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External Quality Cost Sharing Contracts in Supply Chains

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ABSTRACT

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The most important managerial criteria in supply chains are how to manage product, information and cash flows, and how to maximize profits by either increasing the revenue or decreasing the costs. Although the maximum benefits can be achieved if everyone follows the central planner's suggestions; unfortunately, the individual maximum profits may not be guaranteed. Thus, how to regulate the members actions is an interesting topic and a relevant research. Designing contract is a way to control the members actions.

We contribute to the research on the quality cost sharing contacts in several dimensions. First, we expand the definition and modeling of quality on a decentralized supply chain to include product failures resulting from design related imperfections. Secondly, we investigate a larger set of external quality cost sharing contracts than what has been studied in the previous literature. We also discuss how to prepare the right contracts in order to avoid the inefficiency of asymmetrical information. Thirdly, we investigate the profitability under the market competition, quality improvement and the external quality cost sharing contract. We find that the optimal quality improvement effort levels are independent from the sale prices and market competition. No matter in the monopoly market or duopoly market, the manufacturer had better adopt the cost sharing contract with the selective root cause analysis in order to increase profits and the competency. Otherwise, his market share is diminished and his profits are eroded.

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Chapter 1

Introduction

As early as 1958, Forrester proposed that all entities over the supply chain should be viewed as an integrated system. Due to the physical distance and the lengthy delivery, it sounds utopian to interrelate the flows of information, materials, manpower, money, and capital equipment throughout the supply chain. With the advanced information technology and the prevalence of the Internet, the distances among the members on the supply chain have been diminished, and information can be interchanged instantaneously. The research on supply chain management has grown intensively since late 1990. Supply chain management is a colossal subject including many disciplines, and it also employs various quantitative and qualitative tools. Here we divide the research on supply chain management into four categories:

- *Network Design*: facility location, transportation and logistics design and optimization, reverse logistics.
- *Member Collaboration*:

  Upward: collaborative planning, forecasting, and replenishment (CPFR), product design and introduction, production reengineering, supplier and sourcing management.

  Downward: marketing and channel restructuring, revenue and cost sharing, inventory management, customer relationship management (CRM), service and after sale support.
- *Information technology*: electronic data interchange (EDI), business application integration

(EAI), enterprise resource planning (ERP), network and database development, radio frequency identification (RFID).

- *Globalization*: culture, duties, tariffs, legal issues.

The majority of research focuses on how to manage the flows over the supply chain, in terms of money, material, and information. The most important and obvious managerial criteria are how to manage those flows, and how to maximize profits by either increasing the revenue or decreasing the costs. Then the optimal solutions can be used as the guidelines for each member on the supply chain. Although the maximum benefits can be achieved throughout the supply chain if everyone follows the suggestions; unfortunately, the individual maximum profits may not be guaranteed. Thus, how to regulate the members actions is an interesting and relevant topic. Design of contracts is a way to control the members actions. The contract between the suppliers and the retailers deals with how to share the risks for the potential stockout or overstock, how to satisfy all the potential customer demands as much as possible, and how to distribute the total profits on the supply chain.

In an effort to improve production efficiency and product quality, firms have begun to look beyond their own inbound processes and reconsider the overall design of their supply chains (Baiman et al. 2001). A consequence of this move is that firms are outsourcing more of their part and component design and production to other members of their supply chain. Both the automotive and computer industries are leaders in this movement and have come to embrace outsourcing as strategic decision to strengthen a firms competitive advantage. For example, the US automotive industry has steadily increased the number of parts outsourced, while decreasing the number of suppliers. In the computer industry it is not unusual for a computer manufacturer to do only the design and assembly, allocating production to contract manufacturers (Baiman et al. 2001).

As a consequence of the afore mentioned, today the quality of a firm's product has become more dependent not only on its own process capabilities, but also those of its suppliers. *In this dissertation, we investigate the implications of contractual relationships between a supplier and a manufacturer with and without market competition. We also focus on final product quality when the* *quality improvement effort levels are determined by the both parties.* More specifically, we research incentive alignment issues associated with quality improvements in decentralized supply chains.

In his seminal work, Garvin (1988) defined product quality on eight dimensions: Performance, features, serviceability, aesthetics, perceived quality, reliability, conformance, and durability. He related the first five dimensions to the quality of product design, and the latter three to the quality of the production process. We also discuss those two categories in this dissertation. *First, we start by discussing contractual arrangements to coordinate a supplier's and a manufacturer's incentives to improve the design quality of a product. Later, we expand our analysis to quality problems, which may also arise from manufacturing related process imperfections.*

Ensuring high product quality can be a costly exercise for a company. For instance, in 2005, Ford Motor Company reported that its quality-related costs have increased by $500 million in 2005 relative to its quality costs of 2004 (AutoInsider 2005). Operations management literature defines four categories of quality costs (Montgomery 2001): prevention costs, appraisal costs, internal failure costs and external failure costs. Table 1 provides a detailed description of each cost group. *In this dissertation, we particularly discuss the impact of quality improvement investments on a firm's external failure costs.* In chapter 2, we model a product recall context and discuss ways in which the recall costs can be reduced by the design of proper external quality cost sharing contracts between a manufacturer and a supplier. In the following chapters, we expand our analysis to warranty related external quality costs.

A growing literature in operations management discusses the design of quality cost sharing contracts between manufacturers and suppliers. The main focus of this group of papers is on characterization of quality improvement incentive distortions resulting from information asymmetry between the suppliers and the manufacturers which leads to adverse selection and/or moral hazard problems on the manufacturer's and/or the supplier's side. Reyniers and Tapiero (1995a and 1995b) are ones of the early pioneers who model supplier's choice of process quality and manufacturer's choice of inspection strategy in a game theoretic setup. The model characterizes the Nash equilibrium of the

Table 1.1: The Quality Cost

Cost of Control	**Cost of Failure to Control**
Prevention Cost	**Internal Failure Costs**
Quality Planning and Engineering	Scrap
New Product Review	Rework
Product/Process Design	Retest
Process Control	Failure Analysis
Burn-in	Downtime
Training	Yield Losses
Appraisal Costs	**External Failure Costs**
Inspection and testing of incoming material	Complaint Adjustment
Product inspection and text	Returned product/material
Materials and services consumed	Warranty Charges
Maintaining Accuracy of test equipment	Liability Costs
	Indirect Costs

supplier-manufacturer quality game in terms of the cost share mechanisms for internal (incurring rework cost) and external (incurring warranty costs) product failures. They assume a fixed share rate of external failure costs between the parties for the failure. Lim (2001) also develops the optimal contract on the price rebate for incoming part inspection and the share rate for the warranty cost when there is asymmetric information about the part quality. She uses the revelation principle to obtain the equilibrium for the incomplete information. She shows that the full-price rebate is not possible and the supplier should share the damage costs with the manufacturer. Baiman et al. (2000) analyzes the relationship between product quality, cost of quality, and the information that can be contracted on. In a risk neutral setting, the supplier invests in reducing process defect rate and the manufacturer invests in inspection quality of the incoming part. Both the manufacturer's and supplier's decisions are subject to moral hazard. In a subsequent paper (Baiman et al. 2001), authors investigate the linkage between product design choices, the contractible information and investment in process quality. Two product architectures are compared in terms of their incentive implications, separable and non-separable product architecture. The difference between the two is that the separable architecture enables allocation of external failure costs to the responsible party; whereas, in case of the non-separable architecture, the external failures are shared between the two parties irrespective of the responsibility. They argue that the ability to assign the external failure cost to the responsible party ensures first best quality improvement effort levels from the manufacturer and from the supplier. In a recent paper, Baiman et al. (2004) also considered a product structure that exhibited the weakest link property, and investigated how the internal and external failure cost shared mechanism impact supplier selection in the existence of an adverse selection problem. They considered moral hazard only on the supplier side and assumes that the manufacturer cannot identify the responsible supplier for a particular failure, or it is too costly to do so.

