1994 to 2019, as in the first meeting (February 3-4, 1994) a reasonable portion of the discussion centered on the need to make the committee's intentions clear to the public. I then examine ID before and after FOMC meetings following the same approach.

Figure 2.5 plots the time-series average of mean values of ID before and after FOMC meetings. ID before and after behave virtually the same. In particular, the mean ΔID is 0.0007 with a significant Newey and West [1987] insignificant t-statistic of 0.33, which is consistent with the conjecture that ID proxies for firm-specific investor disagreement.

Together, Figure 2.3, Figure 2.4, and Figure 2.5 show that ID is sensitive to firm-specific information disclosure events but indifferent to macroeconomic news. When firm-specific news bring in a sudden influx of information, investors on average tend to slightly disagree more.

2.13. Conclusion

In this chapter, I test the model prediction of Chapter 1. First, investor disagreement (ID) is computed using the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past two months, multiplied by -1.I find that stocks in the highest ID decile significantly outperform stocks in the lowest ID decile by an annualized risk-adjusted return of 9.24%. Bivariate portfolio-level analyses and stock-level cross-sectional regressions that control for firm size, book-to-market ratio, momentum, short-term reversal, turnover ratio, illiquidity, market beta, co-skewness, demand for lottery stocks with extreme positive returns, idiosyncratic volatility, and analyst forecast dispersion provide further support for the positive relation. In addition, the positive relation between ID and future stock returns persists in high and low sentiment periods, recessions and expansions, and high and low macro, financial, real, and policy-related economic uncertainty periods.

Next, I use AR(1) regression and ID transition matrix to show that ID is persistent. In addition, the positive relation between ID and expected stock returns is also robust to different numbers of months in the formation of ID and portfolio holding periods. Besides using monthly returns to examine the asset pricing implications of ID, I also examine the relation between ID and expected stock returns in the earnings announcement setting.

Using portfolio analyses and stock-level cross-sectional regressions, I find that compared to stocks with low ID, stocks with high ID prior to earnings announcements earn significantly higher cumulative abnormal returns around earnings announcements. In addition, I find that ID increases following earnings announcements, with the size of the increase being bigger following bad news. Moreover, ID increases after firm-specific news stories but remains virtually the same following FOMC scheduled announcements.

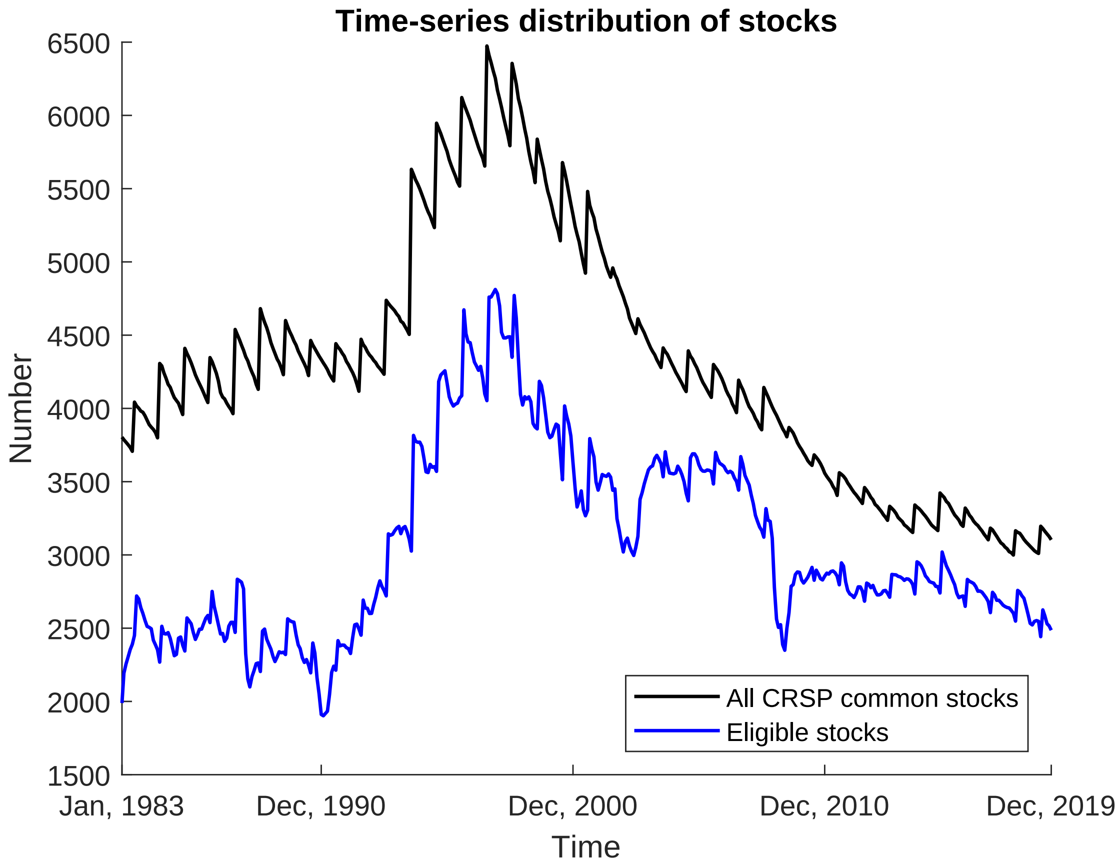


Figure 2.1. **Time-series distribution of stocks** The figure plots the time-series distribution of all CRSP common stocks and eligible stocks. Eligible stocks are stocks with non-missing investor disagreement (ID) at the end of each month. ID at the end of a given month is defined as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past two months (around 44 trading days), multiplied by −1. 3. The sample period is from January 1983 to December 2019 (444 months).

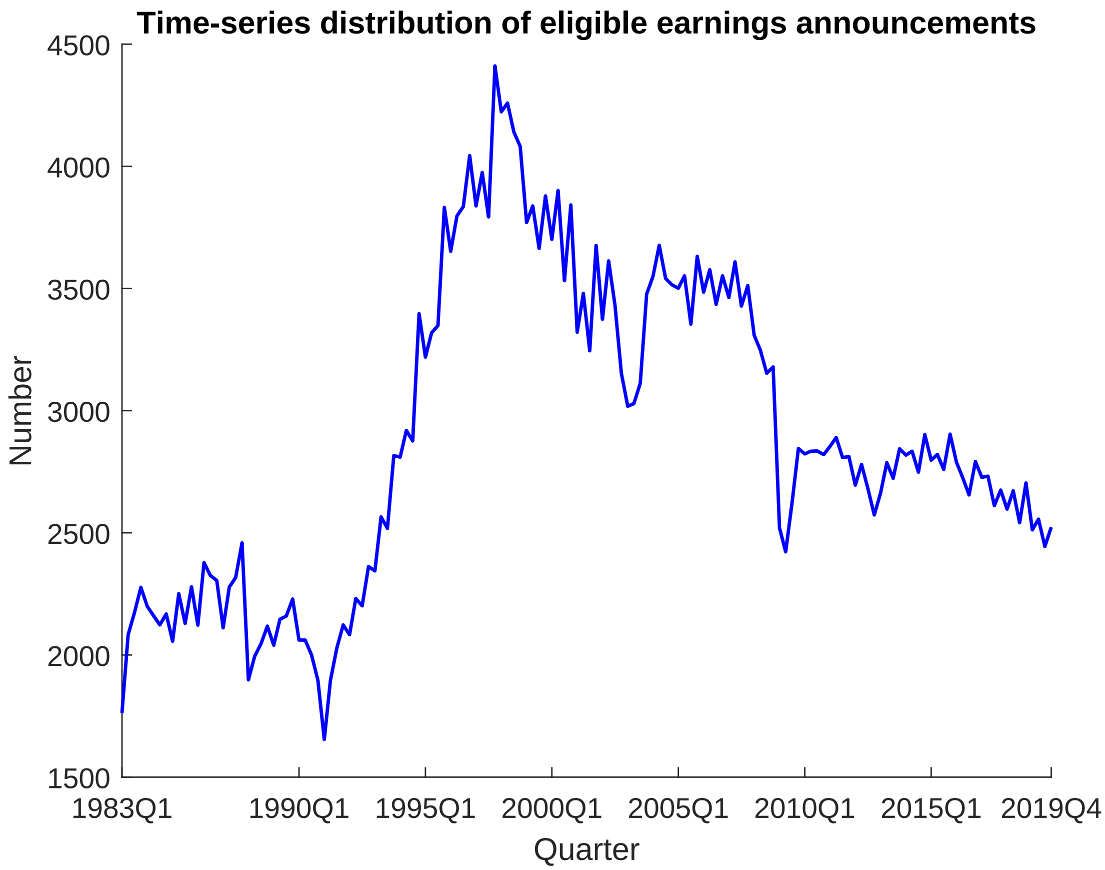


Figure 2.2. **Time-series distribution of eligible earnings announcements.** This figure plots the total number of eligible quarterly earnings announcements over time. It covers all NYSE, NASDAQ, and Amex firms available from the Compustat quarterly file with non-missing earnings. In addition, investor disagreement before the earnings announcement ($\mathrm{ID}_{\mathrm{before}}$) is required to be non-missing. $\mathrm{ID}_{\mathrm{before}}$ is defined as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the window $[-45, -2]$, multiplied by -1, where $t = 0$ is the earnings announcement date. The sample period is from the first quarter of 1983 to the fourth quarter of 2019 (148 quarters).

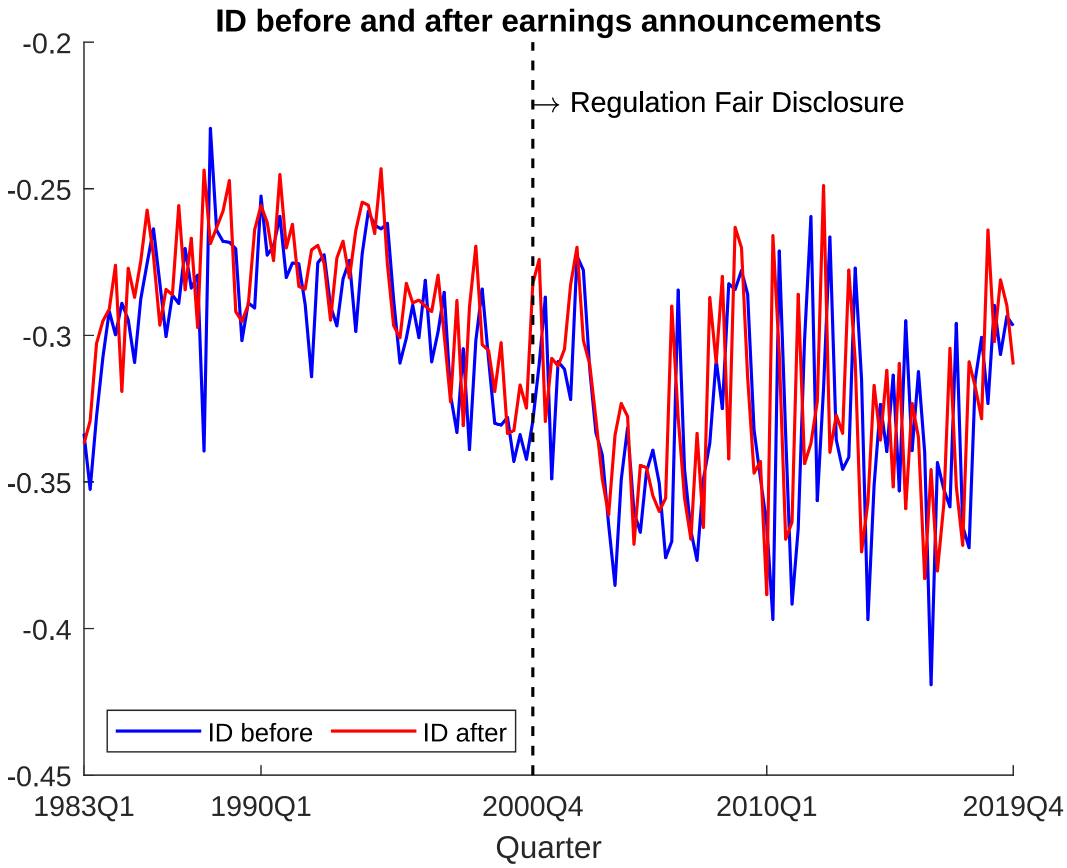
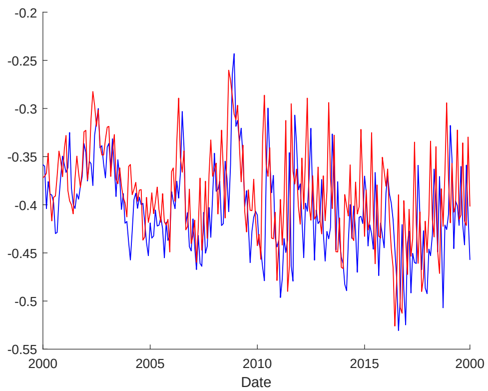
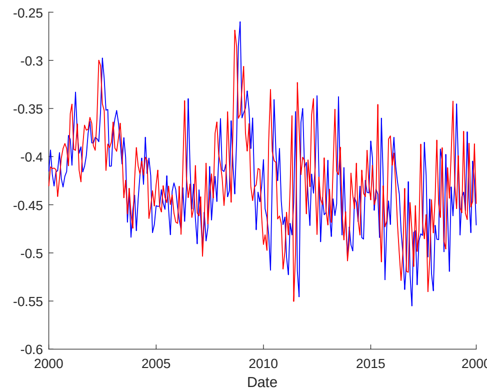


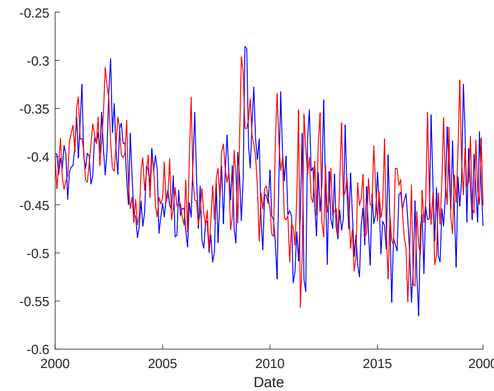
Figure 2.3. **Investor disagreement (ID) before and after earnings announcements.** The figure presents time series of cross-sectional average investor disagreement (ID) before and after the earnings announcement. $\text{ID}_{\text{before}}$ and ID_{after} are defined as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the window $[-45, -2]$ and $[2, 45]$, multiplied by -1, where $t = 0$ is the earnings announcement date. The sample period is from the first quarter of 1983 to the fourth quarter of 2019 (148 quarters). There are 413,454 observations.



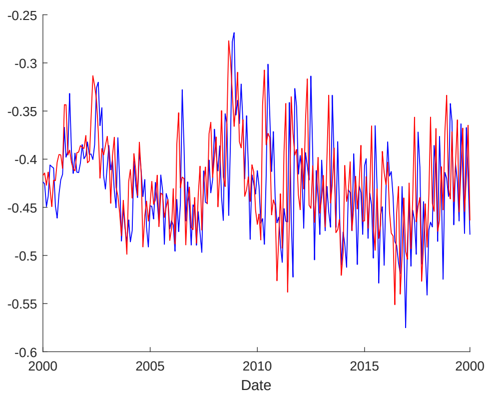
(a) ID before and after financial news



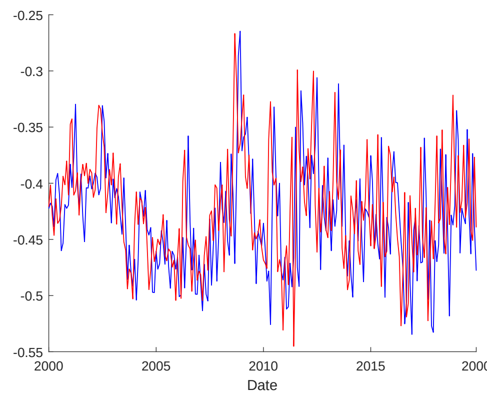
(b) ID before and after legal news



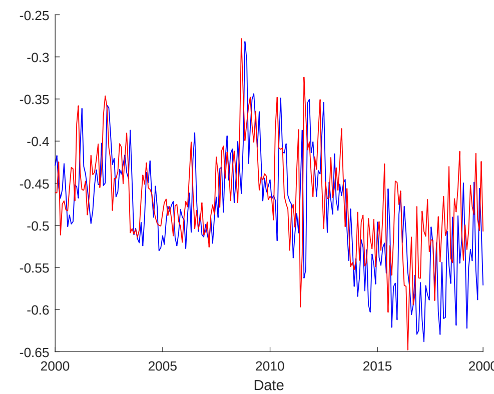
(c) ID before and after M&A news



(d) ID before and after operational news



(e) ID before and after ratings news



(f) ID before and after other news

Figure 2.4. **Investor disagreement (ID) before and after news stories.** The figure presents time series of cross-sectional average investor disagreement (ID) before (blue) and after (red) 6 types (financial, legal, M&A, operational, ratings, and others) of news stories. ID before the news (day 0) is defined as the correlation coefficient between daily trading volume and absolute price change over the 44-day window $[-45, -2]$, multiplied by -1. ID after the earnings announcement is defined similarly over the 44-day window $[2, 45]$. News data is obtained from RavenPack News Analytics on WRDS. The sample period is from January 2000 to December 2019.

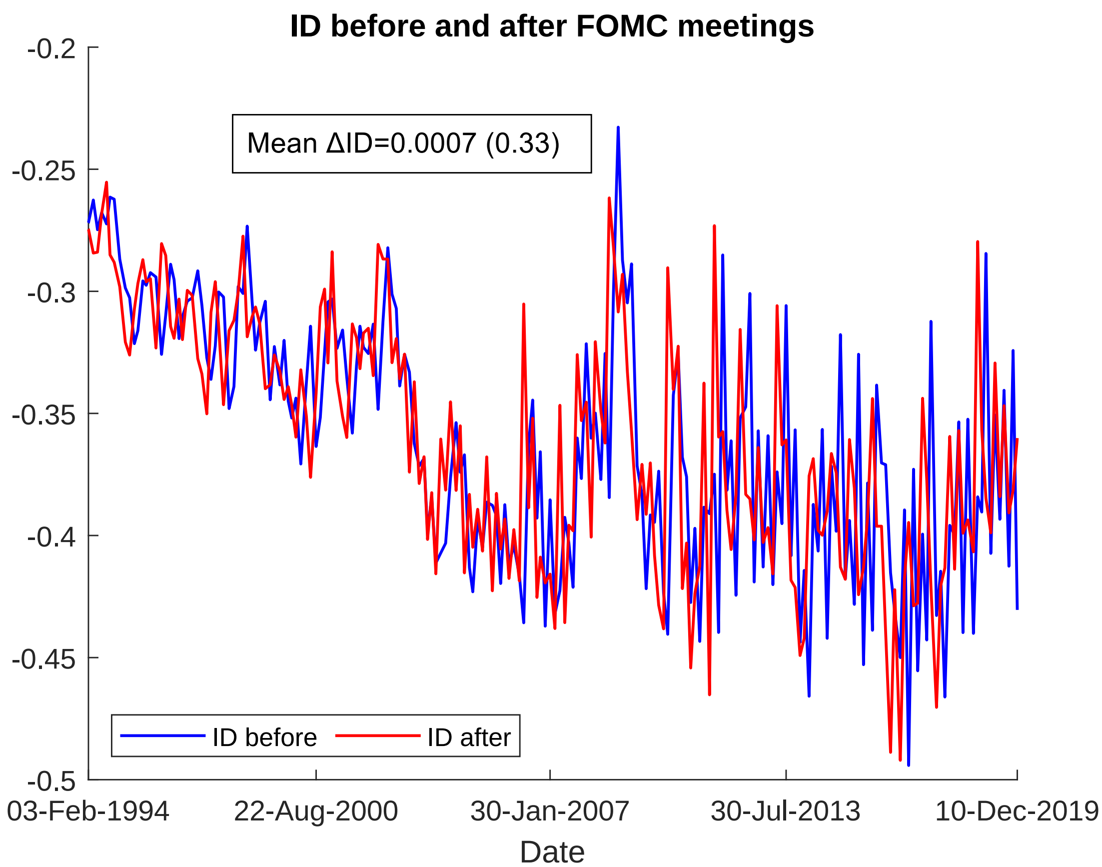


Figure 2.5. **Investor disagreement (ID) before and after FOMC meetings.** The figure presents time series of cross-sectional average investor disagreement before (ID_{before}) and after (ID_{after}) the earnings announcement. ID_{before} and ID_{after} are defined as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the window $[-45, -2]$ and $[2, 45]$, multiplied by -1, where $t = 0$ is the FOMC date. The sample period is from 1994 to 2019 (208 FOMC announcements). There are 665,013 observations.