We contribute to the extant research on this topic in several dimensions. First, we expand the definition and modeling of quality on a decentralized supply chain to include product failures resulting from design related imperfections. To this end, unlike the previous research, we model quality as the survival likelihood of a product under various consumer usage patterns. Secondly, we investigate a larger set of external quality cost sharing contracts than what has been studied in the previous literature. Particularly, we discuss the role and the use of root-cause analysis information while allocating responsibility for external quality costs among supply chain members (Chapter 2). We also discuss how to prepare the right contracts in order to avoid the inefficiency of asymmetrical information. The pooled contract and the menu of contracts will be used to retain the advantages of applying the root cause analysis. Thirdly, we investigate the profitability under the market competition, quality improvement and the external cost-sharing contract. The optimal quality improvement effort levels are independent from the sale prices, whether it is in the monopoly market (Chapter 3) and with market competition (Chapter 4). No matter in the monopoly market or duopoly market, the manufacturer had better adopt the cost-sharing contract in order to increase the profits and the competency. Otherwise, his market share is diminished and his profits are eroded.

In the following chapters, we will discuss the setup and the design of the external cost-sharing contracts in Chapter 2, the pricing strategy and external cost-sharing contract in Chapter 3, the pricing strategy, the manufacturer's movement and the external cost-sharing contract in the market competition in Chapter 4. Finally, we will portrait the potential research works for future development in Chapter 5.

Chapter 2

Quality Improvement Incentives and Product Recall Cost Sharing Contracts

ABSTRACT

As companies outsource more product design and manufacturing activities to other members of the supply chain, improving end-product quality has become an endeavor extending beyond the boundaries of the firms' in-house process capabilities. In this paper, we discuss two contractual agreements by which product recall costs can be shared between a manufacturer and a supplier to induce quality improvement effort. More specifically, we consider (*i*) cost sharing based on selective root cause analysis (Contract S), and (*ii*) partial cost sharing based on complete root cause analysis (Contract P). Using insights from supermodular game theory, for each contractual agreement, we characterize the levels of effort the manufacturer and the supplier would exert in equilibrium to improve their component failure rate when their effort choices are subject to double moral hazard. We show that both Contract S and Contract P can achieve the First Best effort levels; however, Contract S results in higher profits for the supply chain. For the case in which the information about the quality of the supplier product is not revealed to the manufacturer (i.e., the case of information asymmetry), we develop a menu of contracts that can be used to mitigate the impact of information asymmetry. We show that, the menu of contracts not only significantly decreases the manufacturer's cost due to information asymmetry, but also improves product quality.

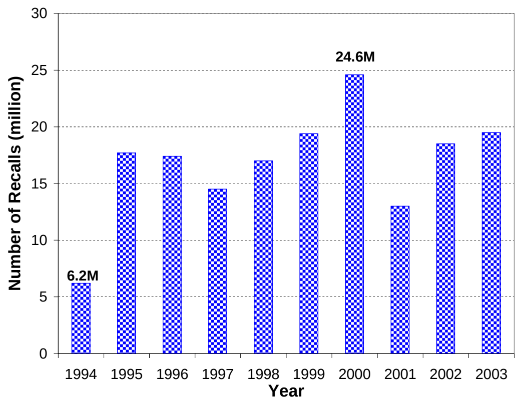
2.1 Introduction

In 2004, a consumer research study in the auto industry reported initial product quality as the second most important factor affecting consumers' purchasing decision after product price (J.D. Power and Associates 2004).[1] The number of product recalls (see Figure 1) and lawsuits in the auto industry demonstrate how undetected quality problems and related production delays can lead to a huge profit loss and be degrading to a company's brand equity. For instance, in 2007, Ford's concerns about a design related quality problem in cruise control switches resulted in a recall of 3.6 million vehicles manufactured between the years 1992 and 2004, increasing the total number of vehicles recalled for the same quality problem to 9.6 million. In the electronics industry, in April 2007, Sanyo agreed to share with the manufacturer Lenovo, the $17 million cost of recalling 205,000 Sanyo made laptop battery packs that can overheat because of a flaw in the product design process. In May 2007, the consumer product safety commission, the National Highway Traffic Safety Administration and Evenflo Company Inc. announced a recall of Evenflo Embrace Infant car seat/carriers because of a malfunctioning handle. A total of 450,000 units, manufactured in the United States and China, were sold nationwide through department stores and baby items stores (http://topics.cnn.com/topics/product_recalls).

These are just a few examples which demonstrate that recalls are common in a variety of industries and often are associated with substantial present and future costs to a company. The cost and scale of recalls necessitate a deeper understanding of how to manage the quality improvement incentives of multiple supply chain partners to ensure better end product performance.

Product recalls result from a lack of quality assurance in the manufacturing and/or design processes of one or many supply chain partners and can affect a large number of products manufactured over extended periods of time. *In this paper, our goal is to develop a modeling framework to capture these critical aspects of recalls and investigate the effect of recall cost sharing schemes on quality im-*

[1] Initial quality is measured by the number quality problems manifesting themselves the first 90 days after the purchase of the product.



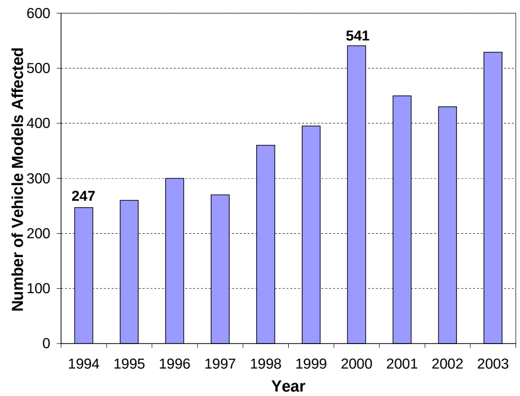


Figure 2.1: Recall Costs in The Automotive Industry (Source: Detroit News and NHSA)

provement incentives as well as on supply chain profit and end product quality. In what follows, we will be referring to recall instances and cost sharing agreements from the automotive industry. Our insights, however, apply to other product categories such as home appliances, children's products, medical devices and electronics.

As companies outsource more product design and manufacturing activities to other members of the supply chain, improving end-product quality in order to avoid product recalls has become an endeavor extending beyond the boundaries of firms' in-house manufacturing capabilities. In a recent study, Ford reported that 76 percent of the company's quality problems stem from its Tier 1 suppliers (Sherefkin 2002).

Today, extended quality improvement efforts take various forms. For instance, manufacturers in the auto industry are more willing to involve suppliers during the product development process to ensure early detection and elimination of quality problems. In addition to preventive initiatives, it is also becoming a common practice among manufacturers to present suppliers with quality cost sharing agreements to ensure accountability of quality problems and to create incentives for process improvement (Balachandran and Radhakrishnan 2005). In this paper, we address the optimal design of a recall cost sharing contract when both the supplier's and the manufacturer's quality improvement efforts are subject to moral hazard and when the manufacturer has uncertainty regarding the quality of supplier's process. In this context, we discuss the optimal use of product failure root cause analysis information in the design of cost sharing schemes.

In this paper, we introduce two contract formats to share the recall costs in the supply chain: (i) the *Selective Root Cause Analysis Contract* (Contract S) which is characterized by a unit part price (p), a fixed recall cost share rate (R) paid by the supplier,[2] and a threshold product failure time ($\overline{T}$), and (ii) the *Partial Cost Allocation Contract* (Contract P) which is characterized by a unit part price (p), a fixed cost share rate (R_m) paid by the supplier if the manufacturer is responsible for the product failure, and a fixed cost share rate (R_s) paid by the supplier if the supplier herself is responsible for the product failure. A critical component of both contract formats is the *root cause analysis information* which reveals the supply chain member who is responsible for the quality problem in the product. Contract S uses this information only if the product failure occurs before a time ($\overline{T}$), which we will refer to as the "root cause analysis threshold," and allocates the total recall cost to the party at fault. Otherwise, the cost is shared according to a fixed rate (R). Under Contract P, root cause analysis information is *always* used in the cost allocation process, and the supply chain member who is responsible for the recall incurs a larger share of the total cost.

Considering a single-manufacturer, single-supplier supply chain structure, we address the following research questions regarding these contractual agreements:

- How effective are Selective Root Cause Analysis (Contract S) and Partial Cost Allocation Contract (Contract P) in coordinating the manufacturer's and supplier's quality improvement efforts, when the effort levels are not observable and therefore are subject to moral hazard? Which contract format achieves higher profits for the manufacturer? How do these contracts affect the quality of the final product?

- How do Contract S and Contract P compare with the cost sharing contracts previously discussed in the literature (e.g., the Fixed Share Rate Contract) in terms of profits and final

[2]The supplier shares R percentage of the total recall cost, while the manufacturer pays the remaining $(1 - R)$ percentage.

product quality?

- If the exact information about the supplier's product quality is not available, can the manufacturer use a menu of Selective Root Cause Analysis Contracts to screen supplier type as well as to induce quality improvement effort? Under what circumstances does knowing the information result in significant savings for the manufacturer? How much does the quality of the final product under the menu of contracts differ from that under perfect information (i.e., when the manufacturer knows the exact quality of supplier's product)?

In the following section, we present the contribution of our paper in the context of the existing literature on quality and supply chain management. Then, we present the basic modeling framework and the assumptions of our model in section 2.3. Section 2.4 develops a detailed analysis of Contract S and Contract P under the complete information assumption. Section 2.5 investigates the case of information asymmetry and discusses the optimal menu of Selective Root Cause Analysis Contracts. Section 2.6 presents a numerical study to compare the cost-efficiency and product quality under different contracts in cases with information asymmetry. A summary of our findings and a list of future research directions are presented in Section 2.7.

2.2 Literature

The main focus of this research is on modeling the process improvement incentives of supply chain members when their effort choices are not observable and there is information asymmetry. This paper contributes to several streams of research each of which we review below.

In operations management, a group of papers discuss the design of quality cost sharing contracts among manufacturers and suppliers. In a game theoretic set-up, Reyniers and Tapiero (1995a and 1995b) and Lim (2001) model supplier's choice of process quality and manufacturer's choice of inspection strategy. Their model characterizes the Nash equilibrium of the supplier-manufacturer quality game in terms of the cost sharing parameters for internal (rework) and external (warranty) quality costs, assuming a fixed rate for sharing external quality costs between the parties. We, however, model a more general contract format for sharing external quality costs resulting from a recall. A special case of our contract of interest is the Fixed Share Rate Contract studied in the above papers.

Baiman et al. (2000) analyze the relationship between product quality, cost of quality and the information that can be contracted on. In a risk neutral setting, the supplier invests in reducing the process defect rate and the manufacturer invests in the inspection quality of the incoming part. Both decisions are subject to moral hazard. Like Reyniers and Tapiero (1995a and 1995b) and Lim (2001), they assume that the external quality costs are shared at a fixed share rate. In a subsequent paper (Baiman et al. 2001), the authors investigate the link between product design, contractible information and the supplier's investment in process quality. Two product designs are compared in terms of their incentive implications: separable and non-separable product architectures. The separable architecture enables the manufacturer to perfectly identify the part that led to product failure. Thus, individual responsibility can be allocated to supply chain members. In contrast, in non-separable product architecture, the cause of product failure cannot be traced back to a particular part/component. Therefore, the cost of external failures is shared by a Fixed Share Rate Contract irrespective of who the responsible party is. They argue that the ability to assign the external failure cost to the responsible party ensures the first best quality improvement effort levels from the manufacturer and from the supplier. In contrast, in this paper, we focus on a broader set of contract formats to share external product failure costs and show that, even though the root cause analysis can perfectly determine the party responsible for product failure, it is not optimal for the supply chain to share quality costs based on this information for all failures occurring during the contract period. We propose a contract with selective root cause analysis which differentiates early failures from late failures in order to coordinate the quality improvement efforts of supply chain members.

In a subsequent paper, Baiman et al. (2003) examine a product structure exhibiting the weakest link property and investigate how the internal and external failure cost sharing mechanisms impact supplier selection when there is an adverse selection problem. The analysis considers moral hazard only on the supplier side, while we focus on moral hazard both on the manufacturer and on the supplier side. Furthermore, like previously cited work, their analysis assumes that the external quality costs are shared at a fixed rate, which is in fact a special case of the contract we investigate.

Balachandran and Radhakrishnan (2005) consider a double moral hazard situation in quality investment effort, in which the final product consists of components made by a buyer and a supplier. While their paper focuses on the best use of incoming *inspection information* to achieve the First Best effort levels from the supply chain partners, in this paper we investigate the best use of *root cause analysis information* about external failures to achieve First Best effort levels from supply chain members. Furthermore, the authors model the Fixed Share Rate Contract for allocating the costs of *internal failures* while we consider a more general contracting arrangement for *external failures.*

In a recent paper, Zhu et al. (2007) look at a buyer who designs a product and owns the brand, yet outsources the production to a supplier. Both the buyer and the supplier incur quality related costs that are shared by a Fixed Share Rate Contract. Their model captures the effect of the buyer's involvement in ensuring product quality. They also endogenously model the effect of operational decisions such as buyer's ordering quantity and supplier's production lot size. Unlike Zhu et al., we look at a setting where the manufacturer is involved in the production process and his effort affects the final quality of the product and discuss two new contract formats to share external quality costs.

A related supply chain management paper by Corbett and Decroix (2001) discusses the use of a shared savings contract (assuming a fixed share rate between a supplier and a buyer) to induce the effort to reduce indirect material consumption. While the modeling of effort in our paper has some similarity to their modeling constructs, we investigate the use of contractual formats to share external quality costs resulting from a recall rather than the cost of indirect materials.

Based on data from the automotive and the pharmaceutical industries, a number of political economy research papers investigate the real total cost of a recall for a manufacturer. For instance, Jarrell and Peltzman (1985), Barber and Darrough (1996) and Rupp (2004) study of the cost attributes of recalls in the US automotive industry and find that the indirect costs such as brand equity loss, consumer goodwill loss and loss in firm value are in fact much larger than the direct costs of a recall such as product collection and repair cost. The findings of this stream of empirical research serve as a basis for some of our assumptions regarding the manufacturer's unit recall cost.

In summary, this paper introduces two new contractual formats for sharing the external quality costs associated with product recalls; in particular, we focus on the best use of root cause analysis information and its impact on the quality of the final product both under *complete* and *asymmetric* information assumptions. In this respect, our findings enrich the growing literature in this area and help managers to better understand the *cost-efficiency* of these contractual agreements and their impact on *product quality.*

2.3 Modeling Framework

To investigate the impact of external quality cost sharing on manufacturer's and supplier's quality improvement efforts and on supply chain profits, we look at a manufacturer[3] who produces a product that consists of two components, one of which he procures from a single supplier at a unit price p. The manufacturer procures a total of M components from the supplier and uses them to manufacture M units of the good to be sold in the market. The product generates a unit revenue of r for the manufacturer. We denote the unit production cost at the manufacturer by u_m, and at the supplier by u_s. At the time of contracting, both the manufacturer and the supplier know M. For example, when an auto manufacturer discusses a contract for, say the oil pump used in the engine of a particular 2008 car model, he has a good estimate of how many of those model cars are planned for production in year 2008. If M represents this number, then the quality cost sharing contract covers M components.