Table 2.1. **Returns of equal-weighted portfolios sorted on investor disagreement.** For each month, decile portfolios are formed by sorting individual stocks based on their investor disagreement (ID) at the end of previous month. Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) ID at the end of last month. Stocks are held for one month and portfolio returns are equal-weighted. ID at the end of a given month is defined as the correlation coefficient of daily trading volume and absolute price change in the past 2 months, multiplied by -1. The second column reports the time series average of monthly excess returns. The third to fifth column report corresponding alphas with respect to the CAPM, Fama-French three-factor model, and Fama-French-Carhart four-factor model. The sixth column reports the alpha of the five-factor model that in addition includes the liquidity factor of Pástor and Stambaugh [2003]. The row labeled "$10-1$" presents the the differences in monthly excess returns and alphas between decile 10 (High ID) and decile 1 (Low ID). Newey and West [1987] t-statistics are reported in parentheses. The sample period is from January 1983 to December 2019.

ID deciles	Excess return	CAPM alpha	3-factor alpha	4-factor alpha	5-factor alpha
1 (Low)	0.18	-0.63	-0.57	-0.50	-0.51
	(0.60)	(-4.07)	(-6.93)	(-6.23)	(-6.43)
2	0.32	-0.49	-0.44	-0.39	-0.40
	(1.13)	(-3.7)	(-7.06)	(-6.47)	(-6.79)
3	0.39	-0.40	-0.39	-0.31	-0.31
	(1.43)	(-3.69)	(-7.26)	(-5.73)	(-5.91)
4	0.46	-0.33	-0.34	-0.25	-0.25
	(1.67)	(-2.88)	(-5.69)	(-4.78)	(-4.80)
5	0.56	-0.19	-0.21	-0.14	-0.14
	(2.20)	(-1.76)	(-4.10)	(-2.62)	(-2.66)
6	0.57	-0.17	-0.21	-0.10	-0.10
	(2.24)	(-1.49)	(-3.48)	(-1.51)	(-1.55)
7	0.69	-0.01	-0.07	0.04	0.05
	(2.79)	(-0.11)	(-1.07)	(0.56)	(0.67)
8	0.73	0.04	-0.02	0.10	0.10
	(3.04)	(0.36)	(-0.27)	(1.22)	(1.37)
9	0.65	0.00	-0.08	0.03	0.03
	(2.72)	(0.02)	(-0.89)	(0.31)	(0.37)
10 (High)	0.82	0.23	0.14	0.25	0.26
	(3.63)	(1.61)	(1.42)	(2.46)	(2.69)
$10-1$	0.65	0.87	0.71	0.75	0.77
	(3.91)	(5.64)	(5.47)	(5.94)	(6.28)
MKT BETA		-0.31	-0.20	-0.21	-0.21
		(-8.25)	(-5.66)	(-6.05)	(-5.80)
SMB BETA			-0.37	-0.37	-0.37
			(-3.04)	(-3.19)	(-3.18)
HML BETA			0.38	0.36	0.36
			(6.17)	(6.81)	(6.73)
UMD BETA				-0.06	-0.06
				(-1.02)	(-1.00)
LIQ BETA					-0.06
					(-1.69)
Adj. R^2		18.82%	48.30%	48.78%	49.08%

Table 2.2. **Returns of value-weighted portfolios sorted on investor disagreement.** For each month, decile portfolios are formed by sorting individual stocks based on their investor disagreement (ID) at the end of previous month. Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) ID at the end of last month. Stocks are held for one month and portfolio returns are value-weighted. ID at the end of a given month is defined as the correlation coefficient of daily trading volume and absolute price change in the past 2 months, multiplied by -1. The second column reports the time series average of monthly excess returns. The third to fifth column report corresponding alphas with respect to the CAPM, Fama-French three-factor model, and Fama-French-Carhart four-factor model. The sixth column reports the alpha of the five-factor model that in addition includes the liquidity factor of Pástor and Stambaugh [2003]. The row labeled "$10-1$" presents the the differences in monthly excess returns and alphas between decile 10 (High ID) and decile 1 (Low ID). Newey and West [1987] t-statistics are reported in parentheses. The sample period is from January 1983 to December 2019.

ID deciles	Excess return	CAPM alpha	3-factor alpha	4-factor alpha	5-factor alpha
1 (Low)	0.50	-0.23	-0.21	-0.07	-0.07
	(2.06)	(-2.55)	(-2.30)	(-0.71)	(-0.74)
2	0.62	-0.08	-0.08	-0.01	-0.00
	(2.91)	(-1.05)	(-1.11)	(-0.08)	(-0.01)
3	0.54	-0.17	-0.19	-0.10	-0.10
	(2.38)	(-2.37)	(-2.76)	(-1.36)	(-1.41)
4	0.67	-0.04	-0.07	0.00	0.01
	(3.03)	(-0.59)	(-1.02)	(0.07)	(0.14)
5	0.67	-0.05	-0.06	0.03	0.04
	(2.92)	(-0.62)	(-0.98)	(0.50)	(0.54)
6	0.61	-0.09	-0.12	-0.04	-0.05
	(2.88)	(-1.31)	(-1.70)	(-0.56)	(-0.67)
7	0.67	-0.04	-0.09	-0.01	-0.01
	(3)	(-0.56)	(-1.27)	(-0.09)	(-0.08)
8	0.76	0.09	0.04	0.10	0.10
	(3.45)	(1.03)	(0.51)	(1.25)	(1.20)
9	0.71	0.05	-0.03	0.09	0.06
	(3.28)	(0.53)	(-0.29)	(0.88)	(0.70)
10 (High)	0.91	0.29	0.18	0.29	0.29
	(4.51)	(2.31)	(1.72)	(2.65)	(2.71)
$10-1$	0.41	0.52	0.40	0.37	0.36
	(2.78)	(3.56)	(3.12)	(2.67)	(2.68)
MKT BETA		-0.15	-0.09	-0.08	-0.08
		(-4.02)	(-2.76)	(-2.26)	(-2.26)
SMB BETA			-0.02	-0.02	-0.02
			(-0.36)	(-0.36)	(-0.36)
HML BETA			0.36	0.37	0.37
			(6.59)	(6.57)	(6.54)
UMD BETA				0.04	0.04
				(0.83)	(0.83)
LIQ BETA					0.01
					(0.23)
Adj. R^2		5.33%	18.45%	18.59%	18.41%

Table 2.3. **Investor disagreement decile: average stock characteristics.** Stocks are sorted into decile portfolios based on investor disagreement (ID) at the end of each month. Decile 1 (10) is the portfolio of stocks with the lowest (highest) investor disagreement at the end of each month. Investor disagreement (ID) at the end of a given month is defined as the correlation coefficient of daily trading volume and absolute price change in the past 2 months, multiplied by -1. The table presents for each ID decile, the time-series average of mean values of stock characteristics, including firm size (SIZE), book-to-market (BM) ratio, the cumulative return (in percent) over the 11 months prior to the portfolio formation month (MOM), the return (in percent) in the portfolio formation month (REV), average turnover ratio (TURN), idiosyncratic volatility (IVOL) as defined in Ang et al. [2006], Amihud [2002] illiquidity ratio (ILLIQ), lottery demand (MAX) as defined in Bali et al. [2011], institutional owernship ratio (IOR) defined the ratio of shares owned by institutions as reported in 13F filings in the last quarter, the stock beta (BETA), co-skewness (COSKEW) as defined in Harvey and Siddique [2000], and analyst forecast dispersion (DISP) as defined in Diether et al. [2002]. The weights are based on the number of observations in each portfolio in each month, and the variables are defined in detail in Appendix C. Newey and West [1987] t-statistics are reported in parentheses. The sample period is from January 1983 to December 2019 and there is an average of 306 stocks per decile portfolio.

	Investor disagreement (ID) decile portfolio									
	1 (Low)	2	3	4	5	6	7	8	9	10 (High)
ID	-0.77	-0.61	-0.52	-0.44	-0.37	-0.3	-0.23	-0.16	-0.07	0.07
ID (next month)	-0.58	-0.49	-0.44	-0.39	-0.35	-0.32	-0.28	-0.24	-0.19	-0.12
SIZE	5.90	6.10	6.16	6.16	6.11	6.04	5.94	5.82	5.66	5.37
BM	0.62	0.63	0.64	0.65	0.67	0.68	0.70	0.72	0.74	0.77
MOM	24.91	26.48	24.7	22.65	20.46	18.83	17.21	15.93	14.62	12.82
REV	2.57	1.75	1.23	0.89	0.69	0.45	0.26	0.15	0.03	-0.19
TURN	1.00	0.70	0.60	0.53	0.47	0.43	0.39	0.35	0.31	0.26
IVOL	3.30	2.56	2.33	2.18	2.09	2.03	1.98	1.95	1.93	1.96
ILLIQ	0.08	0.12	0.16	0.20	0.27	0.24	0.09	0.35	0.43	0.30
MAX	9.99	7.45	6.65	6.12	5.82	5.56	5.38	5.21	5.09	4.99
IOR	0.49	0.48	0.48	0.47	0.47	0.46	0.45	0.44	0.42	0.39
BETA	1.47	1.42	1.37	1.32	1.27	1.22	1.18	1.13	1.09	1.01
COSKEW	0.10	0.14	0.15	0.07	0.02	-0.05	-0.16	-0.20	-0.37	-0.59
DISP	0.02	0.01	0.02	0.03	0.01	0.02	0.04	0.01	0.02	0.01

Table 2.4. **Mean portfolio returns by firm size (SIZE) and investor disagreement (ID).** Each month, individual stocks are first sorted into quintiles based on firm size (SIZE) in the previous month. Next, within each SIZE decile, stocks are further sorted into deciles based on investor disagreement (ID) in the previous month. Stocks are held for one month, and portfolio returns are equal-weighted. The table reports time series averages of monthly excess returns. SIZE is the log of market capitalization in millions of dollars. Investor disagreement (ID) at the end of a given month is defined as the correlation coefficient of daily trading volume and absolute price change in the past 2 months, multiplied by -1. "$10-1$", "CAPM alpha" , "3-factor alpha", "4-factor alpha", and "5-factor alpha" report the difference in excess returns, CAPM alpha, three-factor alpha (MKT, SMB, and HML), four-factor alpha (MKT, SMB, HML, and UMD), and five-factor alpha (MKT, SMB, HML, UMD, and LIQ) between high ID and low ID decile in each SIZE quintile, respectively. The corresponding Newey and West [1987] t-statistics are reported in parentheses. The sample period is from January 1983 to December 2019.

	Size quintiles				
ID deciles	Small caps	2	3	4	Large caps
1 (Low)	-0.39	0.16	0.07	0.37	0.56
2	-0.14	0.16	0.40	0.42	0.68
3	0.06	0.30	0.33	0.56	0.71
4	0.16	0.33	0.57	0.71	0.6
5	0.25	0.49	0.55	0.75	0.80
6	0.37	0.53	0.73	0.63	0.77
7	0.47	0.34	0.67	0.77	0.64
8	0.49	0.56	0.88	0.85	0.77
9	0.63	0.67	0.76	0.69	0.81
High	0.68	0.77	0.83	0.89	0.84
$10-1$	1.07	0.62	0.77	0.52	0.28
	(3.75)	(2.86)	(3.80)	(2.94)	(2.14)
CAPM alpha	1.32	0.81	0.96	0.69	0.41
	(4.68)	(3.92)	(4.81)	(3.97)	(3.37)
3-factor alpha	1.17	0.65	0.77	0.53	0.30
	(4.19)	(3.18)	(4.55)	(3.81)	(2.67)
4-factor alpha	1.27	0.75	0.79	0.50	0.30
	(4.33)	(3.63)	(4.73)	(3.33)	(2.26)
5-factor alpha	1.32	0.78	0.80	0.49	0.29
	(4.57)	(3.82)	(4.82)	(3.28)	(2.29)

Table 2.5. **Mean portfolio returns by book-to-market (BM) ratio and investor disagreement (ID).** Each month, individual stocks are first sorted into quintiles based on book-to-market (BM) ratio in the previous month. Next, within each BM decile, stocks are further sorted into deciles based on investor disagreement (ID) in the previous month. Stocks are held for one month, and portfolio returns are equal-weighted. The table reports time series averages of monthly excess returns. Investor disagreement (ID) at the end of a given month is defined as the correlation coefficient of daily trading volume and absolute price change in the past 2 months, multiplied by -1. "$10-1$", "CAPM alpha" , "3-factor alpha", "4-factor alpha", and "5-factor alpha" report the difference in excess returns, CAPM alpha, three-factor alpha (MKT, SMB, and HML), four-factor alpha (MKT, SMB, HML, and UMD), and five-factor alpha (MKT, SMB, HML, UMD, and LIQ) between high ID and low ID decile in each BM quintile, respectively. The corresponding Newey and West [1987] t-statistics are reported in parentheses. The sample period is from January 1983 to December 2019.

	Book-to-market quintiles				
ID deciles	Low BM	2	3	4	High BM
Low	-0.40	0.31	0.37	0.37	0.38
2	-0.28	0.49	0.50	0.53	0.59
3	-0.11	0.42	0.57	0.61	0.61
4	0.01	0.32	0.67	0.73	0.65
5	0.02	0.61	0.69	0.69	0.80
6	0.17	0.45	0.69	0.84	0.78
7	0.20	0.58	0.69	0.84	0.84
8	0.30	0.58	0.85	0.82	0.82
9	0.38	0.51	0.81	0.79	0.81
High	0.56	0.72	0.76	0.89	0.99
$10-1$	0.97	0.41	0.39	0.52	0.61
	(5.33)	(2.07)	(2.47)	(3.08)	(3.26)
CAPM alpha	1.07	0.56	0.56	0.72	0.82
	(5.85)	(2.91)	(3.78)	(4.68)	(4.66)
3-factor alpha	0.95	0.43	0.44	0.63	0.76
	(5.41)	(2.23)	(3.24)	(4.20)	(4.26)
4-factor alpha	1.01	0.53	0.47	0.66	0.81
	(5.13)	(2.92)	(3.33)	(4.57)	(4.79)
5-factor alpha	1.00	0.56	0.51	0.66	0.84
	(5.17)	(3.15)	(3.65)	(4.71)	(5.01)

Table 2.6. **Mean portfolio returns by momentum (MOM) and investor disagreement (ID).** Each month, individual stocks are first sorted into quintiles based on momentum (MOM) in the previous month. Next, within each MOM decile, stocks are further sorted into deciles based on investor disagreement (ID) in the previous month. Stocks are held for one month, and portfolio returns are equal-weighted. The table reports time series averages of monthly excess returns. MOM is computed as the cumulative return of a stock of 11 months ending one month prior to the portfolio formation month. Investor disagreement (ID) at the end of a given month is defined as the correlation coefficient of daily trading volume and absolute price change in the past 2 months, multiplied by -1. "$10-1$", "CAPM alpha" , "3-factor alpha", "4-factor alpha", and "5-factor alpha" report the difference in excess returns, CAPM alpha, three-factor alpha (MKT, SMB, and HML), four-factor alpha (MKT, SMB, HML, and UMD), and five-factor alpha (MKT, SMB, HML, UMD, and LIQ) between high ID and low ID decile in each MOM quintile, respectively. The corresponding Newey and West [1987] *t*-statistics are reported in parentheses. The sample period is from January 1983 to December 2019.

	Momentum quintiles				
ID deciles	Losers	2	3	4	Winners
Low	-0.69	0.35	0.55	0.50	0.44
2	-0.66	0.40	0.68	0.78	0.56
3	-0.49	0.52	0.70	0.65	0.68
4	-0.40	0.39	0.58	0.82	0.68
5	-0.29	0.53	0.69	0.81	0.83
6	-0.40	0.63	0.76	0.84	0.91
7	0.12	0.75	0.75	0.86	0.77
8	0.25	0.66	0.87	0.83	0.86
9	0.12	0.77	0.78	0.91	0.73
High	0.68	0.74	0.86	0.83	0.94
$10-1$	1.37	0.39	0.32	0.35	0.50
	(5.74)	(2.69)	(1.89)	(2.22)	(3.00)
CAPM alpha	1.54	0.53	0.48	0.54	0.64
	(6.58)	(3.73)	(3.14)	(3.68)	(3.68)
3-factor alpha	1.40	0.44	0.36	0.40	0.48
	(6.44)	(3.44)	(2.54)	(2.68)	(3.33)
4-factor alpha	1.50	0.46	0.36	0.40	0.48
	(6.37)	(3.68)	(2.73)	(2.78)	(3.02)
5-factor alpha	1.54	0.49	0.38	0.42	0.47
	(6.57)	(3.87)	(2.95)	(2.93)	(2.98)

Table 2.7. **Bivariate portfolio sorts on investor disagreement and control variables.** Double-sorted, equally-weighted decile portfolios are formed every month based on investor disagreement (ID) after controlling for market capitalization (SIZE), book-to-market ratio (BM), momentum (MOM), short-term reversals (REV), turnover (TURN), idiosyncratic volatility (IVOL), illiquidity (ILLIQ), demand for lottery stocks (MAX), institutional ownership ratio (IOR), stock market beta (BETA), co-skewness (COSKEW), and analyst forecast dispersion (DISP). ID at the end of a given month is computed as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past 2 months, multiplied by -1. The other control variables are defined in Appendix C. In each case, I first sort stocks into deciles using the control variable, then within each decile I sort stocks into decile portfolios based on ID. The ten ID portfolios are then averaged over each of the ten control deciles to compute excess returns. "$10-1$" and "5-factor alpha" report the differences in average monthly excess returns and alphas with respect to the five-factor model (MKT, SMB, HML, UMD, and LIQ) between the High ID and Low ID decile portfolios, respectively. The sample period is from January 1983 to December 2019. Newey and West [1987] t-statistics are reported in parentheses.