[3] In the rest of the chapter, the manufacturer will be referred to as "he" and the supplier as "she."

After the product is sold to the customer, during its useful life, it can fail to perform its function if any one of its components fails to do so. Baiman et al. (2003) define this product failure behavior as the weakest link property. Component failure can result either from a design or a manufacturing related problem in the supplier's or the manufacturer's process. As an example, in 2002, according to the part returns data of a large North American car company, 25% of external quality problems were found to be design related, but only 15% and 21% of the problems were found to be related to the manufacturer's assembly and supplier's manufacturing processes, respectively. The manufacturing/assembly related external quality problems are often easily fixed by reinstalling a non-defective component in the product. On the other hand, design related external failures are more costly to manufacturers since design flaws often affect multiple product generations (models) and reveal themselves only after the product has been in use with the customer for a period of time. Relative to manufacturing related quality problems, design flaws are more costly to resolve as they may require redesigning multiple components and their interfaces in a product. Root cause analysis bears particular importance for design related recalls since the flawed design decisions may not be readily obvious to supply chain partners. *Given the high cost and challenges in resolving design related quality problems, this paper discusses contracts to share external quality costs, namely recall cost, so as to improve product design failure rates.*

Current literature defines product quality as the likelihood of producing a non-defective unit either from the manufacturer's or from the supplier's process. This way of modeling product quality is more relevant to manufacturing related defects, that exist at the time of product purchase. We consider quality problems which unveil themselves during product usage and which are due to unanticipated and undetected modes of design failures. Therefore, we model quality as *the survival likelihood of the product design subject to varying modes of usage during the useful life of the product.* Ex-ante to procurement and production, the manufacturer and the supplier can choose to exert costly effort to reduce their component's design failure rate by carefully testing the design characteristics. In practice, the process of eliminating the many ways in which a design failure can occur is called

"Failure Mode and Effect Analysis" (FMEA) and is performed by the manufacturer and/or the supplier during the product development stage prior to manufacturing (Stamatis 2004).

A1: We assume that M products are manufactured and sold, and either are with the customers, or are in the distribution channel when a recall is issued. Once a particular problem has revealed itself, and the recall is issued, the manufacturer fixes the particular quality problem in all M products.

A recall can be initiated either by a manufacturer or a governmental agency if there is enough evidence that the quality issue is in fact due to a manufacturing or a design problem rather than a consumer usage problem. In most cases, the collection of product failure data, its preliminary investigation and the decision to issue a recall occurs long after products are sold. In a recent study of the automotive recall data between 1978-1998, Rupp and Taylor (2002) report that the initial investigation stage takes on average 140 days, which is followed by vehicle testing if the recall is initiated by the National Highway Safety Association (NHSA). The testing stage can take up to a year to complete (357 days). From the latest vehicle recall data published by the NHSA, we also find that the average time for a recall of a vehicle model is 3 years while in 80 % of the cases, the recalls are for 1+ year old models. In other words, a recall for a 2008 car model is expected to occur in 2011. By that time, most of the 2008 models (i.e., most of M cars) have been sold and new 2009 and 2010 models have been introduced. This is consistent with our Assumption A.1.

A2: We denote the recall cost *per unit* as ω and assume it to be independent of the root cause of the quality problem. At the time of contracting, both the manufacturer and the supplier agree on an estimate of ω.

Note that there are three major cost components that constitute ω : (i) the unit direct cost of fixing the quality problem in the product upon recall (ω_d), (ii) the unit indirect overhead cost associated with handling a recall process (ω_o), and (iii) the unit indirect cost due to loss of goodwill and firm value (ω_l). (See Baiman et al. 2001 for a similar definition of external quality costs). While ω_d

can *sometimes* be root cause dependent, current research (Jarrell and Peltzman 1985, Barber and Darrough 1996, Rupp 2004) shows that the indirect costs of a recall (in the form of overhead ω_o and goodwill loss ω_l) are independent of the root cause of the problem, and in fact are much larger than the direct costs such as repair or replacement costs. Hence, the assumption that ω $(= \omega_d + \omega_0 + \omega_l)$ is independent of the root cause of a recall is reasonable in many cases. Also note that when the manufacturer and the supplier sign the contract, the general type of quality problem that may result in a recall is often known (e.g., the failure in the air bag system when the supplier provides airbag systems). This allows the parties to get an estimate for the overhead cost ω_0 and the loss of goodwill cost ω_l. However, until the failure occurs and root cause analysis is performed, the nature of the type of repair operations needed to fix the problem is not clear. Therefore, when negotiating a contract, the manufacturer and the supplier usually use an estimate for ω_d (independent of the root cause of the recall, which is unknown at the time when the contract is signed).

In practice, unit recall cost can range between hundreds and thousands of dollars depending on the complexity of the product and the quality problem. In the auto industry, the total average cost of a recall varies between \$5M to \$20M, excluding customer goodwill loss and any liabilities (Sherefkin et al. 2003).

A3: We denote the *unit cost* of root cause analysis by $c_r = \frac{C_r}{M}$, where C_r is the relevant fixed cost of root cause analysis and M is the total number of products subject to a recall. We assume that the root cause analysis perfectly identifies the component that caused the quality problem.

When a recall is initiated, the root cause analysis is performed on failed products to identify *which* component fell short of performing its intended function, and *whose* responsibility the product failure was. As discussed under Assumption A.1, recalls often happen after the product has been in the market for extended periods of time, during which new models are introduced. Therefore, when a quality problem reveals itself leading to a recall, the defective product/component is often replaced with a newer unit, rather than being repaired at the point of return. Therefore, the recall may not initially involve a detailed root cause analysis to identify the party whose process has been responsible for the quality problem. For instance, consider the Lenovo battery recall. In this case, each customer with the defective battery received a new replacement one. At the time of the recall, neither Lenovo nor its supplier knew what caused the over-heating of the batteries. Only after Lenovo wanted to share the recall cost with Sanyo, was a detailed root cause analysis performed to determine if the defect was Sanyo's or Lenovo's fault. A further point is that when the recall involves older product models, unless there is a cost sharing contract requiring root cause analysis information, manufacturers may initially have fewer incentives to identify the part at fault since the new component/product model has already undergone design changes and any information gained from the root cause analysis may not be useful for the current product design. Therefore, in this paper, c_r models the unit root cause analysis cost incurred when the cost sharing contract requires further analysis of the failed product to identify the party at fault for the quality problem.

A4: The manufacturer and the supplier have inherent process capabilities modeled by the initial failure rate of their components due to a design related quality problem. The initial failure rates are common knowledge to both parties and are denoted by $\lambda_m'^0$ and $\lambda_s'^0$ for the manufacturer and the supplier, respectively.

The failure rates are assumed to be time homogeneous (i.e., constant failure rate). Most components, particularly electronics, exhibit a "bathtub" shape failure rate function (Barlow and Proschan 1996). The initial down slope of the curve corresponds to the production and testing phase at the supplier's or the manufacturer's site. The failure rate in that part of the curve (also called the infant mortality rate) is reduced through better design, production and quality control. Even when a product gets out of the assembly line, manufacturers often use "burn-in tests" to detect early failures. A burn-in test is designed to eliminate or reduce infant mortality failures prior to selling the product (see Ebeling 1997). Hence, when a customer receives the product, as modeled in our paper, the product is at the beginning of its useful life with a constant failure rate.

A constant failure rate for each component allows us to model the product's time to a *failure that results in a recall* by the exponential distribution. Modeling product failures by the exponential lifetime distribution is also widely studied in the reliability literature (Barlow and Proschan 1996).

In our model, we assume that the supplier and the manufacturer will exert costly effort to improve the quality of their components and reduce the design failure rate. We use e_m and e_s to denote the amount of quality improvement effort exerted by the manufacturer and the supplier, respectively, where $0 \leq e_m \leq 1$ and $0 \leq e_s \leq 1$.

Under the effort level e_m (effort level e_s) the manufacturer (the supplier) can reduce the failure rate of his (her) component from the initial value $\lambda_m'^0$ (value $\lambda_s'^0$) to $\lambda_m'^0[1 - e_m]$ (to $\lambda_s'^0[1 - e_s]$). Given the constant failure rate assumption, the failure time distribution of the product (after quality improvement) that results in a recall will follow an exponential distribution with a failure rate of $\lambda_m'^0(1-e_m)+\lambda_s'^0(1-e_s)$. The effort choices of the manufacturer and the supplier are not observable. Consequently, neither the manufacturer nor the supplier can enforce a level of effort in the cost sharing contract.

A5: In our model, the contract negotiated between the manufacturer and the supplier covers external quality costs for a duration of T periods, which will denote the duration that the product is in use by the consumers. Without loss of generality, we normalize T to 1.

Given M and the exponential lifetime distribution assumption, the probability of observing the first product failure that results in a recall is given by $1 - e^{-M[\lambda_m'^0(1-e_m)+\lambda_s'^0(1-e_s)]}$. To further simplify the exposition, we define the aggregate failure rates as $\lambda_m^0 = M\lambda_m'^0$ and $\lambda_s^0 = M\lambda_s'^0$. Then the failure time probability distribution simplifies to $1 - e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}$. Note that, if a product fails, the failure is going to be due to the supplier's component with probability $\frac{\lambda_s^0(1-e_s)}{\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)}$. This probability would be $\frac{\lambda_m^0(1-e_m)}{\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)}$ for the manufacturer. We consider the manufacturer's and the supplier's processes to be stochastically independent, i.e., joint failures do not occur.

A6: Improvements in product design failure rate are costly to both parties. More specifically, the efforts of e_s and e_m result in an effort cost of $C_s(e_s)$ and $C_m(e_m)$ (per unit component) for the supplier and the manufacturer, respectively.

In practice, for a particular product line, the *total* design improvement related cost depends on the amount of effort spent during the design process. Let $D_s(e_s)$ and $D_m(e_m)$ denote such design related costs as a function of the supplier's and the manufacturer's effort. Then, the per component unit cost are $C_s(e_s) = \frac{D_s(e_s)}{M}$ and $C_m(e_m) = \frac{D_m(e_m)}{M}$, respectively, for the supplier and the manufacturer.

We consider $C_s(e_s)$ and $C_m(e_m)$ to be twice continuously differentiable on [0,1), and convex increasing in effort so that $C_s^{'}(e_s) > 0$ and $C_m^{'}(e_m) > 0$ for $(e_s, e_m) \in [0,1] \times [0,1]$ and $C_s^{''}(e_s) > 0$, and $C_m^{''}(e_m) > 0$ for $(e_s, e_m) \in [0,1) \times [0,1)$, for the manufacturer and the supplier, respectively.

In what follows, we would like to avoid boundary solutions to the manufacturer's and supplier's effort decisions. To this end, we assume that there are "low hanging" quality improvement opportunities for both parties. This requirement is formalized by $C_s^{'}(0) = 0$ and $C_m^{'}(0) = 0$. Also, given the limited physical and financial resources and the intellectual capacity of firms, we will assume that it is prohibitively more expensive to show incremental effort at high effort levels. More specifically, we will assume $\lim_{e_s \to 1} C_s^{'}(e_s) = \infty$ and $\lim_{e_m \to 1} C_m^{'}(e_m) = \infty$.

2.4 Cost Sharing Contracts under Complete Information

In this section, we investigate the effectiveness of Selective Root Cause Analysis Contract (Contract S) and Partial Cost Allocation Contract (Contract P) in coordinating the manufacturer's and supplier's quality improvement efforts when their effort choices are not contractible. We focus on the case with *complete information.* Specifically, we consider a case where the manufacturer has a good estimate of the supplier's component failure rate. This usually occurs when the supplier has been working with the manufacturer for several years, and therefore the manufacturer has a good idea of the quality of the supplier's component.

As a benchmark, we start by characterizing the First Best effort levels that would be chosen in a centrally coordinated system where both quality improvement decisions are made by a central planner. Next, we discuss how the effort game would be played out under Contract S and Contract P. For each contract, we model the following sequence of events.

Table 2.1: Table of Notation

r	selling price of the manufacturer's product
u_m	unit production cost of the manufacturer
u_s	unit production cost of the supplier
M	total number of products subject to recall
ω	unit recall cost
c_r	unit root cause analysis cost
p_0	unit part procurement price under no cost sharing
p	unit part procurement price
e_s^k	supplier's effort level under contract k
e_m^k	manufacturer's effort level under contract k
T	useful life of the product in the market
$C_s(e\ _s)$	cost of effort for the supplier
$C_m(e_m)$	cost of effort for the manufacturer
λ_s^0	supplier's initial failure rate
λ_m^0	manufacturer's initial failure rate

1. The part price and the cost sharing contract is agreed on by the manufacturer and the supplier, and the contract is signed.

2. The manufacturer and the supplier play the effort game, and select their quality improvement efforts e_s and e_m to maximize their own profits.

3. In case of a recall, the recall cost is realized and shared according to the contract signed in step 1.

In what follows, we will assume that both the manufacturer and the supplier are risk neutral decision makers. To characterize the outcome of the effort game, we will use the concept of Nash equilibrium. A pair (e_s^*, e_m^*) is a Nash equilibrium if neither player achieves higher profits by unilaterally changing his/her effort level. In the rest of the analysis, Π_i^k and e_i^k will denote the profit function and the effort level of supply chain member i under model k. The subscript i will take values of s and m, denoting the supplier and the manufacturer, respectively. The superscript k will take the values of *C, N, S, P* denoting the centrally coordinated (first best), No Sharing, Selective Root Cause Analysis, and Partial Cost Allocation models, respectively. For proofs of all propositions we refer the reader to the On-Line Appendix.

2.4.1 First Best Effort

In this subsection, a central planner maximizes total supply chain profits by jointly selecting the quality improvement efforts e_s^* and e_m^*. We refer to this case as the *First Best effort* since there are no incentive conflicts and both effort decisions are made in a coordinated way. The central planner's optimization problem is given by:

$$\underset{e_s\ ,\ e_m}{Max}\ :\ \Pi^C = r - u_m - u_s - \omega[1 - e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}] - C_m(e_m) - C_s(e_s) \tag{2.1}$$

where r is the product selling price, and u_m and u_s are the unit production costs of the manufacturer and the supplier, respectively.