	Investor disagreement (ID) decile											
	1 (Low)	2	3	4	5	6	7	8	9	10 (High)	$10-1$	5-factor alpha
SIZE	0.16	0.32	0.38	0.47	0.57	0.60	0.64	0.75	0.68	0.80	0.64	0.72
	(0.56)	(1.14)	(1.35)	(1.76)	(2.20)	(2.35)	(2.57)	(3.14)	(2.88)	(3.57)	(4.14)	(6.50)
BM	0.22	0.36	0.45	0.46	0.53	0.61	0.60	0.68	0.65	0.79	0.57	0.71
	(0.77)	(1.30)	(1.68)	(1.76)	(2.04)	(2.42)	(2.42)	(2.86)	(2.68)	(3.45)	(4.19)	(6.32)
MOM	0.26	0.34	0.42	0.41	0.55	0.58	0.69	0.66	0.65	0.81	0.55	0.62
	(0.94)	(1.24)	(1.55)	(1.55)	(2.18)	(2.26)	(2.75)	(2.71)	(2.66)	(3.50)	(4.21)	(6.05)
REV	0.22	0.37	0.48	0.53	0.57	0.58	0.60	0.68	0.62	0.73	0.52	0.60
	(0.78)	(1.40)	(1.77)	(2.00)	(2.16)	(2.25)	(2.38)	(2.79)	(2.54)	(3.16)	(4.51)	(6.57)
TURN	0.20	0.32	0.40	0.50	0.55	0.56	0.62	0.70	0.67	0.84	0.64	0.65
	(0.75)	(1.27)	(1.50)	(1.93)	(2.12)	(2.18)	(2.46)	(2.81)	(2.72)	(3.43)	(6.78)	(6.50)
IVOL	0.40	0.41	0.50	0.53	0.52	0.41	0.60	0.63	0.61	0.75	0.34	0.45
	(1.62)	(1.57)	(1.91)	(1.99)	(1.96)	(1.58)	(2.31)	(2.47)	(2.45)	(3.13)	(3.67)	(5.29)
ILLIQ	0.13	0.31	0.34	0.52	0.56	0.64	0.67	0.67	0.70	0.81	0.68	0.72
	(0.46)	(1.10)	(1.27)	(1.95)	(2.20)	(2.56)	(2.73)	(2.81)	(2.98)	(3.73)	(4.58)	(6.90)
MAX	0.38	0.43	0.49	0.51	0.55	0.51	0.62	0.60	0.57	0.72	0.34	0.45
	(1.45)	(1.62)	(1.89)	(1.96)	(2.12)	(1.98)	(2.38)	(2.45)	(2.27)	(2.96)	(3.41)	(4.65)
IOR	0.24	0.32	0.43	0.51	0.57	0.56	0.66	0.70	0.68	0.79	0.55	0.63
	(0.83)	(1.17)	(1.57)	(1.95)	(2.23)	(2.26)	(2.74)	(2.92)	(2.82)	(3.39)	(3.58)	(5.68)
BETA	0.20	0.30	0.42	0.50	0.52	0.58	0.64	0.72	0.66	0.81	0.61	0.66
	(0.74)	(1.13)	(1.59)	(1.91)	(1.99)	(2.32)	(2.54)	(2.95)	(2.67)	(3.40)	(5.41)	(7.09)
COSKEW	0.23	0.28	0.43	0.47	0.51	0.62	0.61	0.73	0.66	0.83	0.60	0.69
	(0.80)	(1.02)	(1.63)	(1.74)	(1.96)	(2.45)	(2.47)	(3.00)	(2.70)	(3.59)	(4.19)	(6.26)
DISP	0.27	0.34	0.43	0.46	0.59	0.58	0.57	0.62	0.63	0.73	0.45	0.50
	(0.97)	(1.22)	(1.61)	(1.69)	(2.29)	(2.29)	(2.30)	(2.48)	(2.59)	(3.10)	(3.42)	(5.03)

Table 2.8. **Fama-Macbeth Cross-Sectional Regressions.** The table reports the time-series averages of the slope coefficients obtained from regression monthly excess returns on investor disagreement (ID) in the previous month and a set of lagged predictive variables using the Fama-Macbeth (1973) approach. The control variables are the log market capitalization in millions of dollars (SIZE), book-to market (BM) ratio, momentum (MOM), short-term reversal (REV), turnover ratio (TURN), idiosyncratic volatility (IVOL), illiquidity (ILLIQ), demand for lottery stocks (MAX), institutional ownership ratio (IOR), stock beta (BETA), co-skewness (COSKEW), and analyst forecast dispersion (DISP). ID at the end of a given month is computed as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past 2 months, multiplied by -1. The other control variables are defined in Appendix C. The sample period is from January 1983 to December 2019. Newey and West [1987] t-statistics are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
ID	0.780	0.824	0.599	0.367	0.300	0.251
	(3.85)	(4.51)	(4.67)	(3.73)	(3.44)	(2.86)
SIZE		0.095	0.104	0.037	0.032	0.023
		(2.83)	(2.97)	(1.21)	(1.11)	(0.76)
BM		0.208	0.207	0.166	0.111	0.058
		(2.12)	(2.18)	(1.83)	(1.29)	(0.57)
MOM		0.003	0.004	0.003	0.003	0.005
		(1.93)	(2.26)	(2.00)	(2.61)	(3.45)
REV			-0.025	-0.031	-0.035	-0.038
			(-6.13)	(-7.03)	(-8.00)	(7.32)
TURN			-0.241	0.091	0.041	0.075
			(-1.91)	(0.91)	(0.47)	(0.77)
IVOL				-0.361	-0.350	-0.376
				(-6.81)	(-6.88)	(-6.72)
ILLIQ				0.017	0.012	-0.041
				(1.61)	(1.12)	(-0.29)
MAX				0.040	0.041	0.042
				(3.81)	(3.90)	(3.36)
IOR				-0.722	-0.808	-1.297
				(-4.54)	(-5.85)	(-8.79)
BETA					0.050	0.070
					(0.51)	(0.63)
COSKEW					-0.002	0.001
					(-0.44)	(0.16)
DISP						-0.058
						(-3.04)
Intercept	1.084	0.243	0.213	1.237	1.345	1.574
	(4.68)	(0.69)	(0.59)	(4.27)	(5.45)	(4.96)
Adj. R^2	0.33%	2.37%	3.73%	5.09%	6.09%	8.07%
observations	1,355,834	1,355,834	1,355,831	1,293,882	1,178,701	707,856

Table 2.9. **Investor disagreement premium: business cycles and investor sentiment.** Stocks are sorted into decile portfolios based on investor disagreement (ID) at the end of each month. Decile 1 (10) is the portfolio of stocks with the lowest (highest) investor disagreement at the end of each month. The table reports alphas with respect to the five-factor model (MKT, SMB, HML, UMD, and LIQ) in different sample periods. NBER expansion and recession periods are set by the NBER's Business Cycle Dating Committee. A high-sentiment month (low-sentiment) month is one in which the value of the BW (Baker and Wurgler [2006]) sentiment index in the previous month is above (below) the median value for the sample period. The row labeled "10 − 1" reports the five-factor alphas of the long-short ID strategy. The sample period for business cycles is January 1983 to December 2019, and the sampe period for investor sentiment is from January 1983 to December 2018. Newey and West [1987] t-statistics are reported in parentheses.

ID decile	NBER Expansions	NBER Recessions	Low Sentiment	High Sentiment
1 (Low)	-0.49	-0.55	-0.39	-0.65
	(-6.00)	(-1.52)	(-3.78)	(-5.38)
2	-0.39	-0.48	-0.33	-0.46
	(-6.27)	(-2.19)	(-4.27)	(-5.22)
3	-0.32	-0.17	-0.19	-0.42
	(-5.91)	(-0.60)	(-2.56)	(-5.23)
4	-0.24	-0.03	-0.10	-0.35
	(-4.54)	(-0.14)	(-1.58)	(-4.10)
5	-0.14	-0.12	-0.07	-0.18
	(-2.51)	(-0.77)	(-1.01)	(-2.15)
6	-0.09	-0.05	-0.05	-0.10
	(-1.45)	(-0.18)	(-0.17)	(-0.92)
7	0.05	0.14	0.02	0.17
	(0.67)	(0.53)	(0.22)	(1.56)
8	0.09	0.50	0.07	0.18
	(1.17)	(1.73)	(1.00)	(1.41)
9	0.06	0.04	-0.03	0.16
	(0.62)	(0.10)	(-0.39)	(1.08)
10 (High)	0.22	0.69	0.27	0.27
	(2.09)	(1.37)	(3.33)	(1.54)
10 − 1	0.71	1.23	0.65	0.92
	(5.38)	(2.04)	(5.25)	(4.53)
# of months	410	34	215	217

Table 2.10. **Investor disagreement premium: economic uncertainty.** Stocks are sorted into decile portfolios based on investor disagreement (ID) at the end of each month. Decile 1 (10) is the portfolio of stocks with the lowest (highest) investor disagreement at the end of each month. The table reports alphas with respect to the five-factor model (MKT, SMB, HML, UMD, and LIQ) over high and low economic uncertainty periods. The row labeled "$10-1$" reports the five-factor alphas of the long-short ID strategy. A high-sentiment month is one in which the value of the economic uncertainty index is above the median value for the sample period, and the low-sentiment months are those with below-median values. Macro, financial, real economic uncertainty measures are defined in Jurado et al. [2015] and Ludvigson et al. [2015]. Policy-related economic uncertainty is defined in Baker et al. [2016]. The sample period for macro, financial, and real economic uncertainty is from January 1983 to December 2019, and the sample period for policy-related economic uncertainty index is from January 1985 to December 2019. Newey and West [1987] t-statistics are reported in parentheses.

ID decile	Low Macro_UNC	High Macro_UNC	Low Fin_UNC	High Fin_UNC	Low Real_UNC	High Real_UNC	Low Policy_UNC	High Policy_UNC
1 (Low)	-0.58	-0.44	-0.59	-0.46	-0.50	-0.52	-0.59	-0.47
	(-5.64)	(-3.91)	(-6.71)	(-3.74)	(-4.71)	(-4.44)	(-4.80)	(-4.61)
2	-0.47	-0.34	-0.43	-0.36	-0.47	-0.33	-0.35	-0.40
	(-6.20)	(-4.28)	(-5.43)	(-4.16)	(-6.48)	(-4.05)	(-3.67)	(-5.28)
3	-0.34	-0.27	-0.37	-0.27	-0.38	-0.23	-0.34	-0.23
	(-4.48)	(-3.75)	(-5.35)	(-3.33)	(-5.18)	(-3.21)	(-4.2)	(-2.92)
4	-0.30	-0.15	-0.23	-0.22	-0.21	-0.21	-0.21	-0.18
	(-4.15)	(-1.96)	(-3.29)	(-2.84)	(-2.88)	(-2.95)	(-2.83)	(-2.39)
5	-0.14	-0.10	-0.15	-0.14	-0.19	-0.05	-0.07	-0.11
	(-2.36)	(-1.10)	(-2.07)	(-1.58)	(-2.81)	(-0.59)	(-0.96)	(-1.76)
6	-0.11	-0.07	-0.15	-0.08	-0.17	0.02	-0.08	-0.03
	(-1.66)	(-0.69)	(-2.19)	(-0.81)	(-2.76)	(0.20)	(-0.91)	(-0.42)
7	-0.02	0.09	0.05	0.04	0.05	0.08	0.12	0.05
	(-0.24)	(0.92)	(0.85)	(0.34)	(0.70)	(0.84)	(1.52)	(0.52)
8	0.13	0.06	0.03	0.12	0.14	0.09	0.16	0.11
	(-2.33)	(0.55)	(0.47)	(1.03)	(2.36)	(0.83)	(2.00)	(1.04)
9	0.07	0.02	0.06	0.06	0.02	0.10	0.15	0.04
	(1.08)	(0.11)	(0.87)	(0.42)	(0.22)	(0.82)	(1.73)	(0.32)
10 (High)	0.23	0.27	0.15	0.37	0.22	0.31	0.33	0.32
	(3.19)	(1.85)	(2.17)	(2.40)	(2.75)	(2.23)	(3.20)	(2.26)
$10-1$	0.81	0.72	0.74	0.83	0.72	0.83	0.92	0.79
	(6.65)	(3.95)	(6.68)	(4.21)	(5.53)	(4.84)	(5.30)	(5.43)
# of months	222	222	222	222	222	222	210	211

Table 2.11. **Persistence of investor disagreement.** The table examines the persistence of investor disagreement (ID). Panel A reports coefficients of regressing firm-level ID on lagged ID and lagged cross-sectional variables, including firm size (SIZE), book-to market (BM) ratio, momentum (MOM), short-term reversal (REV), turnover ratio (TURN), idiosyncratic volatility (IVOL), illiquidity (ILLIQ), demand for lottery stocks (MAX), institutional ownership ratio (IOR), stock beta (BETA), and co-skewness (COSKEW). Panel B presents the average month-to-month investor disagreement (ID) decile transition matrix. Column (i, j) is the average probability (in percentage) that a stock in ID decile i in one month will be in decile j in the subsequent month. Newey and West [1987] t-statistics are reported in parentheses. The sample period is from January 1983 to December 2019.

Panel A: Predictive regression	
Univariate predictive regression	0.552 (185.20)
Adj. R^2	30.58%
Controlling for lagged variables	0.468 (104.08)
Adj. R^2	34.54%

Panel B: Transition matrix (in %)

	ID deciles in next month									
ID deciles	Low	2	3	4	5	6	7	8	9	High
Low	42.87	18.26	9.99	7.14	5.64	4.48	3.80	3.23	2.57	2.02
2	17.15	24.20	17.38	11.72	8.47	6.47	5.07	4.14	3.10	2.31
3	9.76	16.82	18.61	15.30	11.65	8.86	6.81	5.28	4.10	2.80
4	7.10	11.16	15.07	16.24	14.01	11.48	9.07	6.94	5.28	3.67
5	5.68	8.09	11.39	11.93	14.94	13.83	11.27	9.06	7.09	4.71
6	4.77	6.40	8.62	11.16	13.49	14.76	13.85	11.69	9.13	6.12
7	4.09	5.19	6.71	8.66	11.35	13.66	15.27	14.45	12.13	8.50
8	3.50	4.22	5.21	6.91	8.87	11.41	14.50	16.85	16.27	12.26
9	2.81	3.29	4.12	5.30	6.91	8.99	11.97	16.18	20.96	19.46
High	2.17	2.30	2.89	3.59	4.64	6.17	8.44	12.25	19.48	38.06

Table 2.12. **Investor disagreement premium: formation periods.** At the end of each month, stocks are sorted into deciles based on investor disagreement (ID) and assigned into portfolios. Stocks are then held in the portfolio for the subsequent month. ID at the end of a given month is computed as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past T months, multiplied by -1. Portfolio returns are equal-weighted. The table presents the difference in excess returns, CAPM alpha, three-factor alpha (MKT, SMB, and HML), four-factor alpha (MKT, SMB, HML, and UMD), and five-factor alpha (MKT, SMB, HML, UMD, and LIQ) between the highest and the lowest ID decile. The sample period is from January 1983 to December 2019. Newey and West [1987] t-statistics are reported in parentheses.

Formation period (in months)	Excess return	CAPM alpha	3-factor alpha	4-factor alpha	5-factor alpha
3	0.58	0.82	0.66	0.70	0.72
	(3.44)	(5.29)	(5.10)	(5.57)	(5.85)
4	0.50	0.77	0.59	0.59	0.61
	(2.91)	(4.67)	(4.45)	(4.81)	(5.01)
5	0.56	0.84	0.66	0.65	0.67
	(3.17)	(5.15)	(5.07)	(5.35)	(5.58)
6	0.50	0.80	0.61	0.58	0.60
	(2.73)	(4.60)	(4.42)	(4.52)	(4.73)
7	0.47	0.78	0.57	0.54	0.57
	(2.42)	(4.19)	(3.90)	(4.00)	(4.28)
8	0.50	0.82	0.61	0.58	0.61
	(2.56)	(4.41)	(4.30)	(4.37)	(4.70)
9	0.52	0.85	0.62	0.60	0.63
	(2.57)	(4.42)	(4.35)	(4.46)	(4.81)
10	0.50	0.83	0.60	0.54	0.57
	(2.50)	(4.41)	(4.26)	(3.81)	(4.21)
11	0.49	0.84	0.60	0.53	0.56
	(2.41)	(4.40)	(4.11)	(3.43)	(3.89)
12	0.46	0.81	0.57	0.48	0.52
	(2.20)	(4.21)	(3.86)	(3.19)	(3.59)

Table 2.13. **Investor disagreement premium: holding periods.** At the end of each month, stocks are sorted into deciles based on investor disagreement (ID) and assigned into portfolios. Stocks are then held in the portfolio for T months, with $\frac{1}{T}$ of each portfolio reinvested monthly. ID at the end of a given month is computed as the contemporaneous correlation coefficient of daily trading volume and absolute price change over the past 2 months, multiplied by -1. Portfolio returns are equal-weighted. The table presents the difference in excess returns, CAPM alpha, three-factor alpha (MKT, SMB, and HML), four-factor alpha (MKT, SMB, HML, and UMD), and five-factor alpha (MKT, SMB, HML, UMD, and LIQ) between the highest and the lowest ID decile. The sample period is from January 1983 to December 2019. Newey and West [1987] t-statistics are reported in parentheses.

Holding period (in months)	Excess return	CAPM alpha	3-factor alpha	4-factor alpha	5-factor alpha
2	0.57	0.80	0.65	0.66	0.68
	(3.85)	(5.82)	(5.80)	(6.20)	(6.57)
3	0.48	0.71	0.57	0.55	0.58
	(3.42)	(5.49)	(5.64)	(5.62)	(6.05)
4	0.45	0.69	0.55	0.52	0.55
	(3.33)	(5.57)	(5.76)	(5.4)	(5.85)
5	0.44	0.68	0.54	0.51	0.53
	(3.27)	(5.57)	(5.69)	(5.13)	(5.57)
6	0.49	0.65	0.51	0.47	0.49
	(5.21)	(5.34)	(5.4)	(4.80)	(5.21)
7	0.38	0.63	0.48	0.44	0.46
	(2.83)	(5.07)	(5.11)	(4.51)	(4.91)
8	0.37	0.61	0.47	0.42	0.43
	(2.70)	(4.92)	(4.96)	(4.25)	(4.64)
9	0.34	0.59	0.44	0.38	0.40
	(2.50)	(4.66)	(4.65)	(3.80)	(4.18)
10	0.34	0.58	0.43	0.37	0.39
	(2.46)	(4.60)	(4.59)	(3.61)	(3.99)
11	0.33	0.58	0.44	0.36	0.38
	(2.42)	(4.57)	(4.55)	(3.47)	(3.84)
12	0.33	0.58	0.43	0.35	0.37
	(2.4)	(4.55)	(4.52)	(3.32)	(3.68)

Table 2.14. **Average cumulative abnormal return (CAR) around earnings announcement by investor disagreement (ID).** The table presents time-series average of quarterly mean values of cumulative market-adjusted returns (CAR) within investor disagreement (ID) deciles. The weights are based on the number of observations in each portfolio in each calendar quarter. CAR_{-t_1,t_1} is defined as the compounded return over the $[-t_1, t_1]$ window around the earnings announcement date ($t = 0$) in excess of the compounded value-weighted market return (in percent). The corresponding reference period is defined as the 44-day $[t_1 - 44, t_1 - 1]$ window prior to the earnings announcement date. In each calendar quarter, stocks are sorted into deciles by ID, which is defined as the contemporaneous correlation coefficient of volume and absolute price change in the reference period, multiplied by -1. The row labeled "$10 - 1$" reports the difference in CAR between decile 10 (High ID) and decile 1 (Low ID). The sample period is from the first quarter of 1983 to the fourth quarter of 2019 (444 quarters). Newey and West [1987] t-statistics are reported in parentheses.

ID deciles	$CAR_{-1,1}$ (1)	$CAR_{-2,2}$ (2)	$CAR_{-3,3}$ (3)	$CAR_{-4,4}$ (4)	$CAR_{-5,5}$ (5)
1 (Low)	0.12	-0.54	-0.57	-0.51	-0.51
	(0.45)	(-3.99)	(-8.02)	(-7.12)	(-7.07)
2	0.27	-0.39	-0.44	-0.39	-0.40
	(1.06)	(-3.36)	(-8.24)	(-7.49)	(-7.62)
3	0.37	-0.30	-0.37	-0.29	-0.29
	(1.43)	(-2.87)	(-7.59)	(-6.03)	(-6.13)
4	0.42	-0.23	-0.31	-0.23	-0.22
	(1.65)	(-2.16)	(-6.05)	(-4.87)	(-4.72)
5	0.53	-0.10	-0.19	-0.13	-0.12
	(2.22)	(-1.03)	(-4.06)	(-2.58)	(-2.46)
6	0.52	-0.10	-0.21	-0.11	-0.11
	(2.19)	(-1.01)	(-4.09)	(-2.07)	(-2.01)
7	0.66	0.06	-0.07	0.04	0.05
	(2.84)	(0.62)	(-1.20)	(0.66)	(0.78)
8	0.66	0.08	-0.06	0.07	0.07
	(2.95)	(0.82)	(-0.93)	(1.04)	(1.13)
9	0.69	0.13	-0.02	0.10	0.10
	(3.09)	(1.20)	(-0.25)	(1.29)	(1.41)
10 (High)	0.76	0.25	0.09	0.21	0.22
	(3.56)	(2.15)	(1.05)	(2.53)	(2.72)
$10 - 1$	0.65	0.79	0.65	0.72	0.73
	(4.77)	(6.18)	(5.90)	(6.88)	(7.10)

Table 2.15. **Investor disagreement and earnings announcement returns.** The table presents results of quarterly weighted Fama and MacBeth (1973) regressions using cumulative abnormal returns around the earnings announcement date, CAR_{-t_1,t_1}, as the dependent variable. The weights correspond to the number of observations used in each quarterly cross-sectional regression. CAR_{-t_1,t_1} is defined as the compounded return over the $[-t_1, t_1]$ window around the earnings announcement date ($t = 0$) in excess of the compounded value-weighted market return (in percent). The corresponding reference period defined as the 44-day $[t_1 - 44, t_1 - 1]$ window prior to the earnings announcement date. ID (investor disagreement) is defined as the contemporaneous correlation coefficient of volume and absolute price change in the reference period, multiplied by -1. SIZE is the log of market capitalization in millions of dollars and BM is book-to-market ratio. RET is the return (in percent) compounded over the reference period. TURN and IVOL are the average turnover ratio and idiosyncratic volatility in the reference period, respectively. IOR is the ratio of shares owned by institutions as reported in 13F filings in the last quarter. NUMEST is the number of unique analysts that have eligible fiscal year one earnings estimates on IBES in the reference period. The sample period is from the first quarter of 1983 to the fourth quarter of 2019. Newey and West [1987] t-statistics are reported in parentheses.