To ensure concavity of the central planner's maximization problem, we will require $\frac{\partial^2\Pi^C}{\partial e_s^2} < 0$ and $\frac{\partial^2\Pi^C}{\partial e_s^2} < 0$, which can be translated into lower bounds on $C_m''(e_m)$ and $C_s''(e_s)$. Specifically, let $C_m''(e_m) > \omega(\lambda_m^0)^2$ and $C_s''(e_s) > \omega(\lambda_s^0)^2$ for $\forall(e_s, e_m) \in [0,1] \times [0,1]$ then:

Proposition 1 *If $C_m''(e_m) > \omega(\lambda_m^0)^2$ and $C_s''(e_s) > \omega(\lambda_s^0)^2$ for $\forall(e_s, e_m) \in [0,1] \times [0,1]$ hold, then there exists a unique First Best supplier and manufacturer efforts (e_s^{*C}, e_m^{*C}) that satisfy the following first order optimality conditions:*

$$\begin{aligned}
\frac{\partial \Pi^C}{\partial e_m} &= \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]} - C_m'(e_m^{*C}) = 0 \\
\frac{\partial \Pi^C}{\partial e_s} &= \omega\lambda_s^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]} - C_s'(e_s^{*C}) = 0
\end{aligned}$$

Note that $\frac{\partial\Pi^C}{\partial e_m \partial e_s} = \omega\lambda_m^0\lambda_s^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]} > 0$, which implies complementarity between effort choices, i.e., e_m^{*C} (e_s^C) is increasing with e_s^C, for all $e_s^C \in [0,1]$, and vice versa. The complementarity between effort choices increases with the unit recall cost ω and the initial failure rates of the supplier and the manufacturer. This implies that the interaction effect between effort choices is more significant for newer products, which are more likely to have a higher number of design flaws (higher λ_m^0 or λ_s^0) than products which have been on the market for a period of time and have already undergone multiple design improvements. Since a stronger interaction between effort decisions demands more coordinated decision making, the effort coordinating contracts discussed in this paper may be of greater importance for new product introductions.

Next, we examine *decentralized* supply chain models under different cost sharing agreements (i.e., No Cost Sharing, Selective Root Cause Analysis, and Partial Cost Allocation), where the supplier and the manufacturer maximize his/her own profits. The fundamental question we address is, under which contractual agreement(s) can a decentralized supply chain attain the First Best quality improvement effort levels? We first start with the No Cost Sharing case.

2.4.2 No Cost Sharing (N)

Here, we consider a decentralized setting in which the manufacturer internalizes total recall costs, even though in some cases the supplier may be at fault. Under this setting, while the manufacturer exerts some effort to improve his component failure rate, the supplier has no incentives to improve the quality of her component.

The manufacturer's and the supplier's optimization problems are respectively given by:

$$\underset{e_m}{Max} : \Pi_m^N = r - u_m - p_0 - \omega \left[1 - e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}\right] - C_m(e_m)$$

$$\underset{e_s}{Max} : \Pi_s^N = p_0 - u_s - C_s(e_s)$$

where p_0 is the manufacturer's part procurement cost. We consider $p_0 \geq u_s$ to ensure that the supplier makes a profit under the No Cost Sharing scenario. Note that p_0 does not affect the effort choices of the manufacturer or the supplier but allocates the supply chain profits between the two parties.

The next proposition summarizes optimal supplier and manufacturer effort levels under No Cost Sharing.

Proposition 2 *Under No Cost Sharing, the supplier exerts zero effort. The manufacturer underinvests in effort relative to the First Best (i.e., $e_m^{*N} < e_m^{*C}$) even though he fully internalizes all costs associated with his effort choice.*

Under No Cost Sharing, the optimal manufacturer effort increases with the effort exerted by the supplier. Specifically, the manufacturer's best response function $e_m^{*N}(e_s^{*N})$ is increasing in e_s^{*N}.

This observation, which follows from the positive cross partial derivative of the manufacturer's profit function in e_s and e_m, shows that there is complementarity between the effort choices of the manufacturer and the supplier. Under No Cost Sharing, the supplier exerts minimum effort because she does not internalize any costs. Therefore $e_s^{*C} > e_s^{*N}$. From the complementarity of effort decisions, it follows that $e_m^{*N}(e_s^{*N} = 0) < e_m^{*N}(e_s^{*N} = e_s^{*C}) = e_m^{*C}$. Hence, the analysis of this case particularly demonstrates that even though the manufacturer internalizes all costs associated with his effort choice, due to complementarity between effort choices, he *underinvests* in effort in equilibrium. Consequently, No Sharing scheme not only *directly* affects the effort exerted by the supplier, but also *indirectly* leads to less than First Best level of effort from the manufacturer.

2.4.3 Selective Root Cause Analysis Contract (Contract S)

Under Contract S, the cost allocation rule is defined as a function of the product's time to failure that results in product recall. If product failure occurs before the root cause analysis threshold time $\overline{T}$, the party responsible for the quality problem is identified through root cause analysis and incurs total recall costs. Otherwise (i.e., if product failure occurs after $\overline{T}$), the supplier only shares a percentage R of the total recall cost.

In practice, we observe a trend toward differentiating quality problems based on their time of occurrence. For example, by centralizing part failure data collected from dealerships, General Motors was one of the first to develop a monitoring system to differentiate early failures from late failures. Early failures are classified as special cause quality problems. For these type of product failures, the company pursues a detailed root cause analysis. This information is instantly fed into the design process to eliminate design faults (White 1999). In this paper, we identify ways in which this information can be used for cost sharing purposes.

Contract S is a more general and flexible contract format than the Fixed Share Rate Contract, previously modeled in the supply chain management literature. In fact, the Fixed Share Rate Contract is a special case of Contract S when $\overline{T} = 0$. As will be discussed below, unlike the Fixed

Share Rate Contract, the flexibility in the structure of Contract S is critical to obtaining the first best level of effort from the supply chain members.

Under Contract S, the manufacturer and the supplier solve the following optimization problems, respectively:

$$\underset{e_m}{Max} \ : \ \Pi_m^S = r - u_m - p - (\omega + c_r)G[1 - e^{-\Lambda_T \overline{T}}] - \omega(1-R)[e^{-\Lambda_T \overline{T}} - e^{-\Lambda_T}] - C_m(e_m)$$
$$\underset{e_s}{Max} \ : \ \Pi_s^S = p - u_s - (\omega + c_r)(1-G)[1 - e^{-\Lambda_T \overline{T}}] - R\omega[e^{-\Lambda_T \overline{T}} - e^{-\Lambda_T}] - C_s(e_s)$$

where $\Lambda_T = \lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)$ and $G = \lambda_m^o(1-e_m)/[\lambda_m^o(1-e_m) + \lambda_s^o(1-e_s)]$. Note that p is the price under contract S, where $p \geq p_0$ and $p - p_0$ is the incentive given to the supplier to accept the cost sharing contract.

Our interest is in understanding whether one can design a Contract S that achieves the First Best effort levels from supply chain partners. The next proposition describes the equilibrium outcome of the effort game.

Proposition 3 *Positive cross partial derivatives of the manufacturer's and the supplier's objective functions ensure that the effort game is supermodular under the Selective Root Cause Analysis Contract. Furthermore:*

(a) Supermodularity ensures the existence of at least one Nash equilibrium.

(b) The best response functions, $e_m^{*S}(e_s)$ *and* $e_s^{*S}(e_m)$ *are both increasing in their arguments.*

(c) The set of equilibria is a chain: i.e., if there are multiple equilibria, they can be ordered as follows: for any pair of equilibria $(\widehat{e}_m^{*S}, \widehat{e}_s^{*S})$ *and* $(\widetilde{e}_m^{*S}, \widetilde{e}_s^{*S})$ *either* $\widehat{e}_m^{*S} \geq \widetilde{e}_m^{*S}$ *and* $\widehat{e}_s^{*S} \geq \widetilde{e}_s^{*S}$ *or* $\widehat{e}_m^{*S} \leq \widetilde{e}_m^{*S}$ *and* $\widehat{e}_s^{*S} \leq \widetilde{e}_s^{*S}$.

(d) If there are multiple equilibria, then for any pair of equilibria $(\widehat{e}_m^{*S}, \widehat{e}_s^{*S})$ *and* $(\widetilde{e}_m^{*S}, \widetilde{e}_s^{*S})$*, where* $\widehat{e}_m^{*S} \geq \widetilde{e}_m^{*S}$ *and* $\widehat{e}_s^{*S} \geq \widetilde{e}_s^{*S}$ *then* $\Pi_m^{*S}(\widehat{e}_m^{*S}, \widehat{e}_s^{*S}) \geq \Pi_m^{*S}(\widetilde{e}_m^{*S}, \widetilde{e}_s^{*S})$ *and* $\Pi_s^{*S}(\widehat{e}_m^{*S}, \widehat{e}_s^{*S}) \geq \Pi_s^{*S}(\widetilde{e}_m^{*S}, \widetilde{e}_s^{*S})$.

While supermodularity ensures the existence of at least one Nash equilibrium, it does not rule out multiple equilibria. However, the equilibria are Pareto rankable and there is a most preferred and a least preferred equilibrium by both parties. From part (d) of Proposition 3, it follows that both parties prefer the equilibrium where they both show higher effort. Therefore, in what follows, we will be focusing on the most preferred equilibrium (Pareto optimal equilibrium) and avoid the issues associated with multiple equilibria when analyzing the effort coordinating contract (Cachon and Netessine, 2004).

The next proposition presents closed-form solutions to the effort coordinating contract parameters that achieve the First Best effort levels in the asymmetric effort game with $c_r = 0$ and the symmetric effort game[4] with $c_r > 0$. The optimal contract parameters for the asymmetric effort game with $c_r > 0$ cannot be characterized in closed-form solutions; therefore, for clarity of exposition, we present a detailed analysis of this case in the On-Line Appendix.

Proposition 4 *(i) In the asymmetric effort game with $c_r > 0$, there exists a unique effort coordinating contract defined by $(R^*, \overline{T}^*)$ that achieves the First Best effort levels from the manufacturer and the supplier.*

(ii) In the asymmetric effort game with $c_r = 0$, the effort coordinating contract is given by: $R^* = \frac{\lambda_s^0(1-e_s^{*C})}{\Lambda_T^{*C}}$ and $\overline{T}^* = -\frac{Ln(1-\Lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}}$, where $\Lambda_T^{*C} = \lambda_m^0(1-e_m^{*C}) + \lambda_s^0(1-e_s^{*C})$ and $\overline{T}^* < 1$.

(iii) In the symmetric effort game with $c_r > 0$, the effort coordinating contract is given by: $\overline{T}^* = \frac{-Ln(1-\Lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}}$, where $\overline{T}^* < 1$ and $R^* = 0.5$.

We gain the following insights from Proposition 4. First, notice that, when both agents exert their First Best effort levels, the share rate R is proportional to each party's product failure rate at the First Best effort level. Secondly, one can easily show that the Fixed Share Rate Contract ($\overline{T}^* = 0$) and the cost sharing contract that *always* uses root cause analysis information ($\overline{T}^* = 1$) cannot attain First Best effort levels. We summarize these insights in the following corollaries.

Corollary 1: *The effort coordinating optimal $\overline{T}^* > 0$; therefore, the Fixed Share Rate Contract which is a special case of the Selective Root Cause Analysis Contract when $\overline{T} = 0$, cannot*

[4]In the symmetric game, the manufacturer and the supplier have identical effort cost functions and initial product failure rates.

achieve the First Best effort level and coordinate the supply chain. Furthermore, setting $\overline{T}^* = 0$ *(i.e., the Fixed Share Rate Contract) leads to underinvestment in effort by the manufacturer and the supplier.*

Corollary 1 shows that the Fixed Share Rate Contract cannot coordinate the quality improvement effort levels. More interestingly, in the next corollary, we point out that to achieve the First Best quality level, one does not need to always use the root cause analysis information, even if it were costless and could perfectly identify the party at fault.

Corollary 2: *The effort coordinating optimal* $\overline{T}^* \neq 1$; *therefore, always performing a root cause analysis and allocating the quality costs to the party responsible for the quality problem – even if it were costless to do so – would not attain the First Best effort levels. Furthermore, setting* $\overline{T}^* = 1$ *would lead to an overinvestment in effort by the manufacture and the supplier.*

Lastly, we observe that $\frac{d\overline{T}^*}{d\Lambda_T^{*C}} < 0$. This implies that, when the coordinated total aggregate failure rate is smaller, (i.e., high effort is exerted at the First Best solution), then the root cause analysis threshold, $\overline{T}^*$ is larger, resulting in a higher likelihood of sharing recall costs based on root cause analysis information. In other words, the more both parties improve the quality of their components, the more both sides are likely to determine the party at fault in case of a failure that results in a recall.

In the next section, we present an alternative cost sharing scheme, namely the Partial Cost Allocation Contract (Contract P) and discuss ways in which the manufacturer's and supplier's efforts can be coordinated.

2.4.4 Partial Cost Allocation Contract (Contract P)

In this section, we introduce an alternative cost allocation rule which also achieves the First Best effort levels from the manufacturer and the supplier. Under this cost allocation scheme, the cost is always shared between the manufacturer and the supplier. However, the sharing rates are adjusted according to the root cause analysis information. More specifically, we denote R_m and $(1 - R_m)$ as the supplier's and the manufacturer's share of the recall cost, respectively, when *the manufacturer is at fault*. Similarly R_s and $(1 - R_s)$ are the supplier's and the manufacturer's share of the recall cost, respectively, when *the supplier is at fault*. Under the Partial Cost Allocation scheme, we have $0 \leq R_m \leq 1$ and $0 \leq R_s \leq 1$ and $R_s \geq R_m$. The last inequality ensures that the supplier assumes a larger fraction of the recall cost when she is at fault compared to the case when the manufacturer is at fault.

Analogous to the Selective Root Cause Analysis Contract, we will consider an effort game under Contract P, in which the manufacturer's and the supplier's strategies are complements. Furthermore, we will focus on the Pareto optimal equilibrium outcome of this game, where both parties show high effort. (See On-Line Appendix for a discussion of the supermodularity condition and elimination of multiple equilibria.)

Since the analysis of the asymmetric effort game does not lead to closed form solutions regarding the effort coordinating contract parameters, here we present our analytical results for the symmetric effort game and the effort coordinating parameters for contract P. For details, please see our On-Line Appendix.

Proposition 5 *In the symmetric effort game with $c_r \geq 0$, if there exists R_s^* and R_m^* such that:*

$$\begin{aligned} R_m^* &= \frac{A^* D^* - B^* \lambda_m^0 e^{-\Lambda_T^{*C}} + A^* \lambda_s^0 e^{-\Lambda_T^{*C}}}{B^* C^* + A^* D^*} \\ \Delta^* &= R_s^* - R_m^* \text{ and } \Delta^* = \frac{R_s C^* - \lambda_m^0 e^{-\Lambda_T^{*C}}}{A^*} \end{aligned}$$

and $0 \leq R_s^*, R_m^* \leq 1$, then there exists a coordinating Partial Cost Allocation Contract that achieves the First Best level of effort. The expressions $A^* = -(1 + \frac{c_r}{\omega})(G_m^{'} H - G H_m^{'})$, $B^* = (1 + \frac{c_r}{\omega})(G_s^{'} H - G H_s^{'})$, $C^* = (1 + \frac{c_r}{\omega}) H_m^{'}$ and $D^* = (1 + \frac{c_r}{\omega}) H_m^{'}$ are evaluated at (e_m^{*C}, e_s^{*C}).