	$CAR_{-1,1}$ (1)	$CAR_{-2,2}$ (2)	$CAR_{-3,3}$ (3)	$CAR_{-4,4}$ (4)	$CAR_{-5,5}$ (5)
ID	0.337	0.379	0.431	0.493	0.539
	(3.96)	(4.33)	(4.18)	(4.35)	(4.58)
SIZE	0.031	0.04	0.058	0.061	0.064
	(1.52)	(1.83)	(2.35)	(2.28)	(2.18)
BM	0.189	0.198	0.220	0.198	0.189
	(4.46)	(3.78)	(3.78)	(2.80)	(2.13)
RET	-0.013	-0.015	-0.017	-0.019	-0.022
	(-7.84)	(-7.21)	(-6.68)	(-6.64)	(-6.66)
TURN	-9.405	-7.964	-6.324	-5.409	1.008
	(-1.75)	(-1.20)	(-0.78)	(-0.56)	(0.09)
IVOL	-0.121	-0.158	-0.175	-0.186	-0.212
	(-4.44)	(-5.16)	(-4.77)	(-4.43)	(-4.79)
IOR	-0.650	-0.710	-0.779	-0.827	-0.904
	(-6.57)	(-6.48)	(-5.99)	(-5.79)	(-5.84)
NUMEST	0.011	0.012	0.012	0.015	0.016
	(2.68)	(2.55)	(2.29)	(2.6)	(2.59)
Intercept	0.331	0.352	0.274	0.279	0.323
	(2.52)	(2.55)	(1.54)	(1.31)	(1.39)
Adj. R^2	0.87%	0.98%	1.34%	1.64%	1.93%
# of observations	428,194	428,003	427,892	427,764	427,648

CHAPTER 3

Investor Disagreement and Volume-Return Relation

This chapter builds a similar multi-asset disagreement model with ambiguity with a view to further speaking to the joint behavior of stock prices and trading volume. Without some source of investor disagreement involved, it would be very difficult to explain why trading volume exists. Kim and Verrecchia [1991] and Kim and Verrecchia [1991] propose that trading volume is proportional to absolute price change when there is dispersion of risk tolerance coefficients and prior precision among investors. In addition, Shalen [1993] and Harris and Raviv [1993] indicate that dispersion in beliefs can be a factor contributing to the positive correlation between volume and absolute price changes. Kandel and Pearson [1995], on the other hand, provide evidence of high trading volume accompanied by close to zero price change around some earnings announcements. They set up a model where traders have differences of opinion about the meaning of the announcements. Scheinkman and Xiong [2003] and Hong et al. [2006] build dynamic models of investors continually updating their valuations based on their personal interpretations of incoming signals and argue that in the presence of short-sale constraints, a positive correlation exists between trading volume and the degree of overpricing. Banerjee and Kremer [2010] develop a dynamic model in which investors disagree about the interpretation of public information and show that when investors have infrequent but major disagreements, there is positive autocorrelation in volume and positive correlation between volume and volatility.

In this chapter, this assumption is that both types of investors simply form a convex combination of the two interpretations. That is, with a weight parameter $\alpha \in [0, 1]$, traders form a revised interpretation of the stock by assigning α weight to their own interpretation and $(1-\alpha)$ weight to the other type's interpretation. When $\alpha > \frac{1}{2}$ $(\alpha < \frac{1}{2})$, traders believe that they are superior (inferior) to others in terms of processing signals and hence assign disproportionately higher (lower) weight to their own interpretation.

Ambiguity in the paper means that α can take in a set of values, i.e., $\alpha \in [\underline{\alpha}, \overline{\alpha}] \subseteq [0, 1]$.[1] As a result, instead of forming a single revised interpretation, traders investors have in mind a set of revised interpretations. In reality, as investors typically have incomplete knowledge about other investors, it seems plausible that they experience ambiguity when thinking about interpretations belong to others. Furthermore, different values of $\underline{\alpha}$ and $\overline{\alpha}$ allow us to study investors' different ways of processing others' interpretations. In particular, investors can be unbiased, slightly overconfident, slightly underconfident, overconfident, and underconfident.

First, unbiased investors are impartial when picking the set of weight parameters, i.e., $\underline{\alpha} + \overline{\alpha} = 1$, $0 < \underline{\alpha} < 0.5 < \overline{\alpha} < 1$. Second, slightly overconfident investors ($\underline{\alpha} + \overline{\alpha} > 1$, $0 < \underline{\alpha} < 0.5 < \overline{\alpha} \leq 1$) believe that most of the time they are superior in processing

[1]Most ambiguity models such as Epstein and Schneider [2007], Epstein and Schneider [2008], Easley and O'Hara [2010], and Illeditsch [2011] assume that news or signals are of uncertainty quality. In this model, ambiguity lies in others' interpretations of the signal rather than the signal itself.

the signal but others may have a upper hand from time to time. Slightly underconfident investors ($\underline{\alpha}+\overline{\alpha} < 1$, $0 \leq \underline{\alpha} < 0.5 < \overline{\alpha} < 1$) are the exact opposite of slightly overconfident investors. Third, overconfident investors ($\underline{\alpha}+\overline{\alpha} > 1$, $0.5 \leq \underline{\alpha} < \overline{\alpha} \leq 1$), always place themselves above others in terms of processing the signal. That is, they never assign a weight below $\frac{1}{2}$ to their own interpretation. Finally, underconfident investors ($\underline{\alpha}+\overline{\alpha} < 1$, $0 \leq \underline{\alpha} < \overline{\alpha} \leq 0.5$) are the exact opposite of overconfident investors.

As before, the essential behavioral assumption is that investors are ambiguity-averse. There are many interesting properties in equilibrium. First, when investors are unbiased, both optimistic and pessimistic traders go long in the asset, price is decreasing in investor disagreement, and trading volume is zero. When investors are slightly overconfident, optimistic ones always go long in the asset while pessimistic ones go long in the asset when disagreement is low but step out of the market when disagreement is high. Price is always decreasing in disagreement, and trading volume is increasing in investor disagreement when both types of investors participate in the market but becomes fixed when only optimistic ones are present.[2]

Next, when investors are overconfident, optimistic ones always go long in the asset, while pessimistic ones go long in the asset at low disagreement, step out the market at medium disagreement, and go short in the asset at high disagreement. Price is decreasing in disagreement when both types of investors go long, and increasing in disagreement when only optimistic ones are present. When pessimistic ones go short in the asset,

[2]When investors are slightly underconfident, everything is reversed. That is, pessimistic investors always go long in the asset while optimistic ones can either go long or step out of the market.

disagreement is unrelated to price. Volume, on the other hand, is increasing in investor disagreement when both types of investors participate in the market and is independent of disagreement when only optimistic ones are present.[3]

The disagreement model along with investors being ambiguity-averse provides a good framework to explain the joint behavior of stock returns and trading volume. First, past literature has documented a positive contemporaneous relation of volume and stock returns.[4] That is, stock prices rise on high volume but decline on low volume. In addition, Kandel and Pearson [1995] document that high trading volume can coexists with small price changes around earnings announcements. The above empirical evidence can be explained by my model when investors are either overconfident or underconfident.

Second, trading volume tends to rise around earning earnings announcements.[5] Beaver [1968], Bamber [1987], Ajinkya et al. [1991], and Garfinkel [2009] all argue that high trading volume is somehow related to high divergence of opinion around earnings announcements. In addition, Frazzini and Lamont [2007] and Savor and Wilson [2016] both find that firms scheduled to report earnings experience positive abnormal returns. Again, my model can explain the above empirical evidence under the assumption that disagreement is higher around announcement periods.

[3]When investors are underconfident, everything is reversed. That is, optimistic investors always go long in the asset while pessimistic ones can go long, go short, or step out of the market.

[4]See for example, Clark [1973], Tauchen and Pitts [1983], Karpoff [1987], Gallant et al. [1992], and Andersen [1996].

[5]See Bamber et al. [2010] for detailed literature.

An advantage of the model is that it directly characterizes the position (long, short, or non-participating) investors take under different levels of disagreement. This is useful when we discuss the role of short-sale constraints in asset pricing. For example, Mayshar [1983] and Miller [1977] argue that when there are constraints on short selling, pessimists cannot freely trade on their negative information. Hence, disagreement leads to overpricing in the presence of short-sale constraints. However, my model predicts that overpricing occurs when optimistic traders go long in the market and pessimistic investors step out of the market. On the other hand, when optimistic traders go long and pessimistic ones go short, price is independent of disagreement. As a result, short selling restriction is independent of overpricing.

3.1. The model

This section develops the model. In the market there are $I+1$ assets: a risk-free asset, money, which has a constant price of one, and I risky assets, denoted by $i = 1, ..., I$. There are N investors and three dates.

At date 0, each investor begins with the same endowments of money and risky assets, $(\bar{m}, \bar{x}^1, ..., \bar{x}^I)$, and has identical prior beliefs for each risky asset.[6] All investors believe that the value of each risky asset, V^i, follows a normal distribution of mean v^i and variance σ^i, and that V^i's are independent. Each investor has CARA utility of his wealth, w, with

[6]In fact, under the CARA-normal structure, investors end up having the same holdings of money and risky assets even if they begin with different endowments but are allowed to trade at date 0.

risk aversion parameter set equal to one:

$$u(w) = -exp(-w). \tag{3.1}$$

Investors' budget constraints are given by

$$w = \bar{m} + \sum_i p^i \bar{x}^i, \tag{3.2}$$

where p^i is the price of risky asset i.

At date 1, there is a public signal for each risky asset and all signals are independent and observed by investors. Let S^i denote the public signal of risky asset i, where $S^i = V^i + \epsilon^i$, $\epsilon^i \perp V^i$, and $\epsilon^i \sim N(\mu^i, \phi^i)$. Investors know everything about S^i except for μ^i. In particular, half of the investors (type A) believe that $\mu^i = \mu^i_A$, while the other half (type B) believe that $\mu^i = \mu^i_B$. In addition, type A investors observe μ^i_B and type B investors observe μ^i_A.

I refer to μ^i_A and μ^i_B as type A's and type B's interpretation of S^i. Disagreement for risky asset i is defined as the absolute difference between two types of interpretations, $|\mu^i_A - \mu^i_B|$. If $\mu^i_A = \mu^i_B$, then there is no investor disagreement for risky asset i. If $\mu^i_B > \mu^i_A$, then type A investors are considered optimistic and type B investors are considered pessimistic for risky asset i, and vice versa.[7]

[7] "Pessimistic" and "optimistic" traders are in a relative sense. For example, when $0 > \mu^i_B > \mu^i_A$, both types of investors think that V_i is higher than S_i. However, type A investors are relatively more optimistic than type B investors.

In the next section, I introduce how investors update their estimates of μ^i after observing the other type's interpretation. At date 2, V^i's are realized and investors consume their wealth.

3.2. Updating under ambiguity

At date 1, after observing the other type's interpretation, I assume that investors' updated estimates of μ^i are weighted arithmetic means of μ^i_A and μ^i_B with non-negative weights $\alpha \in [\underline{\alpha}, \overline{\alpha}] \subseteq [0,1]$. That is, investors' updated estimates of μ^i are given by

$$\begin{cases} \{\widehat{\mu^i_A} \mid \widehat{\mu^i_A} = \alpha\mu^i_A + (1-\alpha)\mu^i_B, \alpha \in [\underline{\alpha}, \overline{\alpha}]\}, & \text{if type A investors} \\ \{\widehat{\mu^i_B} \mid \widehat{\mu^i_B} = \alpha\mu^i_B + (1-\alpha)\mu^i_A, \alpha \in [\underline{\alpha}, \overline{\alpha}]\}, & \text{if type B investors} \end{cases} \tag{3.3}$$

First, the set of weights $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ indicates that investors experience ambiguity when taking account of the other type's interpretation, which results in a set of updated estimates of μ^i. If, however, $\underline{\alpha} = \overline{\alpha}$, then investors experience no uncertainty. For example, when $\underline{\alpha} = \overline{\alpha} = 0.3$, type A investors' updated estimate of μ^i is $0.3\mu^i_A + 0.7\mu^i_B$ and type B investors' updated estimate of μ^i is $0.3\mu^i_B + 0.7\mu^i_A$.

Second, higher α indicates that investors value their own interpretation more when computing the updated estimates of μ^i for each risky asset i. As a result, the values of $\underline{\alpha}$ and $\overline{\alpha}$ enable us to characterize different levels of ambiguity. In particular, there are five categories of ambiguity:

$$
(3.4) \qquad \begin{cases} \text{unbiased,} & \text{if } (\underline{\alpha} + \overline{\alpha} = 1,\ 0 < \underline{\alpha} < 0.5 < \overline{\alpha} < 1) \\ \text{slightly overconfident,} & \text{if } (\underline{\alpha} + \overline{\alpha} > 1,\ 0 < \underline{\alpha} < 0.5 < \overline{\alpha} \leq 1) \\ \text{slightly underconfident,} & \text{if } (\underline{\alpha} + \overline{\alpha} < 1,\ 0 \leq \underline{\alpha} < 0.5 < \overline{\alpha} < 1) \\ \text{overconfident,} & \text{if } (\underline{\alpha} + \overline{\alpha} > 1,\ 0.5 \leq \underline{\alpha} < \overline{\alpha} \leq 1) \\ \text{underconfident,} & \text{if } (\underline{\alpha} + \overline{\alpha} < 1,\ 0 \leq \underline{\alpha} < \overline{\alpha} \leq 0.5) \end{cases}
$$

Following Gilboa and Schmeidler [1989], traders choose a portfolio to maximize their minimum expected utility over the set of weights, $[\underline{\alpha}, \overline{\alpha}]$.

3.3. Asset demands

We now solve for investors' asset demands. First, after taking account of S^i and the other type's interpretation, investors' posterior beliefs of V^i are given by

$$
(3.5) \qquad \begin{cases} V^i \sim N(\dfrac{\delta^i v^i + \gamma^i (S^i - \widehat{\mu^i_A})}{\delta^i + \gamma^i}, \dfrac{1}{\delta^i + \gamma^i}) := N(A^i, \dfrac{1}{\delta^i + \gamma^i}), & \text{if type } A \text{ investors} \\ V^i \sim N(\dfrac{\delta^i v^i + \gamma^i (S^i - \widehat{\mu^i_B})}{\delta^i + \gamma^i}, \dfrac{1}{\delta^i + \gamma^i}) := N(B^i, \dfrac{1}{\delta^i + \gamma^i}), & \text{if type } B \text{ investors} \end{cases}
$$

where $\delta^i = 1/\sigma^i$ and $\gamma^i = 1/(\phi^i + \theta^i)$. Let A^i and B^i denote type A and type B investors' possible posterior means of V^i, respectively. In addition, denote the minimum and maximum value of A^i and B^i by A^i_{min}, A^i_{max}, B^i_{min}, and B^i_{max}, respectively. Note that if $\mu^i_B > \mu^i_A$, then A^i_{min} and A^i_{max} are achieved at $\alpha = \underline{\alpha}$ and at $\alpha = \overline{\alpha}$, while B^i_{min}

and B^i_{max} are achieved at $\alpha = \overline{\alpha}$ and at $\alpha = \underline{\alpha}$. If, on the other hand, $\mu^i_A > \mu^i_B$, then everything is reversed due to symmetry. That is, when $\mu^i_B > \mu^i_A$,

$$
(3.6) \qquad \begin{cases}
A^i_{min} = \frac{\delta^i v^i + \gamma^i [S^i - (\underline{\alpha}^i \mu^i_A + (1-\underline{\alpha}^i)\mu^i_B)]}{\delta^i + \gamma^i} \\
B^i_{min} = \frac{\delta^i v^i + \gamma^i [S^i - (\overline{\alpha^i} \mu^i_B + (1-\overline{\alpha^i})\mu^i_A)]}{\delta^i + \gamma^i} \\
A^i_{max} = \frac{\delta^i v^i + \gamma^i [S^i - (\overline{\alpha^i} \mu^i_A + (1-\overline{\alpha^i})\mu^i_B)]}{\delta^i + \gamma^i} \\
B^i_{max} = \frac{\delta^i v^i + \gamma^i [S^i - (\underline{\alpha}^i \mu^i_B + (1-\underline{\alpha}^i)\mu^i_A)]}{\delta^i + \gamma^i}.
\end{cases}
$$

When $\mu^i_A > \mu^i_B$,

$$
(3.7) \qquad \begin{cases}
A^i_{min} = \frac{\delta^i v^i + \gamma^i [S^i - (\overline{\alpha^i} \mu^i_A + (1-\overline{\alpha^i})\mu^i_B)]}{\delta^i + \gamma^i} \\
B^i_{min} = \frac{\delta^i v^i + \gamma^i [S^i - (\underline{\alpha}^i \mu^i_B + (1-\underline{\alpha}^i)\mu^i_A)]}{\delta^i + \gamma^i} \\
A^i_{max} = \frac{\delta^i v^i + \gamma^i [S^i - (\underline{\alpha}^i \mu^i_A + (1-\underline{\alpha}^i)\mu^i_B)]}{\delta^i + \gamma^i} \\
B^i_{max} = \frac{\delta^i v^i + \gamma^i [S^i - (\overline{\alpha^i} \mu^i_B + (1-\overline{\alpha^i})\mu^i_A)]}{\delta^i + \gamma^i}.
\end{cases}
$$

Let $(m_A, {x_A}^1, ..., {x_A}^I)$ and $(m_B, {x_B}^1, ..., {x_B}^I)$ denote type A and B investors' per capital asset demands, respectively. Demands for risky assets can be positive (go long) or negative (go short). Each investor solves the following decision problem:

(3.8)

$$
\begin{cases}
\max\limits_{(m_A, {x_A}^1, ..., {x_A}^I)} \min\limits_{\alpha \in [\underline{\alpha}, \overline{\alpha}]} \quad E\big[-exp(-(w_A + \sum\limits_i (V^i - p^i)x^i_A))\big], \quad \text{if type } A \text{ investors} \\
\max\limits_{(m_B, {x_B}^1, ..., {x_B}^I)} \min\limits_{\alpha \in [\underline{\alpha}, \overline{\alpha}]} \quad E\big[-exp(-(w_B + \sum\limits_i (V^i - p^i)x^i_B))\big], \quad \text{if type } B \text{ investors,}
\end{cases}
$$

which can be rewritten as[8]

(3.9)

$$\begin{cases} \max\limits_{(m_A, {x_A}^1, \ldots, {x_A}^I)} \min\limits_{\alpha \in [\underline{\alpha}, \overline{\alpha}]} \quad w_A + \sum\limits_i (A^i - p^i) x_A^i - 1/2 (\dfrac{1}{\delta^i + \gamma^i})^2 (x_A^i)^2), & \text{if type } A \text{ investors} \\ \max\limits_{(m_B, {x_B}^1, \ldots, {x_B}^I)} \min\limits_{\alpha \in [\underline{\alpha}, \overline{\alpha}]} \quad w_B + \sum\limits_i (B^i - p^i) x_B^i - 1/2 (\dfrac{1}{\delta^i + \gamma^i})^2 (x_B^i)^2), & \text{if type } B \text{ investors,} \end{cases}$$

where the minimum is taken over the set $[\underline{\alpha}, \overline{\alpha}]$. Equivalently, the minimum is taken over the possible posterior means of V_i, $[A^i_{min}, A^i_{max}]$ and $[B^i_{min}, B^i_{max}]$, for type A and type B investors, respectively.

3.4. Conservatism under ambiguity aversion

An ambiguity-averse investor contemplating either a long or a short position in risky asset i evaluates it using the posterior mean of V_i that yields the smallest expected utility before maximization. To figure out investor's demand function for each risky asset i, it is essential to consider whether an investor would prefer a long position, a short position, or zero position (non-participating).

For example, suppose $A^i_{min} > p^i$, a type A investor contemplating a long position ($x^i_A > 0$) would evaluate it using A^i_{min} before maximization as $(A^i - p^i)x^i_A$ is minimized at $A^i = A^i_{min}$. In contrast, a type A investor contemplating a short position ($x^i_A < 0$) would evaluate it using A^i_{max} before maximization as $(A^i - p^i)x^i_A$ is minimized at $A^i = A^i_{max}$.

[8]This is because $E[-e^{-(w_A + \sum_i (V^i - p^i) x^i_A)}]$ is a strictly increasing transformation of $w_A + \sum_i (A^i - p^i) x^i_A - 1/2(\frac{1}{\delta^i + \gamma^i})^2 (x^i_A)^2$ and $E[-e^{-(w_B + \sum_i (V^i - p^i) x^i_B)}]$ is a strictly increasing transformation of $w_B + \sum_i (B^i - p^i) x^i_B - 1/2(\frac{1}{\delta^i + \gamma^i})^2 (x^i_B)^2$.

However, a long position of $(A^i_{min} - p^i)(\delta^i + \gamma^i)$ generates the highest expected utility, which is higher than the expected utility generated by any short positions or zero position.

Following this logic, each type A investor's demand function for risky asset i is

$$x^{i*}_A = \begin{cases} (A^i_{min} - p^i)(\delta^i + \gamma^i), & \text{if } A^i_{min} > p^i \\ 0, & \text{if } A^i_{min} \leq p^i \leq A^i_{max} \\ (A^i_{max} - p^i)(\delta^i + \gamma^i), & \text{if } A^i_{max} < p^i. \end{cases} \tag{3.10}$$

Similarly, Each type B investor's demand function for risky asset i is given by

$$x^{i*}_B = \begin{cases} (B^i_{min} - p^i)(\delta^i + \gamma^i), & \text{if } B^i_{min} > p^i \\ 0, & \text{if } B^i_{min} \leq p^i \leq B^i_{max} \\ (B^i_{max} - p^i)(\delta^i + \gamma^i), & \text{if } B^i_{max} < p^i. \end{cases} \tag{3.11}$$

Equation (3.10) and (3.11) describes how ambiguity aversion affects an investor's demand. In particular, if the price of risky asset i is higher than his minimum posterior mean of V_i, than an investor goes long in risky asset i. If the price of risky asset i is higher than his maximum posterior mean of V_i, than an investor goes short in risky asset i. If the price of risky asset i is between than his minimum and posterior mean of V_i, then an investor will not participate in the market for risky asset i. In other words, ambiguity-averse investors are conservative to the fullest. An ambiguity-averse investor contemplating either a long or a short position in a risky asset evaluates it using an estimate of the stock that yields the smallest expected utility before maximization. Hence, investors go long in the asset only if the price is above their minimum possible estimate and go short in the asset only if the price is below their maximum possible estimate. In addition, if the price is above the minimum possible estimate and below the maximum possible estimate, investors will not participate in the market. These results are consistent with Easley and O'Hara [2009] and Easley and O'Hara [2010].

In equilibrium the per capita demand for each risky asset must equal its per capita supply. That is, for each risky asset i:

$$1/2\ x_A^{i^*} + 1/2\ x_B^{i^*} = \bar{x}^i \tag{3.12}$$

Next, I study how ambiguity plays a role in the equilibrium. The reason is that, under different values of $\underline{\alpha}$ and $\overline{\alpha}$, the relative magnitude of A^i_{min}, A^i_{max}, B^i_{min}, and B^i_{max} will be different and hence affect the equilibrium price of risky asset i.

3.5. Benchmark case: no ambiguity

As a benchmark, I first solve the case where investors face no ambiguity, i.e., $\alpha = \underline{\alpha} = \overline{\alpha}$. Each type A and type B investor's demand function for risky asset i are:

$$\begin{cases} x_A^{i^*} = (\delta^i v^i + \gamma^i [S^i - (\alpha\mu_A^i + (1-\alpha)\mu_B^i)] - p^i(\delta^i + \gamma^i) \\ x_B^{i^*} = (\delta^i v^i + \gamma^i [S^i - (\alpha\mu_B^i + (1-\alpha)\mu_A^i)] - p^i(\delta^i + \gamma^i) \end{cases} \tag{3.13}$$

Using equation (3.12), the market-clearing price is

$$p^i_{benchmark} = \frac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A+\mu^i_B)}{2}] - \bar{x}^i}{\delta^i + \gamma^i}, \tag{3.14}$$

In addition, since there are only two types of investors, trading volume for risky asset i, $Volume^i$, is the absolute change in equilibrium holdings for either type A or type B investors from before to after they observe the public signal S^i and the other type's interpretation. In particular, when investors face no ambiguity,

(3.15)

$$Volume^i = \frac{N}{2}|x^{i*}_A(p^i_{benchmark}) - \bar{x}^i| = \frac{N}{2}|x^{i*}_B(p^i_{benchmark}) - \bar{x}^i| = \frac{N}{2}|(0.5-\alpha)||\mu^i_A - \mu^i_B|.$$

Since the average interpretation of the public signal for risky asset i, $(\mu^i_A + \mu^i_B)/2$, clearly affects the market-clearing price, it is essential to hold the average interpretation fixed when we study the relationship between disagreement and price.[9]

I define the abnormal price as the difference between the equilibrium price p^{i^*} and the benchmark price, i.e, $\widehat{p^i} = p^{i^*} - p^i_{benchmark}$. In this paper, I'll focus on

$$\frac{\partial \widehat{p^i}}{\partial|\mu^i_A - \mu^i_B|}, \tag{3.16}$$

which represents how price of risky asset i increases or decreases with respect to a unit increase in investor disagreement, holding the average interpretation of the public signal constant.

[9]For example, higher average interpretation by setup implies a more negative view of the signal and naturally generates a lower price, and vice versa.

Equation (3.14) indicates that when investors face no ambiguity, price of any risky asset is not related to disagreement. In addition, equation (3.15) indicates that $Volume^i$ is increasing in disagreement. However, when investors face ambiguity, disagreement and price can be either positively related, negatively related, or unrelated, while disagreement and volume can be either positively related or unrelated.

For the following analysis, it is useful to denote the benchmark posterior mean of V_i by

$$V^i_{benchmark} = \frac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2}]}{\delta^i + \gamma^i}. \tag{3.17}$$

It can be shown that $V^i_{benchmark} = (A^i_{min} + B^i_{max})/2 = (B^i_{min} + A^i_{max})/2$. As a result, on the real line A^i_{min} and B^i_{max} are symmetric with respect to $V^i_{benchmark}$, and so are B^i_{min} and A^i_{max}. In addition, A^i_{min}, A^i_{max}, B^i_{min}, and B^i_{max} can all be written as $V^i_{benchmark}$ plus some function of disagreement, $|\mu^i_A - \mu^i_B|$.

3.6. Unbiased investors

In this section I study the case where investors are unbiased ($\underline{\alpha} + \overline{\alpha} = 1$ and $0 < \underline{\alpha} < 0.5 < \overline{\alpha} < 1$) when thinking about the other type's interpretation. Figure 3.1 shows that $A^i_{max} = B^i_{max} > V^i_{benchmark} > A^i_{min} = B^i_{min}$.

First, if at a price lower than $A^i_{min} = B^i_{min}$, both types of investors go long in risky asset i with their posterior means of V_i being A^i_{min} and B^i_{min}, respectively. Second, if $A^i_{min} = B^i_{min} \leq p^i \leq A^i_{max} = B^i_{max}$, then both types of investors won't participate in the market. Lastly, if at a price higher than $A^i_{max} = B^i_{max}$, then both types of investors go short in risky asset i, which can't be the equilibrium as risky asset i is in positive supply.

Hence, equilibrium only exists if $p^i < A^i_{min} = B^i_{min}$ and the average posterior mean of V_i in the market is equal to $(A^i_{min} + B^i_{min})/2$, which is smaller than $V^i_{benchmark}$ and thus a decreasing function of disagreement, $|\mu^i_A - \mu^i_B|$.

Proposition 3. *When investors are unbiased, the abnormal price for risky asset i is*

$$\widehat{p^i} = \frac{-\gamma^i}{\delta^i + \gamma^i} \frac{(\overline{\alpha} - \underline{\alpha})}{2} |\mu^i_A - \mu^i_B|. \tag{3.18}$$

In addition, there's no trading for risky asset i, i.e., $Volume^i = 0$.

Proof. See Appendix D. □

Proposition 3 indicates that the equilibrium price for risky asset i is decreasing in investor disagreement, $|\mu^i_A - \mu^i_B|$, holding fixed the average interpretation of the public signal for risky asset i is $(\mu^i_A + \mu^i_B)/2$. That is, price reflects a more pessimistic valuation as disagreement increases. Figure 3.2 plots the relationship between the abnormal price of risky asset i and disagreement when investors are unbiased.

In particular, when disagreement increases by 1, price decreases by $\frac{\gamma^i}{\delta^i + \gamma^i} \frac{(\overline{\alpha} - \underline{\alpha})}{2}$, which is between 0 and 1/4. In addition, there's no trading volume for risky asset i as investors' prior and posterior means of V^i are the same.[10]

[10] $A^i_{min} = B^i_{min} = (A^i_{min} + B^i_{min})/2$

3.7. Slightly overconfident investors

I then study the equilibrium where investors are slightly overconfident ($\underline{\alpha}+\overline{\alpha}>1$ and $0<\underline{\alpha}<0.5<\overline{\alpha}\leq 1$) when thinking about the other type's interpretation. Without loss of generality, assume $\mu_B^i>\mu_A^i$, so type A investors are optimistic and type B investors are pessimistic. Figure 3.3 shows that $A_{max}^i>B_{max}^i>V_{benchmark}^i>A_{min}^i>B_{min}^i$ under $\mu_B^i>\mu_A^i$.

First, if at a price lower than B_{min}^i, both types of investors go long in risky asset i with their posterior means of V_i being A_{min}^i and B_{min}^i, respectively. Second, if $B_{min}^i\leq p^i<A_{min}^i$, only type A investors participate in the market with their posterior mean of V_i being A_{min}^i. In the above two cases, the average posterior means of V_i in the market equal to $(A_{min}^i+B_{min}^i)/2$ and A_{min}^i, which are both smaller than $V_{benchmark}^i$ and thus are decreasing in disagreement. The corresponding market-clearing prices are $p^i=\frac{(A_{min}^i+B_{min}^i)}{2}-\frac{\bar{x}^i}{(\delta^i+\gamma^i)}$ and $p^i=A_{min}^i-\frac{2\bar{x}^i}{(\delta^i+\gamma^i)}$, respectively.[11]

Third, if $A_{min}^i\leq p^i\leq B_{max}^i$, both types of investors won't participate in the market so no equilibrium exists. Fourth, if $B_{max}^i<p^i\leq A_{max}^i$, only type B investors want to go short in risky asset i and thus no equilibrium exists. Lastly, if $p^i>A_{max}^i$, both types of investors want to go short in risky asset i and thus no equilibrium exists.

To summarize, when investors are slightly overconfident, optimistic traders always go long in the risky asset. Pessimistic traders, on the other hand, either go long in the risky asset or don't participate in the market.

[11]The binding constraints are $\frac{(A_{min}^i+B_{min}^i)}{2}-\frac{\bar{x}^i}{(\delta^i+\gamma^i)}<B_{min}^i$ and $B_{min}^i\leq A_{min}^i-\frac{2\bar{x}^i}{(\delta^i+\gamma^i)}<A_{min}^i$.

Proposition 4. *When investors are slightly overconfident, the abnormal price for risky asset i is*

$$(3.19)\qquad \widehat{p^i} = \begin{cases} \dfrac{-\gamma^i}{\delta^i+\gamma^i}\dfrac{(\overline{\alpha}-\underline{\alpha})}{2}|\mu_A^i-\mu_B^i|, & \text{if } |\mu_A^i-\mu_B^i| < \dfrac{2\bar{x}^i}{\gamma^i(\overline{\alpha}+\underline{\alpha}-1)} \\ \dfrac{-\gamma^i}{\delta^i+\gamma^i}\dfrac{(1-2\underline{\alpha})}{2}|\mu_A^i-\mu_B^i| - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, & \text{if } |\mu_A^i-\mu_B^i| \geq \dfrac{2\bar{x}^i}{\gamma^i(\overline{\alpha}+\underline{\alpha}-1)}. \end{cases}$$

In addition, trading volume for risky asset i, is given by

$$(3.20)\qquad Volume^i = \begin{cases} \dfrac{N\gamma^i(\overline{\alpha}+\underline{\alpha}-1)}{4}|\mu_A^i-\mu_B^i|, & \text{if } |\mu_A^i-\mu_B^i| < \dfrac{2\bar{x}^i}{\gamma^i(\overline{\alpha}+\underline{\alpha}-1)} \\ \dfrac{N}{2}\,\bar{x}^i, & \text{if } |\mu_A^i-\mu_B^i| \geq \dfrac{2\bar{x}^i}{\gamma^i(\overline{\alpha}+\underline{\alpha}-1)}. \end{cases}$$

Proof. See Appendix E. □

Figure 3.4 illustrates the idea of Proposition 4. First, when investors are slightly overconfident, the equilibrium price for risky asset i is decreasing in investor disagreement, $|\mu_A^i-\mu_B^i|$, holding fixed the average interpretation of the public signal for risky asset i is $(\mu_A^i+\mu_B^i)/2$. In addition, disagreement has a more negative effect on price when both types of investors participate in the market compared to the case where only optimistic traders are present, since $|\frac{-\gamma^i}{\delta^i+\gamma^i}\frac{(\overline{\alpha}-\underline{\alpha})}{2}| > |\frac{-\gamma^i}{\delta^i+\gamma^i}\frac{(1-2\underline{\alpha})}{2}|$.

On the other hand, trading volume is increasing in disagreement when both optimistic and pessimistic investors go long in risky asset i. However, when disagreement is high enough so that pessimistic traders step out of the market, volume is fixed at $\frac{N}{2}\bar{x}^i$ as optimistic traders buy out positions from pessimistic investors.

3.8. Slightly underconfident investors

I then study the equilibrium where investors are slightly underconfident ($\underline{\alpha} + \overline{\alpha} < 1$ and $0 \leq \underline{\alpha} < 0.5 < \overline{\alpha} < 1$) when thinking about the other type's interpretation. Without loss of generality, assume $\mu_B^i > \mu_A^i$, so type A investors are optimistic and type B investors are pessimistic. Figure 3.5 shows that $B_{max}^i > A_{max}^i > V_{benchmark}^i > B_{min}^i > A_{min}^i$ under $\mu_B^i > \mu_A^i$.

First, if at a price lower than A_{min}^i, both types of investors go long in risky asset i with their posterior means of V_i being A_{min}^i and B_{min}^i, respectively. Second, if $A_{min}^i \leq p^i < B_{min}^i$, only type B investors participate in the market with their posterior mean of V_i being B_{min}^i. In the above two cases, the average posterior means of V_i in the market equal to $(A_{min}^i + B_{min}^i)/2$ and B_{min}^i, which are both smaller than $V_{benchmark}^i$ and thus are decreasing in disagreement. The corresponding market-clearing prices are $p^i = \frac{(A_{min}^i + B_{min}^i)}{2} - \frac{\bar{x}^i}{(\delta^i + \gamma^i)}$ and $p^i = B_{min}^i - \frac{2\bar{x}^i}{(\delta^i + \gamma^i)}$, respectively.[12]

Third, if $B_{min}^i \leq p^i \leq A_{max}^i$, both types of investors won't participate in the market so no equilibrium exists. Fourth, if $A_{max}^i < p^i \leq B_{max}^i$, only type A investors want to go short in risky asset i and thus no equilibrium exists. Lastly, if $p^i > B_{max}^i$, both types of investors want to go short in risky asset i and thus no equilibrium exists.

To summarize, when investors are slightly underconfident, pessimistic traders always go long in the risky asset. Optimistic traders, on the other hand, either go long in the risky asset or don't participate in the market.

[12]The binding constraints are $\frac{(A_{min}^i + B_{min}^i)}{2} - \frac{\bar{x}^i}{(\delta^i + \gamma^i)} < B_{min}^i$ and $A_{min}^i \leq A_{min}^i - \frac{2\bar{x}^i}{(\delta^i + \gamma^i)} < B_{min}^i$.

Proposition 5. *When investors are slightly underconfident, the abnormal price for risky asset i is*

$$\widehat{p^i} = \begin{cases} \dfrac{-\gamma^i}{\delta^i+\gamma^i}\dfrac{(\overline{\alpha}-\underline{\alpha})}{2}|\mu_A^i-\mu_B^i|, & \text{if } |\mu_A^i-\mu_B^i| < \dfrac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \\ \dfrac{-\gamma^i}{\delta^i+\gamma^i}\dfrac{(2\overline{\alpha}-1)}{2}|\mu_A^i-\mu_B^i| - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, & \text{if } |\mu_A^i-\mu_B^i| \geq \dfrac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \end{cases} \tag{3.21}$$

In addition, trading volume for risky asset i, is given by

$$Volume^i = \begin{cases} \dfrac{N\gamma^i(1-\underline{\alpha}-\overline{\alpha})}{4}|\mu_A^i-\mu_B^i|, & \text{if } |\mu_A^i-\mu_B^i| < \frac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \\ \dfrac{N}{2}\,\bar{x}^i, & \text{if } |\mu_A^i-\mu_B^i| \geq \frac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \end{cases} \tag{3.22}$$

Proof. Proof is similar to that of Proposition 4. □

Figure 3.6 illustrates the idea of Proposition 5. First, when investors are slightly underconfident, the equilibrium price for risky asset i is decreasing in investor disagreement, $|\mu_A^i - \mu_B^i|$, holding fixed the average interpretation of the public signal for risky asset i is $(\mu_A^i + \mu_B^i)/2$. In addition, disagreement has a more negative effect on price when both types of investors participate in the market compared to the case where only optimistic traders are present, since $|\frac{-\gamma^i}{\delta^i+\gamma^i}\frac{(\overline{\alpha}-\underline{\alpha})}{2}| > |\frac{-\gamma^i}{\delta^i+\gamma^i}\frac{(2\overline{\alpha}-1)}{2}|$.

On the other hand, trading volume is increasing in disagreement when both optimistic and pessimistic investors go long in risky asset i. However, when disagreement is high enough so that optimistic traders step out of the market, volume is fixed at $\frac{N}{2}\bar{x}^i$ as pessimistic traders buy out positions from optimistic investors.

So far I've examines the cases where investors are slightly overconfident and slightly underconfident. In both cases, there always exists a negative relationship between disagreement and price. In addition, trading volume and disagreement are either positively related or unrelated. I proceed to examine the cases where investors are overconfident and underconfident.

3.9. Overconfident investors

I then study the equilibrium where investors are overconfident ($\underline{\alpha} + \overline{\alpha} > 1$ and $0.5 \leq \underline{\alpha} < \overline{\alpha} \leq 1$) when thinking about the other type's interpretation. Without loss of generality, assume $\mu_B^i > \mu_A^i$, so type A investors are optimistic and type B investors are pessimistic. Figure 3.7 shows that $A_{max}^i > A_{min}^i > V_{benchmark}^i > B_{max}^i > B_{min}^i$ under $\mu_B^i > \mu_A^i$.

First, if at a price lower than B_{min}^i, both types of investors go long in risky asset i with their posterior means of V_i being A_{min}^i and B_{min}^i, respectively. Second, if $B_{min}^i \leq p^i \leq B_{max}^i$, only type A investors participate in the market with their posterior mean of V_i being A_{min}^i. Third, if $B_{max}^i < p^i \leq A_{min}^i$, type A investors go long in risky asset i while type B investors go short in risky asset i, with their posterior means of V_i being A_{min}^i and B_{max}^i, respectively.

In the above three cases, the average posterior means of V_i in the market equal to $(A_{min}^i + B_{min}^i)/2$, A_{min}^i, and $(A_{min}^i + B_{max}^i)/2$, which are lower than, higher than, and equal to $V^i_{benchmark}$, respectively. The corresponding market-clearing prices are $p^i = \frac{(A^i_{min}+B^i_{min})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)}$, $p^i = A^i_{min} - \frac{2\bar{x}^i}{(\delta^i+\gamma^i)}$, and $p^i = \frac{(A^i_{min}+B^i_{max})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)}$, respectively.[13]

Fourth, if $A^i_{min} \leq p^i \leq A^i_{max}$, only type B investors want to go short in risky asset i and thus no equilibrium exists. Lastly, if $p^i > A^i_{max}$, both types of investors want to go short in risky asset i and thus no equilibrium exists.

To summarize, when investors are overconfident, optimistic traders always go long in the risky asset. Pessimistic traders, on the other hand, can go long in the risky asset, go short in the risky asset, or don't participate in the market at all. The following proposition presents the equilibrium abnormal price and trading volume.

Proposition 6. *When investors are overconfident, the abnormal price for risky asset i is*

(3.23)

$$\widehat{p^i} = \begin{cases} \dfrac{-\gamma^i}{\delta^i+\gamma^i}\dfrac{(\overline{\alpha}-\underline{\alpha})}{2}|\mu^i_A - \mu^i_B|, & \text{if } |\mu^i_A - \mu^i_B| < \dfrac{2\bar{x}^i}{\gamma^i(\underline{\alpha}+\overline{\alpha}-1)} \\ \dfrac{\gamma^i}{\delta^i+\gamma^i}\dfrac{(2\underline{\alpha}-1)}{2}|\mu^i_A - \mu^i_B| - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, & \text{if } \dfrac{2\bar{x}^i}{\gamma^i(\underline{\alpha}+\overline{\alpha}-1)} \leq |\mu^i_A - \mu^i_B| \leq \dfrac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}-1)} \\ 0, & \text{if } |\mu^i_A - \mu^i_B| > \dfrac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}-1)} \end{cases}$$

[13]The binding constraints are $\frac{(A^i_{min}+B^i_{min})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)} < B^i_{min}$, $B^i_{min} \leq A^i_{min} - \frac{2\bar{x}^i}{(\delta^i+\gamma^i)} \leq B^i_{max}$, and $B^i_{max} < \frac{(A^i_{min}+B^i_{max})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)} \leq A^i_{min}$.

In addition, trading volume for risky asset i, is given by

(3.24)

$$Volume_i = \begin{cases} \dfrac{N\gamma^i(\overline{\alpha}+\underline{\alpha}-1)}{4}|\mu_A^i - \mu_B^i|, & \text{if } |\mu_A^i - \mu_B^i| < \dfrac{2\bar{x}^i}{\gamma^i(\underline{\alpha}+\overline{\alpha}-1)} \\ \dfrac{N}{2}\ \bar{x}^i, & \text{if } \dfrac{2\bar{x}^i}{\gamma^i(\underline{\alpha}+\overline{\alpha}-1)} \le |\mu_A^i - \mu_B^i| \le \dfrac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}-1)} \\ \dfrac{N\gamma^i(2\underline{\alpha}-1)}{4}|\mu_A^i - \mu_B^i|, & \text{if } |\mu_A^i - \mu_B^i| > \dfrac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}-1)} \end{cases}$$

Proof. See Appendix F. □

Figure 3.8 illustrates the idea of Proposition 6. First, when investors are overconfident, the equilibrium price for risky asset i can be decreasing in, increasing in, or unrelated to investor disagreement, $|\mu_A^i - \mu_B^i|$, holding fixed the average interpretation of the public signal for risky asset i is $(\mu_A^i + \mu_B^i)/2$.

In particular, when disagreement is low, i.e, $|\mu_A^i - \mu_B^i| < \frac{2\bar{x}^i}{\gamma^i(\underline{\alpha}+\overline{\alpha}-1)}$, price for risky asset i is decreasing in disagreement. In this case, both optimistic and pessimistic traders go long in risky asset i and both use their most conservative estimates of V^i. While A_{min}^i of optimistic traders exceeds $V_{benchmark}^i$, B_{min}^i of pessimistic traders is much lower than $V_{benchmark}^i$ and thus the average $(A_{min}^i + B_{min}^i)/2$ is lower than $V_{benchmark}^i$.

When disagreement is medium, i.e, $\frac{2\bar{x}^i}{\gamma^i(\underline{\alpha}+\overline{\alpha}-1)} \le |\mu_A^i - \mu_B^i| \le \frac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}-1)}$, price for risky asset i is increasing in disagreement. In this case, only optimistic traders participate in the market and their most conservative estimate, A_{min}^i, is higher than $V_{benchmark}^i$.

When disagreement is high enough, i.e., $|\mu_A^i - \mu_B^i| > \frac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}-1)}$, price for risky asset i is unrelated to disagreement. Optimistic traders go long in risky asset i while pessimistic go short in risky asset i. In particular, their average estimates of V^i equal to $V^i_{benchmark}$, so disagreement doesn't play a role in affecting the price.

On the other hand, trading volume is increasing in disagreement when both optimistic and pessimistic investors participate in the market. In addition, the model predicts that extremely high volume is caused by heavy selling. However, when disagreement is medium so that only optimistic traders participate in the market, volume is fixed at $\frac{N}{2}\bar{x}^i$ as optimistic traders buy out positions from pessimistic investors.

3.10. Underconfident investors

I then study the equilibrium where investors are underconfident ($\underline{\alpha} + \overline{\alpha} < 1$ and $0 \leq \underline{\alpha} < \overline{\alpha} \leq 0.5$) when thinking about the other type's interpretation. Without loss of generality, assume $\mu^i_B > \mu^i_A$, so type A investors are optimistic and type B investors are pessimistic. Figure 3.9 shows that $B^i_{max} > B^i_{min} > V^i_{benchmark} > A^i_{max} > A^i_{min}$ under $\mu^i_B > \mu^i_A$.

First, if at a price lower than A^i_{min}, both types of investors go long in risky asset i with their posterior means of V_i being A^i_{min} and B^i_{min}, respectively. Second, if $A^i_{min} \leq p^i \leq A^i_{max}$, only type B investors participate in the market with their posterior mean of V_i being B^i_{min}. Third, if $A^i_{max} < p^i \leq B^i_{min}$, type B investors go long in risky asset i while type A investors go short in risky asset i, with their posterior means of V_i being B^i_{min} and A^i_{max}, respectively.

In the above three cases, the average posterior means of V_i in the market equal to $(A^i_{min} + B^i_{min})/2$, B^i_{min}, and $(B^i_{min} + A^i_{max})/2$, which are lower than, higher than, and equal to $V^i_{benchmark}$, respectively. The corresponding market-clearing prices are $p^i = \frac{(A^i_{min}+B^i_{min})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)}$, $p^i = B^i_{min} - \frac{2\bar{x}^i}{(\delta^i+\gamma^i)}$, and $p^i = \frac{(B^i_{min}+A^i_{max})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)}$, respectively.[14]

Fourth, if $B^i_{min} \leq p^i \leq B^i_{max}$, only type A investors want to go short in risky asset i and thus no equilibrium exists. Lastly, if $p^i > B^i_{max}$, both types of investors want to go short in risky asset i and thus no equilibrium exists.

To summarize, when investors are overconfident, pessimistic traders always go long in the risky asset. Optimistic traders, on the other hand, can go long in the risky asset, go short in the risky asset, or don't participate in the market at all. The following proposition presents the equilibrium abnormal price and trading volume.

Proposition 7. *When investors are underconfident, the abnormal price for risky asset i is*

(3.25)

$$\widehat{p^i} = \begin{cases} \dfrac{-\gamma^i}{\delta^i+\gamma^i}\dfrac{(\overline{\alpha}-\underline{\alpha})}{2}|\mu^i_A-\mu^i_B|, & \text{if } |\mu^i_A-\mu^i_B| < \dfrac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \\ \dfrac{\gamma^i}{\delta^i+\gamma^i}\dfrac{(1-2\overline{\alpha})}{2}|\mu^i_A-\mu^i_B| - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, & \text{if } \dfrac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \leq |\mu^i_A-\mu^i_B| \leq \dfrac{2\bar{x}^i}{\gamma^i(1-2\overline{\alpha})} \\ 0, & \text{if } |\mu^i_A-\mu^i_B| > \dfrac{2\bar{x}^i}{\gamma^i(1-2\overline{\alpha})} \end{cases}$$

[14]The binding constraints are $\frac{(A^i_{min}+B^i_{min})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)} < A^i_{min}$, $A^i_{min} \leq A^i_{min} - \frac{2\bar{x}^i}{(\delta^i+\gamma^i)} \leq A^i_{max}$, and $A^i_{max} < \frac{(B^i_{min}+A^i_{max})}{2} - \frac{\bar{x}^i}{(\delta^i+\gamma^i)} \leq B^i_{min}$, respectively.

In addition, trading volume for risky asset i, is given by

(3.26)

$$Volume_i = \begin{cases} \dfrac{N\gamma^i(1-\underline{\alpha}-\overline{\alpha})}{4}|\mu_A^i - \mu_B^i|, & if \;\; |\mu_A^i - \mu_B^i| < \dfrac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \\ \dfrac{N}{2}\bar{x}^i, & if \;\; \dfrac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \leq |\mu_A^i - \mu_B^i| \leq \dfrac{2\bar{x}^i}{\gamma^i(1-2\overline{\alpha})} \\ \dfrac{N\gamma^i(1-2\overline{\alpha})}{4}|\mu_A^i - \mu_B^i|, & if \;\; |\mu_A^i - \mu_B^i| > \dfrac{2\bar{x}^i}{\gamma^i(1-2\overline{\alpha})} \end{cases}$$

Proof. Proof is similar to that of Proposition 6. □

Figure 3.10 illustrates the idea of Proposition 7. First, when investors are underconfident, the equilibrium price for risky asset i can be decreasing in, increasing in, or unrelated to investor disagreement, $|\mu_A^i - \mu_B^i|$, holding fixed the average interpretation of the public signal for risky asset i is $(\mu_A^i + \mu_B^i)/2$.

In particular, when disagreement is low, i.e, $|\mu_A^i - \mu_B^i| < \frac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})}$, price for risky asset i is decreasing in disagreement. In this case, both optimistic and pessimistic traders go long in risky asset i and both use their most conservative estimates of V^i. While B_{min}^i of optimistic traders exceeds $V_{benchmark}^i$, A_{min}^i of pessimistic traders is much lower than $V_{benchmark}^i$ and thus the average $(A_{min}^i + B_{min}^i)/2$ is lower than $V_{benchmark}^i$.

When disagreement is medium, i.e, $\frac{2\bar{x}^i}{\gamma^i(1-\underline{\alpha}-\overline{\alpha})} \leq |\mu_A^i - \mu_B^i| \leq \frac{2\bar{x}^i}{\gamma^i(1-2\overline{\alpha})}$, price for risky asset i is increasing in disagreement. In this case, only pessimistic traders participate in the market and their most conservative estimate, B_{min}^i, is higher than $V_{benchmark}^i$.

When disagreement is high enough, i.e., $|\mu_A^i - \mu_B^i| > \frac{2\bar{x}^i}{\gamma^i(1-2\overline{\alpha})}$, price for risky asset i is unrelated to disagreement. Pessimistic traders go long in risky asset i while optimistic traders go short in risky asset i. In particular, their average estimates of V^i equal to $V^i_{benchmark}$, so disagreement doesn't play a role in affecting the price.

On the other hand, trading volume is increasing in disagreement when both optimistic and pessimistic investors participate in the market. However, when disagreement is medium so that only pessimistic traders participate in the market, volume is fixed at $\frac{N}{2}\bar{x}^i$ as pessimistic traders buy out positions from optimistic investors.

3.11. Empirical predictions

The equilibrium with overconfident or underconfident investors can explain many interesting empirical facts. However, as overconfidence is more likely to happen in reality, I use the case where investors are overconfident to discuss the predictions of the model.[15]

3.11.1. Volume-return relationship

Empirical evidence shows a positive correlation between volume and returns in the stock market. In particular, stock prices tend to rise on high volume but decline on low volume.[16]

I first define low and high trading volume as the volume that is below and above half of the outstanding shares. That is, for risky asset i, low volume is when $Volume^i < \frac{N\bar{x}^i}{2}$ and high volume is when $Volume^i \geq \frac{N\bar{x}^i}{2}$. From Figure 3.8, low volume occurs when disagreement is below $\frac{2\bar{x}^i}{\gamma^i(\underline{\alpha}^i+\overline{\alpha}^i-1)}$. Under this range of disagreement, abnormal price is

[15]See, for example, Oskamp [1965], Scheinkman and Xiong [2003], Daniel et al. [2005], Van den Steen [2011], and Ortoleva and Snowberg [2015].
[16]See, for example,Ying [1966], Karpoff [1987], and Harris [1987].

decreasing in investor disagreement. Taken together, it means that stock price declines on low volume.

The intuition is that both optimists and pessimists go long in risky asset i and thus turnover ratio is never more than a half. In addition, optimists' most conservative estimate of the stock is not optimistic enough to balance that of pessimists, which eventually leads to a decrease in abnormal price.

Next, again from Figure 3.8, high volume occurs when disagreement is above $\frac{2\bar{x}^i}{\gamma^i(\underline{\alpha}^i+\overline{\alpha}^i-1)}$. Under this range of disagreement, price is either increasing in or unrelated to investor disagreement. Taken together, it means that stock price on average rises on high volume.

The intuition is that when optimistic investors go long in risky asset i while pessimistic ones sit out of the market, trading volume is fixed at one half of the total outstanding shares. In addition, market-clearing price is solely determined by optimists' most conservative estimate of the stock, which eventually generates an increase in abnormal price.

When investor disagreement is above $\frac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}^i-1)}$ so that optimists go long and pessimists go short, there exists extremely large trading volume accompanied by zero price change. Intuitively, optimist' most conservative estimate of the stock cancels out with pessimists' boldest estimate of the stock, and thus leaves the abnormal price unchanged. This situation corresponds to the findings in Kandel and Pearson [1995], where they document that stocks experience little or no price change at the time of their massive trading volume around earnings announcements. In addition, Bamber and Cheon [1995] document that trading is high relative to the magnitude of the price reaction when analysts forecasts are more diverse. Bailey et al. [2003] also find that after Regulation Fair Disclosure many earnings announcements have large trading reactions despite small price reactions.

3.11.2. Volume and earnings announcements

Beaver [1968], Kiger [1972], and Morse [1981] document that trading volume tends to rise around earnings announcements. Graham et al. [2006] find that both anticipated and unanticipated announcements generate high trading volume. Several studies including Holthausen and Verrecchia [1990], Lee et al. [1993], Krinsky and Lee [1996], and Bamber et al. [1997] suggest that earnings announcements most likely generate dispersion in beliefs. While volume may proxy for more than disagreement, Garfinkel [2009] provides evidence that unexplained trading volume, which controls for both liquidity effect and informedness effect in volume, is high around earnings announcments.

Hence, I make a time-varying assumption on investor disagreement: disagreement is lower around non-announcement period but higher around announcements. Based on this assumption, as shown in Figure 3.8, volume is monotonically increasing in disagreement. In particular, when both optimists and pessimists participate in the market, volume is strictly increasing in disagreement.

3.12. Conclusion

In this chapter, I set up a multi-asset model in which optimistic and pessimistic investors interpret signals differently and characterize the corresponding equilibrium. The disagreement model provides insights on how disagreement shapes volume and returns when ambiguity-averse investors face ambiguity in a simple and tractable way.

For instance, when investors always value their own interpretations more, disagreement generates under-pricing at low disagreement while overpricing at medium disagreement. When disagreement is extremely high, price is unrelated to disagreement. Trading volume, on the other hand, is strictly increasing in disagreement at low and high disagreement level, but remains fixed under medium disagreement level. Investors' long, short, or non-participating decisions are also specified.

The model can also speak to several empirical evidence. For example, stock prices rise on high volume but decline on volume, while extremely high volume can be accompanied by little price change. In addition, trading volume on average increases prior to or around earnings announcements.

The framework can be extended to a multi-period, multi-asset model that studies return and volume dynamics when additional assumptions on disagreement evolution are imposed. I leave this for future research.

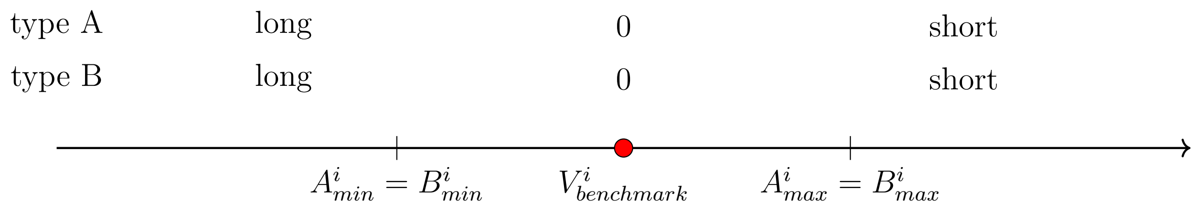


Figure 3.1. **Unbiased investors.** This figure plots the relative magnitude of A^i_{min}, A^i_{max}, B^i_{min}, B^i_{max}, and $V^i_{benchmark}$ when investors are unbiased. In addition, the figure shows the long, short, or non-participating positions investors take in each interval when the price p^i falls in that given interval.

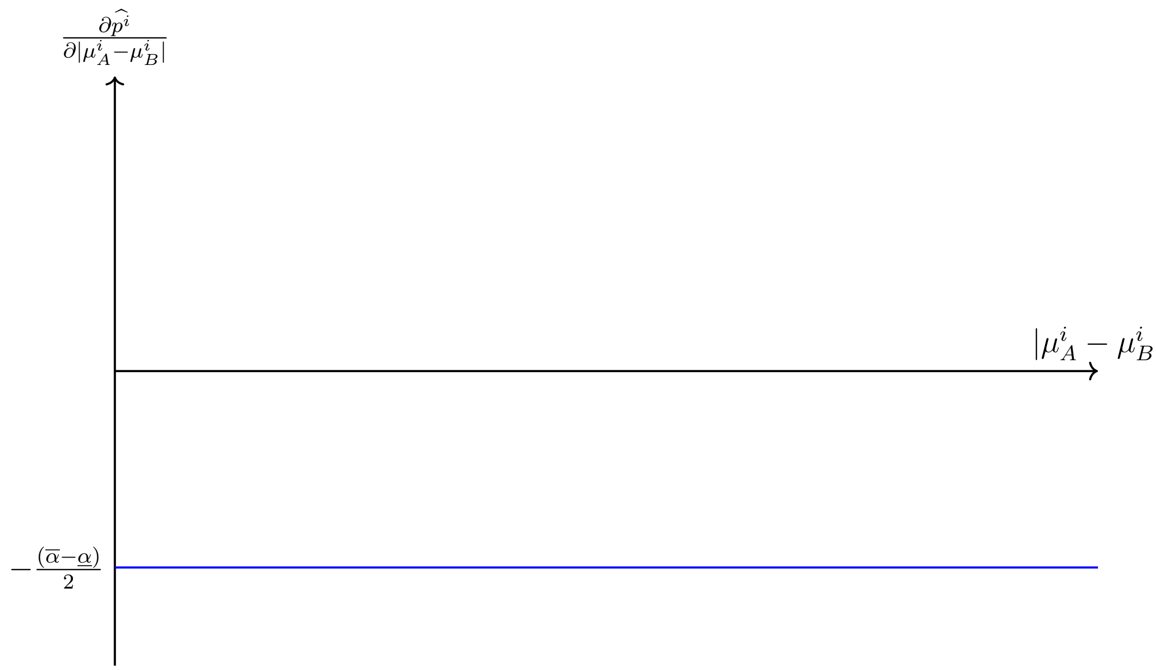


Figure 3.2. **Unbiased investors: disagreement and price.** This figure shows the relationship between risky asset i's price and disagreement when investors are unbiased. $\frac{\partial \widehat{p^i}}{\partial |\mu^i_A - \mu^i_B|}$ measures how price of risky asset i increases or decreases with respect to a unit increase in investor disagreement, holding the average interpretation of the public signal, $\frac{\mu^i_A + \mu^i_B}{2}$, constant. $|\mu^i_A - \mu^i_B|$ is disagreement for risky asset i.

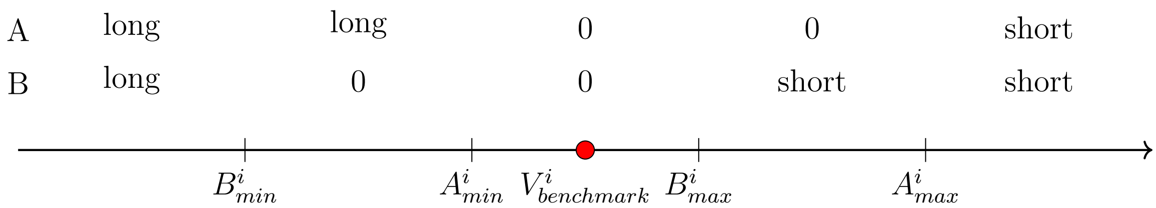


Figure 3.3. **Slightly overconfident investors with $\mu^i_B > \mu^i_A$.** This figure plots the relative magnitude of A^i_{min}, A^i_{max}, B^i_{min}, B^i_{max}, and $V^i_{benchmark}$ when investors are slightly overconfident under $\mu^i_B > \mu^i_A$ (type A investors are optimistic while type B investors are pessimistic). In addition, the figure shows the long, short, or non-participating positions investors take in each interval when the price p^i falls in that given interval.

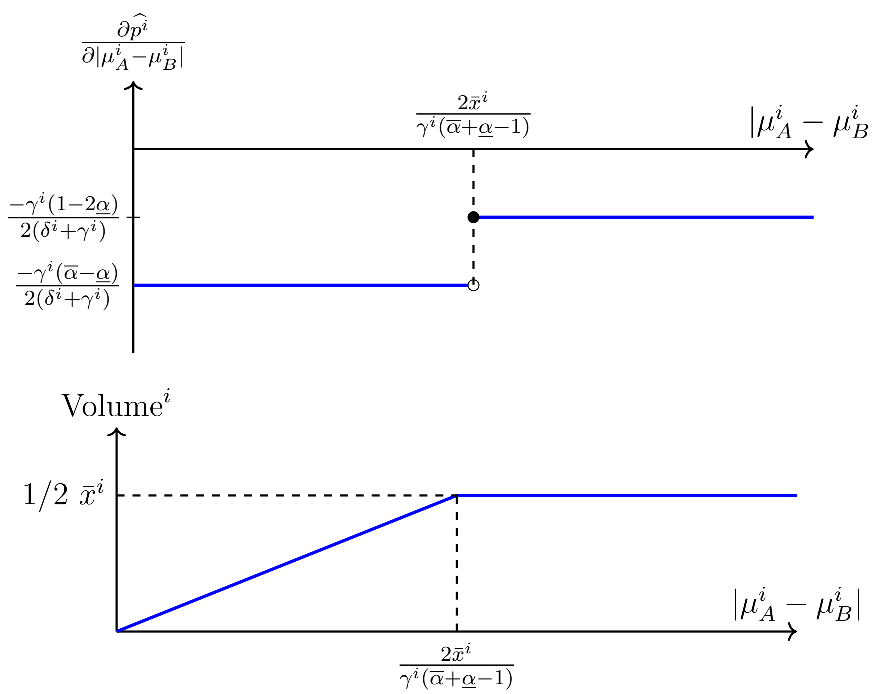


Figure 3.4. **Slightly overconfident investors: disagreement, price, and volume.** This figure shows the relationship between risky asset i's price and disagreement when investors are slightly overconfident. $\frac{\partial \widehat{p^i}}{\partial |\mu^i_A - \mu^i_B|}$ measures how price of risky asset i increases or decreases with respect to a unit increase in investor disagreement, holding the average interpretation of the public signal, $\frac{\mu^i_A + \mu^i_B}{2}$, constant. $|\mu^i_A - \mu^i_B|$ is disagreement for risky asset i.

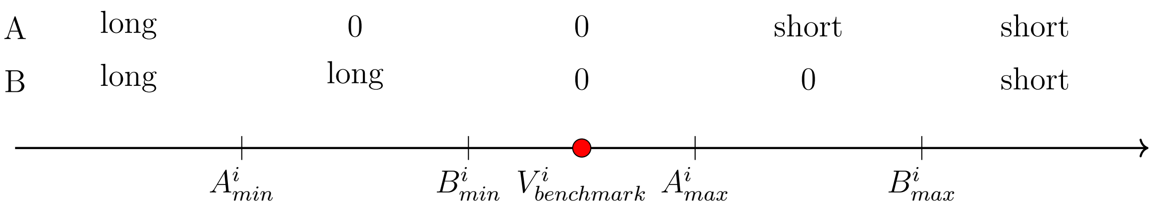


Figure 3.5. **Slightly underconfident investors with $\mu^i_B > \mu^i_A$.** This figure plots the relative magnitude of A^i_{min}, A^i_{max}, B^i_{min}, B^i_{max}, and $V^i_{benchmark}$ under $\mu^i_B > \mu^i_A$ (type A investors are optimistic while type B investors are pessimistic) when investors are slightly underconfident. In addition, the figure shows the long, short, or non-participating positions investors take in each interval when the price p^i falls in that given interval.

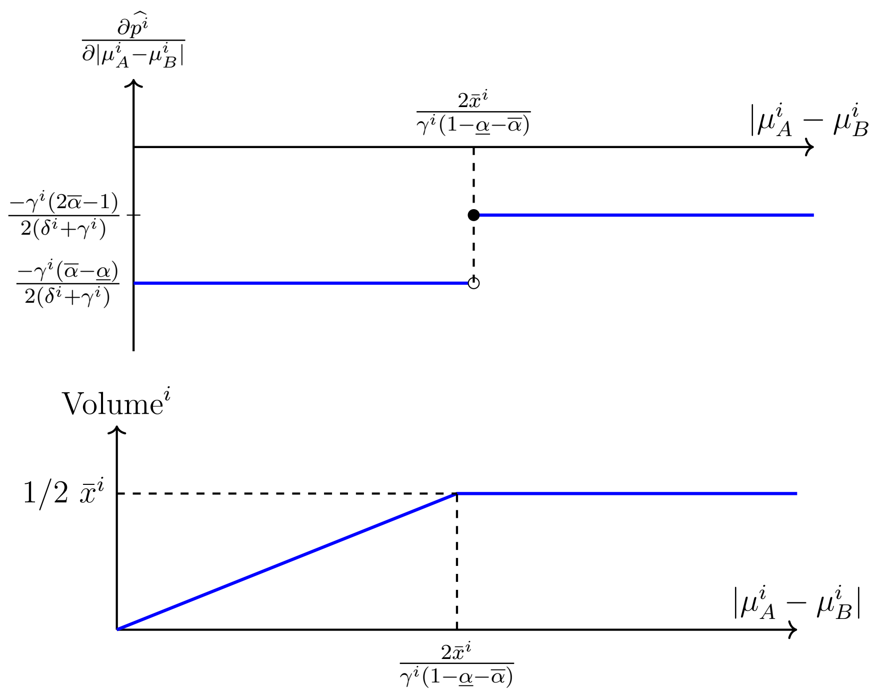


Figure 3.6. **Slightly underconfident investors: disagreement, price, and volume.** This figure shows the relationship between risky asset i's price and disagreement when investors are slightly underconfident. $\frac{\partial \widehat{p^i}}{\partial|\mu^i_A - \mu^i_B|}$ measures how price of risky asset i increases or decreases with respect to a unit increase in investor disagreement, holding the average interpretation of the public signal, $\frac{\mu^i_A + \mu^i_B}{2}$, constant. $|\mu^i_A - \mu^i_B|$ is disagreement for risky asset i.

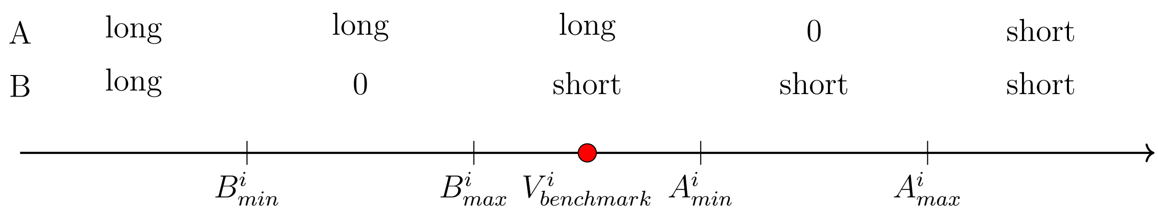


Figure 3.7. **Overconfident investors with $\mu^i_B > \mu^i_A$.** This figure plots the relative magnitude of A^i_{min}, A^i_{max}, B^i_{min}, B^i_{max}, and $V^i_{benchmark}$ under $\mu^i_B > \mu^i_A$ when investors are underconfident. In addition, the figure shows the long, short, or non-participating positions investors take in each interval when p^i falls in that given interval.

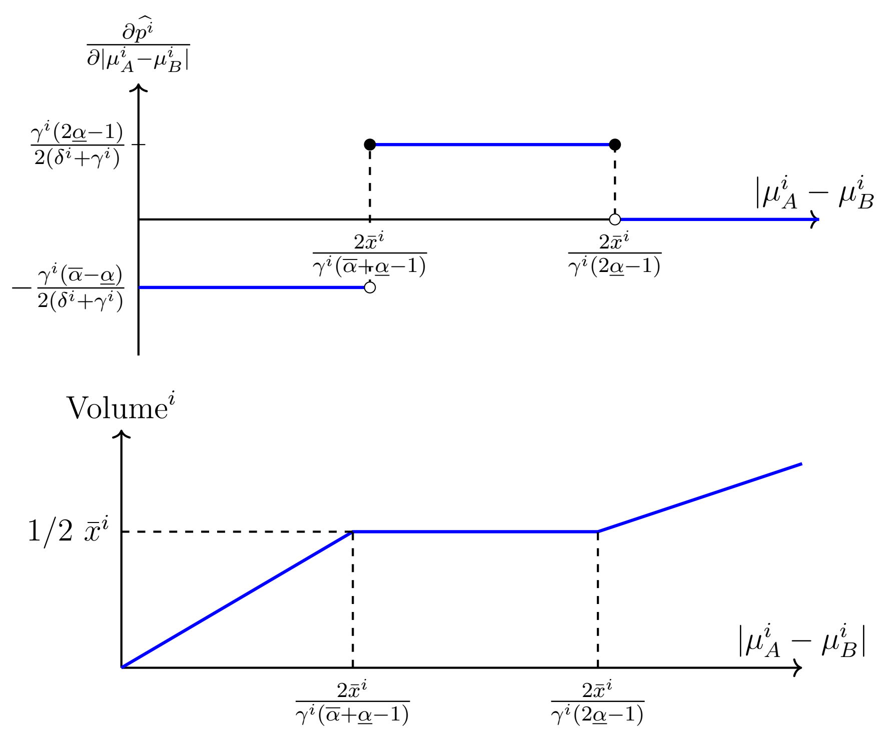


Figure 3.8. **Overconfident investors: disagreement, price, and volume.** This figure shows the relationship between risky asset i's price and disagreement when investors are overconfident. $\frac{\partial \widehat{p^i}}{\partial|\mu^i_A - \mu^i_B|}$ measures how price of risky asset i increases or decreases with respect to a unit increase in investor disagreement, holding the average interpretation of the public signal, $\frac{\mu^i_A + \mu^i_B}{2}$, constant. $|\mu^i_A - \mu^i_B|$ is disagreement for risky asset i.

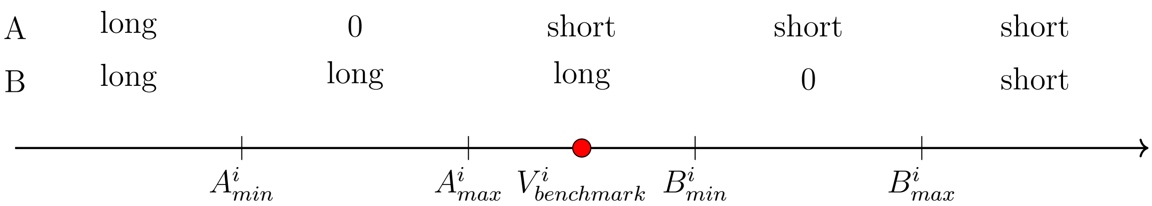


Figure 3.9. **Underconfident investors with $\mu^i_B > \mu^i_A$.** This figure plots the relative magnitude of A^i_{min}, A^i_{max}, B^i_{min}, B^i_{max}, and $V^i_{benchmark}$ under $\mu^i_B > \mu^i_A$ when investors are underconfident. In addition, the figure shows the long, short, or non-participating positions investors take in each interval when p^i falls in that given interval.

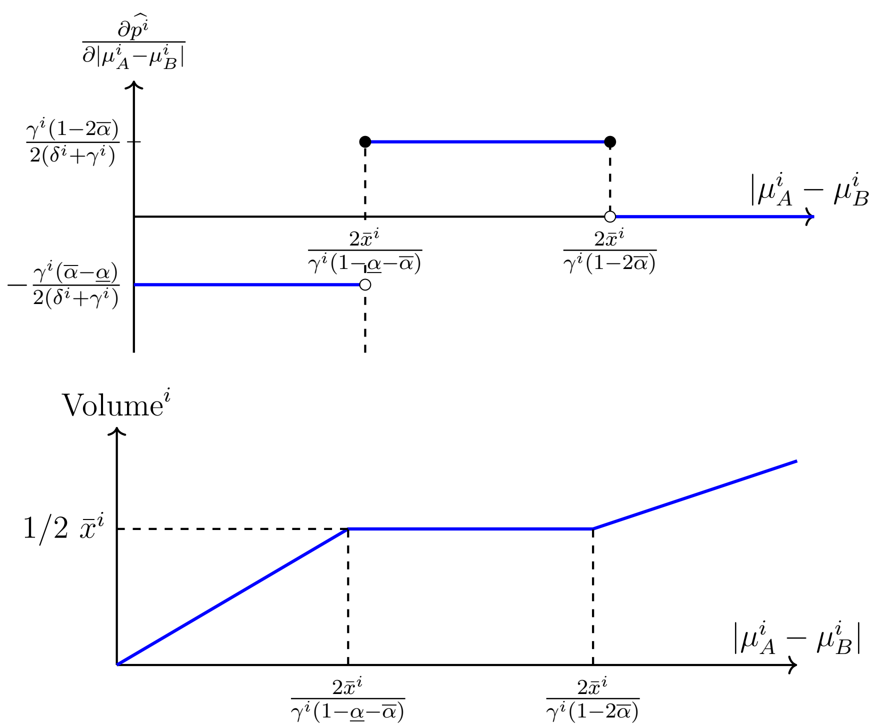


Figure 3.10. **Underconfident investors: disagreement, price, and volume.** This figure shows the relationship between risky asset i's price and disagreement when investors are underconfident. $\frac{\partial \widehat{p^i}}{\partial|\mu^i_A-\mu^i_B|}$ measures how price of risky asset i increases or decreases with respect to a unit increase in investor disagreement, holding the average interpretation of the public signal, $\frac{\mu^i_A+\mu^i_B}{2}$, constant. $|\mu^i_A - \mu^i_B|$ is disagreement for risky asset i.

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APPENDIX A

Proof of Proposition 1

The posterior beliefs of type i traders on X at $t=1$ are represented by

(A.1)
$$\begin{cases} X \sim N\left(\frac{Z_iX_i+(\frac{1-\alpha}{2-\alpha}\sigma^2+\sigma_\eta^2)^{-1}(S-\frac{(1-\alpha)\mu_i+\mu_{-i}}{2-\alpha})}{Z_i+(\frac{1-\alpha}{2-\alpha}\sigma^2+\sigma_\eta^2)^{-1}}, (Z_i+(\frac{1-\alpha}{2-\alpha}\sigma^2+\sigma_\eta^2)^{-1})^{-1}\right) & \text{if } \mu_{-i}-\mu_i>0 \\ X \sim N\left(\frac{Z_iX_i+(\frac{1+\alpha}{2+\alpha}\sigma^2+\sigma_\eta^2)^{-1}(S-\frac{(1+\alpha)\mu_i+\mu_{-i}}{2+\alpha})}{Z_i+(\frac{1+\alpha}{2+\alpha}\sigma^2+\sigma_\eta^2)^{-1}}, (Z_i+(\frac{1+\alpha}{2+\alpha}\sigma^2+\sigma_\eta^2)^{-1})^{-1}\right) & \text{if } \mu_{-i}-\mu_i<0 \end{cases}$$

Since $\sigma^2 << \sigma_\eta^2$, we have $\frac{1-\alpha}{2-\alpha}\sigma^2 < \frac{1+\alpha}{2+\alpha}\sigma^2 << \sigma_\eta^2$. Therefore, $(\frac{1+\alpha}{2+\alpha}\sigma^2+\sigma_\eta^2) \approx \sigma_\eta^2$ and $(\frac{1-\alpha}{2-\alpha}\sigma^2+\sigma_\eta^2) \approx \sigma_\eta^2$. Using the above properties, (20) is given by

$$\text{(A.2)} \qquad \begin{cases} X \sim N\left(\frac{Z_iX_i+\sigma_\eta^{-2}(S-\frac{(1-\alpha)\mu_i+\mu_{-i}}{2-\alpha})}{Z_i+\sigma_\eta^{-2}}, (Z_i+\sigma_\eta^{-2})^{-1}\right) & \text{if } \mu_{-i}-\mu_i>0 \\ X \sim N\left(\frac{Z_iX_i+\sigma_\eta^{-2}(S-\frac{(1+\alpha)\mu_i+\mu_{-i}}{2+\alpha})}{Z_i+\sigma_\eta^{-2}}, (Z_i+\sigma_\eta^{-2})^{-1}\right) & \text{if } \mu_{-i}-\mu_i<0. \end{cases}$$

The resulting demand for type i traders is given by

$$\text{(A.3)} \qquad \begin{cases} m_{i,1} = \left(\frac{Z_iX_i+\sigma_\eta^{-2}(S-\frac{(1-\alpha)\mu_i+\mu_{-i}}{2-\alpha})}{Z_i+\sigma_\eta^{-2}} - P_1\right)\frac{Z_i+\sigma_\eta^{-2}}{\lambda} & \text{if } \mu_{-i}-\mu_i>0 \\ m_{i,1} = \left(\frac{Z_iX_i+\sigma_\eta^{-2}(S-\frac{(1+\alpha)\mu_i+\mu_{-i}}{2+\alpha})}{Z_i+\sigma_\eta^{-2}} - P_1\right)\frac{Z_i+\sigma_\eta^{-2}}{\lambda} & \text{if } \mu_{-i}-\mu_i<0. \end{cases}$$

Using the market-clearing condition ($0.5m_{1,1}+0.5m_{2,1}=0$), the market-clearing price at $t=1$ is given by

$$\text{(A.4)} \qquad P_1^* = \frac{Z_1X_1+Z_2X_2+\sigma_\eta^{-2}\{(S-\mu_1)+(S-\mu_2)-\frac{2\alpha}{(2-\alpha)(2+\alpha)}|\mu_1-\mu_2|\}}{Z_1+Z_2+2\sigma_\eta^{-2}}.$$

Since X is revealed at $t=2$, $P_2^*=X$. Denote $R=P_2^*-P_1^*$. Then,

$$\text{(A.5)} \qquad R = X - \left(\frac{Z_1X_1+Z_2X_2+\sigma_\eta^{-2}\{(S-\mu_1)+(S-\mu_2)-\frac{2\alpha}{(2-\alpha)(2+\alpha)}|\mu_1-\mu_2|\}}{Z_1+Z_2+2\sigma_\eta^{-2}}\right)$$

Since $0 < \alpha < 1$, R is increasing in investor disagreement, $|\mu_1 - \mu_2|$, holding the average interpretation of the signal $(\frac{\mu_1+\mu_2}{2})$ fixed.

APPENDIX B

Proof of Proposition 2

From (1.5) and (1.6), we have the equilibrium holdings at $t = 0$, $m_{1,0}(P_0^*)$ and $m_{2,0}(P_0^*)$. From (A.3) and (A.4), we have the equilibrium holdings at $t = 1$, $m_{1,1}(P_1^*)$ and $m_{2,1}(P_1^*)$. Since there are only two types of traders in the market, the equilibrium trading volume from $t = 0$ to $t = 1$, $V_{0,1}^*$, is the absolute change in traders' aggregate equilibrium holdings from $t = 0$ to $t = 1$. That is,

$$V_{0,1}^* = |\frac{1}{2}m_{1,1}(P_1^*) - \frac{1}{2}m_{1,0}(P_0^*)| = |\frac{1}{2}m_{2,1}(P_1^*) - \frac{1}{2}m_{2,0}(P_0^*)|. \tag{B.1}$$

Next, define $\Delta P_{0,1}^* = (P_1^* - P_0^*)$. Using

$$\Delta P_{0,1}^* = P_1^* - P_0^* = \frac{2\sigma_\eta^{-2}(S - P_0^*) - \sigma_\eta^{-2}(\mu_1 + \mu_2 + \frac{2\alpha}{(2-\alpha)(2+\alpha)}|\mu_1 - \mu_2|)}{Z_1 + Z_2 + 2\sigma_\eta^{-2}}, \tag{B.2}$$

it is straightforward to show that

$$V_{0,1}^* = |A + B\Delta P_{0,1}^*|, \tag{B.3}$$

where

$$A = \frac{\sigma_\eta^{-2}}{4\lambda}\frac{\alpha^2}{(2-\alpha)(2+\alpha)}(\mu_2 - \mu_1), \tag{B.4}$$

and

$$B = \frac{1}{4\lambda}(Z_1 - Z_2). \tag{B.5}$$

APPENDIX C

Variable definitions

In this section, I define variables used in Chapter 2. Following Fama and French [1992], Fama and French [1993], and Davis et al. [2000], firm size (SIZE) for July of year t to June of t+1 is defined as the natural logarithm of market value of equity at the end of December of year $t-1$, and the book-to-market (BM) ratio from July of year t through June of year $t+1$, is computed as the shareholders' book value of equity plus deferred taxes and investment tax credit (if available) minus the book value of preferred stock at the end of the last fiscal year, $t-1$, divided by the market value of equity at the end of December of year $t-1$. Depending on availability, the redemption, liquidation, or par value is used to estimate the book value of preferred stock. Following Daniel and Titman [2006], the minimum 6-month lag is to ensure the firm's annual report is publicly available information.

Following Jegadeesh and Titman [1993], momentum (MOM) is computed as the cumulative return of a stock of 11 months ending one month prior to the given month. Following Jegadeesh [1990], short-term reversal (REV) is defined as the stock return over the portfolio formation month. Turnover ratio (TURN) is computed as the percentage of trading volume divided by the total number of shares outstanding shares over the portfolio formation month. A minimum of 15 daily observations in the given month is required to calculate TURN.

Following Amihud [2002], stock illiquidity for each stock in month m as the ratio of the absolute monthly stock return to its dollar trading volume, multiplied by 10^6:

$$ILLIQ_{i,m} = 10^6 \times Avg\left[\frac{|R_{i,d}|}{DTV_{i,d}}\right], \tag{C.1}$$

where $R_{i,d}$ and $DTV_{i,d}$ are the daily return and dollar trading volume for stock i on day d, respectively. A minimum of 15 daily observations in the given month is required to calculate ILLIQ.

Stock beta (BETA), is computed by regressing the stock's monthly excess return on monthly market excess return and lagged market excess return to accommodate non-synchronous trading effects:

$$R_{i,m} = \alpha_i + \beta_{i,1}R_{M,m} + \beta_{i,2}R_{M,m-1} + \epsilon_{i,m}, \tag{C.2}$$

where $R_{i,m}$ and $R_{M,m}$ are the monthly excess returns on stock i and the CRSP value-weighted market index, respectively. Following Fama and French [1992], I run the regression each month over a moving window covering the most recent 60 months, requiring at least 36 months of non-missing data. The stock's monthly beta is defined as $\widehat{\beta_{i,1}} + \widehat{\beta_{i,2}}$.

Following Bali et al. [2011] and Bali et al. [2017], demand for lottery-like stocks (MAX) is defined as the average of the five highest daily daily returns of the stock during the portfolio formation month. A minimum of 15 daily observations in the given month is required to calculate MAX.

Following Harvey and Siddique [2000], the co-skewness (COSKEW) of stock i in month m is defined as the estimated slope $\widehat{\gamma_{i,m}}$ in the following regression:

$$R_{i,m} = \alpha_i + \beta_i R_{M,m} + \gamma_i R_{M,m}^2 + \epsilon_{i,m}. \tag{C.3}$$

Similar to stock beta, regression are performed over a moving window covering the most recent 60 months, requiring at least 36 months of non-missing data.

Following Ang et al. [2006], the monthly idiosyncratic volatility of stock i (IVOL) is computed as the standard deviation of the daily residuals estimated from the following regression:

$$R_{i,d} = \alpha_i + \beta_i MKT_{M,d} + \gamma_i SMB_d + \phi_i HML_d + \gamma_i UMD_d \epsilon_{i,d}, \tag{C.4}$$

where $R_{i,d}$ and $MKT_{M,d}$ are the daily excess returns on stock i and the CRSP value-weighted market index, respectively. SMB_d and HML_d are the daily size and book-to-market factors of Fama and French [1996], respectively. UMD_d is the momentum factor.

Following Diether et al. [2002], Analyst forecast dispersion (DISP) is defined as the standard deviation of fiscal year one earnings forecasts scaled by the absolute value of the mean earnings forecast in a given month. To compute analyst forecast dispersion, each stock must be covered by two or more analysts during that month.

APPENDIX D

Proof of Proposition 3

To begin with, assume $\mu_B^i > \mu_A^i$. First, if at a price lower than $A_{min}^i = B_{min}^i$, demand for risky asset i for type A and type B investors are

$$x_A^{i^*} = (A_{min}^i - p^i)(\delta^i + \gamma^i) \text{ and } x_B^{i^*} = (B_{min}^i - p^i)(\delta^i + \gamma^i), \tag{D.1}$$

respectively. Using equation (10), the market-clearing price is given by

$$p^{i^*} = \frac{1}{2}(A_{min}^i + B_{min}^i - 2\frac{\bar{x}^i}{\delta^i + \gamma^i}) \tag{D.2}$$

Thus, p^{i^*} will be the equilibrium market-clearing price for risky asset i if $p^{i^*} \leq A_{min}^i = B_{min}^i$, which automatically holds since $\bar{x}^i > 0$.

Second, if at a price between $A_{min}^i = B_{min}^i$ and $A_{max}^i = B_{max}^i$, both types of investors won't participate in the market for risky asset i. Hence, no equilibrium exists in this case. Lastly, if at a price higher than $A_{max}^i = B_{max}^i$, both types of traders want to go short so no equilibrium exists.

Plug in A_{min}^i and B_{min}^i into equation (28), the market-clearing price is

$$p^{i^*} = \frac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i + \mu_B^i)}{2} - \frac{(\overline{\alpha^i} - \underline{\alpha^i})}{2}(\mu_B^i - \mu_A^i)]}{\delta^i + \gamma^i} - \frac{\bar{x}^i}{\delta^i + \gamma^i}. \tag{D.3}$$

Similarly, if $\mu_A^i > \mu_B^i$, then the market-clearing price is

$$p^{i^*} = \frac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i + \mu_B^i)}{2} - \frac{(\overline{\alpha^i} - \underline{\alpha^i})}{2}(\mu_A^i - \mu_B^i)]}{\delta^i + \gamma^i} - \frac{\bar{x}^i}{\delta^i + \gamma^i}. \tag{D.4}$$

Combining the two cases, we have

$$p^{i^*} = \frac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i + \mu_B^i)}{2} - \frac{(\overline{\alpha^i} - \underline{\alpha^i})}{2}|\mu_A^i - \mu_B^i|]}{\delta^i + \gamma^i} - \frac{\bar{x}^i}{\delta^i + \gamma^i}. \tag{D.5}$$

Since investors have same the same priors and posteriors on V_i, there's no trading activity.

APPENDIX E

Proof of Proposition 4

To begin with, assume $\mu^i_B > \mu^i_A$. Then,

(E.1) $$B^i_{min} < A^i_{min} < B^i_{max} < A^i_{max}.$$

First, if $p^{i^*} < B^i_{min}$, type A and type B investors' demands for risky asset i are

(E.2) $$x^{i^*}_A = (A^i_{min} - p^i)(\delta^i + \gamma^i) \text{ and } x^{i^*}_B = (B^i_{min} - p^i)(\delta^i + \gamma^i),$$

respectively. The market-clearing price is given by

(E.3) $$p^{i^*} = \frac{1}{2}(A^i_{min} + B^i_{min} - 2\frac{\bar{x}^i}{\delta^i + \gamma^i}).$$

Note that p^{i^*} will be the market-clearing price for risky asset i if $p^{i^*} < A^i_{min}$, which is equivalent to

(E.4) $$\gamma^i(\overline{\alpha^i} + \underline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) < 2\bar{x}^i.$$

Second, if $B^i_{min} \leq p^i < A^i_{min}$, only type A investors participate in the market as type B investors prefer zero position:

(E.5) $$x^{i^*}_A = (A^i_{min} - p^i)(\delta^i + \gamma^i),\ x^{i^*}_B = 0.$$

The market-clearing price is given by

(E.6) $$p^{i^*} = A^i_{min} - 2\frac{\bar{x}^i}{\delta^i + \gamma^i}$$

Note that p^{i^*} will be the market-clearing price for risky asset i if $B^i_{min} \leq p^i < A^i_{min}$, which is equivalent to

(E.7) $$\gamma^i(\overline{\alpha^i} + \underline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) \geq 2\bar{x}^i.$$

Third, if $A^i_{min} \leq p^i \leq B^i_{max}$, both types of investors will not participate in the market, so no equilibrium exists. Fourth, if $B^i_{max} < p^i \leq A^i_{max}$, only type B investors go short in risky asset i as type A investors prefer zero position. Hence, no equilibrium exists in this case. Lastly, if $p^i > A^i_{max}$, both types of investors go short so there's no equilibrium.

Based on the above analysis, when $\mu_B^i > \mu_A^i$, the market-clearing price is

$$(E.8)\qquad p^{i^*} = \begin{cases} \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i+\mu_B^i)}{2} - \frac{(\overline{\alpha^i}-\underline{\alpha^i})}{2}(\mu_B^i - \mu_A^i)]}{\delta^i+\gamma^i} - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, \\ \text{if } \gamma^i(\overline{\alpha^i}+\underline{\alpha^i}-1)(\mu_B^i-\mu_A^i) < 2\bar{x}^i \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i+\mu_B^i)}{2} - (\frac{1}{2}-\underline{\alpha^i})(\mu_B^i - \mu_A^i)]}{\delta^i+\gamma^i} - 2\dfrac{\bar{x}^i}{\delta^i+\gamma^i}, \\ \text{if } \gamma^i(\overline{\alpha^i}+\underline{\alpha^i}-1)(\mu_B^i-\mu_A^i) \geq 2\bar{x}^i. \end{cases}$$

Similarly, if $\mu_A^i > \mu_B^i$, then the market-clearing price is

$$(E.9)\qquad p^{i^*} = \begin{cases} \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i+\mu_B^i)}{2} - \frac{(\overline{\alpha^i}-\underline{\alpha^i})}{2}(\mu_A^i - \mu_B^i)]}{\delta^i+\gamma^i} - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, \\ \text{if } \gamma^i(\overline{\alpha^i}+\underline{\alpha^i}-1)(\mu_A^i-\mu_B^i) < 2\bar{x}^i \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i+\mu_B^i)}{2} - (\frac{1}{2}-\underline{\alpha^i})(\mu_A^i - \mu_B^i)]}{\delta^i+\gamma^i} - 2\dfrac{\bar{x}^i}{\delta^i+\gamma^i}, \\ \text{if } \gamma^i(\overline{\alpha^i}+\underline{\alpha^i}-1)(\mu_A^i-\mu_B^i) \geq 2\bar{x}^i. \end{cases}$$

Combining the two cases, we have
(E.10)

$$p^{i^*} = \begin{cases} \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i+\mu_B^i)}{2} - \frac{(\overline{\alpha^i}-\underline{\alpha^i})}{2}|\mu_A^i - \mu_B^i|]}{\delta^i+\gamma^i} - \dfrac{\bar{x}^i}{\delta^i+\gamma^i}, & \text{if } |\mu_A^i-\mu_B^i| < \frac{2\bar{x}^i}{\gamma^i(\overline{\alpha^i}+\underline{\alpha^i}-1)} \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu_A^i+\mu_B^i)}{2} - (\frac{1}{2}-\underline{\alpha^i})|\mu_A^i - \mu_B^i|]}{\delta^i+\gamma^i} - 2\dfrac{\bar{x}^i}{\delta^i+\gamma^i}, & \text{if } |\mu_A^i-\mu_B^i| \geq \frac{2\bar{x}^i}{\gamma^i(\overline{\alpha^i}+\underline{\alpha^i}-1)} \end{cases}$$

Trading volume can be computed accordingly using

$$(E.11)\qquad Volume^i = \frac{N}{2}|x_A^{i^*}(p^{i^*}) - \bar{x}^i| = \frac{N}{2}|x_B^{i^*}(p^{i^*}) - \bar{x}^i|.$$

APPENDIX F

Proof of Proposition 6

To begin with, assume $\mu_B^i > \mu_A^i$.Then,

$$B_{min}^i < B_{max}^i < A_{min}^i < A_{max}^i. \tag{F.1}$$

First, if $p^{i^*} < B_{min}^i$, type A and type B investors' demands for risky asset i are

$$x_A^{i^*} = (A_{min}^i - p^i)(\delta^i + \gamma^i) \text{ and } x_B^{i^*} = (B_{min}^i - p^i)(\delta^i + \gamma^i), \tag{F.2}$$

respectively. The market-clearing price is given by

$$p^{i^*} = \frac{1}{2}(A_{min}^i + B_{min}^i - 2\frac{\bar{x}^i}{\delta^i + \gamma^i}). \tag{F.3}$$

Note that p^{i^*} will be the market-clearing price for risky asset i if $p^{i^*} < B_{min}^i$, which is equivalent to

$$\gamma^i(\underline{\alpha^i} + \overline{\alpha^i} - 1)(\mu_B^i - \mu_A^i) < 2\bar{x}^i. \tag{F.4}$$

Second, if $B_{min}^i \leq p^i \leq B_{max}^i$, only type A investors participate in the market as type B investors prefer zero position:

$$x_A^{i^*} = (A_{min}^i - p^i)(\delta^i + \gamma^i),\ x_B^{i^*} = 0. \tag{F.5}$$

The market-clearing price is given by

$$p^{i^*} = A_{min}^i - 2\frac{\bar{x}^i}{\delta^i + \gamma^i}. \tag{F.6}$$

Note that p^{i^*} will be the market-clearing price for risky asset i if $B_{min}^i \leq p^i < B_{max}^i$, which is equivalent to

$$\gamma^i(2\underline{\alpha^i} - 1)(\mu_B^i - \mu_A^i) \leq 2\bar{x}^i \leq \gamma^i(\underline{\alpha^i} + \overline{\alpha^i} - 1)(\mu_B^i - \mu_A^i). \tag{F.7}$$

Third, if $B_{max}^i < p^i < A_{min}^i$, type A and type B investors' demands for risky asset i are

$$x_A^{i^*} = (A_{min}^i - p^i)(\delta^i + \gamma^i) \text{ and } x_B^{i^*} = (B_{max}^i - p^i)(\delta^i + \gamma^i), \tag{F.8}$$

respectively. The market-clearing price is given by

$$p^{i^*} = \frac{1}{2}(A^i_{min} + B^i_{max} - 2\frac{\bar{x}^i}{\delta^i + \gamma^i}). \tag{F.9}$$

Note that p^{i^*} will be the market-clearing price for risky asset i if $B^i_{max} < p^i < A^i_{min}$, which is equivalent to

$$\gamma^i(2\underline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) > 2\bar{x}^i. \tag{F.10}$$

Fourth, if $A^i_{min} < p^i \leq A^i_{max}$, only type B investors go short in risky asset i in the market as type A investors prefer zero position. Hence, no equilibrium exists in this case. Lastly, if $p^i > A^i_{max}$, both types of investors want to go short in risky asset i so no equilibrium exists. Based on the above analysis, when $\mu^i_B > \mu^i_A$, the market-clearing price is

$$p^{i^*} = \begin{cases} \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2}]}{\delta^i + \gamma^i} - \dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ 2\bar{x}^i < \gamma^i(2\underline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2} + (\underline{\alpha}^i - \frac{1}{2})(\mu^i_B - \mu^i_A)]}{\delta^i + \gamma^i} - 2\dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ \gamma^i(2\underline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) \leq 2\bar{x}^i \leq \gamma^i(\underline{\alpha}^i + \overline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2} - \frac{(\overline{\alpha}^i - \underline{\alpha}^i)}{2}(\mu^i_B - \mu^i_A)]}{\delta^i + \gamma^i} - \dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ 2\bar{x}^i > \gamma^i(\underline{\alpha}^i + \overline{\alpha}^i - 1)(\mu^i_B - \mu^i_A) \end{cases} \tag{F.11}$$

Similarly, if $\mu^i_A > \mu^i_B$, then the market-clearing price is

$$p^{i^*} = \begin{cases} \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2}]}{\delta^i + \gamma^i} - \dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ 2\bar{x}^i < \gamma^i(2\underline{\alpha}^i - 1)(\mu^i_A - \mu^i_B) \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2} + (\underline{\alpha}^i - \frac{1}{2})(\mu^i_A - \mu^i_B)]}{\delta^i + \gamma^i} - 2\dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ \gamma^i(2\underline{\alpha}^i - 1)(\mu^i_A - \mu^i_B) \leq 2\bar{x}^i \leq \gamma^i(\underline{\alpha}^i + \overline{\alpha}^i - 1)(\mu^i_A - \mu^i_B) \\ \dfrac{\delta^i v^i + \gamma^i[S^i - \frac{(\mu^i_A + \mu^i_B)}{2} - \frac{(\overline{\alpha}^i - \underline{\alpha}^i)}{2}(\mu^i_A - \mu^i_B)]}{\delta^i + \gamma^i} - \dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ 2\bar{x}^i > \gamma^i(\underline{\alpha}^i + \overline{\alpha}^i - 1)(\mu^i_A - \mu^i_B) \end{cases} \tag{F.12}$$

Combining the two cases, we have
(F.13)

$$p^{i^*} = \begin{cases} \dfrac{\delta^i v^i + \gamma^i [S^i - \frac{(\mu_A^i + \mu_B^i)}{2}]}{\delta^i + \gamma^i} - \dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \text{ if } \ |\mu_A^i - \mu_B^i| > \dfrac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}^i - 1)} \\ \dfrac{\delta^i v^i + \gamma^i [S^i - \frac{(\mu_A^i + \mu_B^i)}{2} + (\underline{\alpha}^i - \frac{1}{2})|\mu_A^i - \mu_B^i|]}{\delta^i + \gamma^i} - 2\dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \\ \text{if } \ \frac{2\bar{x}^i}{\gamma^i(\underline{\alpha}^i + \overline{\alpha}^i - 1)} \leq |\mu_A^i - \mu_B^i| \leq \frac{2\bar{x}^i}{\gamma^i(2\underline{\alpha}^i - 1)} \\ \dfrac{\delta^i v^i + \gamma^i [S^i - \frac{(\mu_A^i + \mu_B^i)}{2} - \frac{(\overline{\alpha}^i - \underline{\alpha}^i)}{2}|\mu_A^i - \mu_B^i|]}{\delta^i + \gamma^i} - \dfrac{\bar{x}^i}{\delta^i + \gamma^i}, \text{ if } \ |\mu_A^i - \mu_B^i| < \dfrac{2\bar{x}^i}{\gamma^i(\underline{\alpha}^i + \overline{\alpha}^i - 1)} \end{cases}$$

Trading volume can be computed accordingly using

(F.14) $$Volume^i = \frac{N}{2}|x_A^{i^*}(p^{i^*}) - \bar{x}^i| = \frac{N}{2}|x_B^{i^*}(p^{i^*}) - \bar{x}^i|.$$