In practice, it is not unusual to encounter situations in which a supplier, particularly if it is a small-sized company, faces budget constraints that limit the maximum cost allocated to the firm (Sherefkin et al. 2003). In such situations, even if the supplier is at fault, the manufacturer may choose to refrain from allocating total costs to the supplier, as it may lead to her bankruptcy. One example is Bremi Auto-Elektrik, a supplier of ignition coils for Volkswagen AG. In February of 2003, VW recalled about 500,000 VW and Audi vehicles in the United States from 2004 and 2002 model years to replace the faulty ignition coils. The recall cost VW $83 million, while Bremi's annual revenue was estimated at about $40 million.

Under such circumstances, our findings are particularly interesting in the sense that even if the supplier does not internalize the full liability and costs associated with her part failure, we show that the First Best effort levels and quality can still be attained if the share rates are set optimally as demonstrated in the above proposition.

In recent years, we observe a growing trend among original equipment manufacturers, particularly in the automotive industry, to push more of the external quality costs to their upstream partners by arguing more of their accountability in product failures. This movement inevitably resulted in a number of "bitter" manufacturer-supplier relationships (Sherefkin and Armstrong 2003). Our analysis shows that reflecting the total accountability to a supplier does not result in the coordination of effort levels, and that the First Best product quality is in fact attainable by establishing a collaborative relationship based on cost sharing. Specifically, if the manufacturer and the supplier incur the total recall cost whenever the product failure is due to their process quality (*i.e.*, $R_m = 0$ and $R_s = 1$), both parties invest in quality improvement effort not only to reduce the total product failure rate but also to decrease the likelihood of the failure being due to their own processes. This leads to overinvestment in quality improvement when $R_m = 0$ and $R_s = 1$. Therefore, establishing a cost sharing relationship between the supplier and the manufacturer ($0 < R_m < 1$ and $0 < R_s < 1$) can eliminate the overinvestment incentives of both parties and ensure coordinated effort levels.

Corollary 3: *At the coordinated first best effort level, Contract S results in higher total supply chain profits than Contract P.*

While both Contract S and Contract P can be designed to achieve the First Best level of effort from the manufacturer and the supplier, they result in different total supply chain profits. Notice that $\overline{T}^* < 1$, which implies that at the First Best level of effort there is a smaller likelihood that a root cause analysis will be performed under Contract S than under Contract P. This results in smaller expected root cause analysis costs for Contract S than for Contract P. Since under the First Best effort levels, the effort and the expected recall costs are the same for both contracts, a smaller expected root cause analysis cost leads to higher total supply chain profits for Contract S. As the root cause analysis threshold $\overline{T}^*$ gets larger, which happens when it is critical to induce higher effort and achieve lower failure rate at the First Best effort, the cost difference between the two contracts diminishes.

2.4.5 A Numerical Study of Contract Comparison and Discussion

In this subsection, our goal is to develop an understanding of how Contract S and Contract P perform relative to the Fixed Share Rate Contract previously studied as a means of sharing external quality costs. Hence, to answer our second research question, we perform an extensive numerical analysis, and develop insights as to why and under what situations the contracts studied in this paper are valuable for a supply chain. To this end, we study the effect of a number of factors such as unit recall cost (ω), the convexity of the effort cost function, the manufacturer's and the supplier's initial failure rates (λ_m^0 and λ_s^0), and the market size (M) on the performance of contracts.

We measure contract performance along two dimensions: (i) the total expected supply chain cost[5] (the expected product recall cost and quality improvement effort cost), and (ii) the final product quality, measured by the final product's failure rate after quality improvement efforts. We define a *Cost Inefficiency Index* (C_I) and a *Quality Inefficiency Index* (Q_I) to compare the decentralized supply chain performance to that of the centrally coordinated system. More specifically, we calculate:

$$\begin{aligned}
\textit{Cost}\ \text{Inefficiency Index}\ (\mathrm{C}_I^i) &= \frac{SC^i - SC^C}{SC^C} \times 100\% \\
\textit{Quality}\ \text{Inefficiency Index}\ \ (\mathrm{Q}_I^i) &= \frac{\Lambda_T^i - \Lambda_T^C}{\Lambda_T^C} \times 100\%
\end{aligned}$$

where SC^i and Λ_T^i are the total expected supply chain cost and the final product's failure rate under Contract i, and SC^C and Λ_T^C are the total supply chain cost and final product's failure rate of

[5]Since the revenues of the total supply chain is constant and equals rM, it is sufficient to compare the total supply chain cost.

Table 2.2: Parameter Values for Numerical Analysis

Parameter	Values	Parameter	Values
ω	$1, 10, 100, 1000$	c_r	$0,\ 0.1\omega,\ 0.5\omega,\ \omega$
γ_m	$10, 100, 500, 1000$	γ_s	$0.2\gamma_m,\ 0.5\gamma_m,\ \gamma_m,\ 2\gamma_m,\ 5\gamma_m,\ 10\gamma_m$
λ_m^0	$0.05, 0.4, 0.8$	λ_s^0	$0.2\lambda_m^0,\ 0.5\lambda_m^0,\ \lambda_m^0,\ 2\lambda_m^0,\ 5\lambda_m^0$
M	$1000, 8000, 16000$		

the corresponding centrally coordinated supply chain. Note that lower values of C_I and Q_I report a performance closer to the centrally coordinated system. To optimize SC^i, we choose contract parameters that maximizes the manufacturer's expected profit (i.e., minimizes manufacturer's expected cost) and provides just enough incentives to the supplier to accept the contract and exert optimal effort.

To perform our numerical analysis, following the existing literature, we assume that the manufacturer's and the supplier's effort cost functions have the following functional forms (Corbett and DeCroix, 2001): $C_m(e_m) = \gamma_m[-Ln(1-e_m) - e_m]$ and $C_s(e_s) = \gamma_s[-Ln(1-e_s) - e_s]$. The parameters γ_m and γ_s model the convexity of the effort cost functions, i.e., a larger value of γ_m (value of γ_s) is associated with a faster increasing effort cost function for the manufacturer (for the supplier).

Table 2.2 lists the range of parameter values for our numerical analysis. We consider 17280 combinations of parameter values while evaluating our cost and quality index for each contract. Table 2.3[6] reports the average and the maximum values of C_I and Q_I across 17280 parameter settings.

From our numerical analysis, we gain the following insights regarding the comparison between the three contracts: Fixed Share Rate, Contract S and Contract P.

- We found that the Selective Root Cause Analysis Contract consistently performs better than the Fixed Share Rate and the Partial Cost Allocation Contracts in terms of the average and the maximum total supply chain cost. For instance, its average and maximum cost and quality

[6]Note that a negative quality index reports overinvestment in quality effort.

Table 2.3: Cost and Quality Index Comparisons

	Fixed Share Rate	Contract S	Contract P
Average Cost Inefficiency Index	0.76 %	0.39 %	37.1%
Maximum Cost Inefficiency Index	15.75%	6.05%	40%
Average Quality Inefficiency Index	6.4%	3.1%	-8.45%
Maximum Quality Inefficiency Index	227%	125%	-0.04%

indices are around half of those of the Fixed Share Rate Contract.

- The gap between the performance of the Selective Root Cause Analysis Contract and Fixed Share Rate Contract increases, as: (i) the gap between the unit recall cost (ω) and the unit root cause analysis cost (c_r) increases, (ii) the initial failure rate increases leading to higher investment in effort, and (iii) the sales volume of the product increases, leading to an increase in the likelihood of observing product failure. We also observed that when the convexity of the effort cost function increases, leading to higher effort levels and higher quality in equilibrium, the Selective Root Cause Analysis Contract performs significantly better than the Fixed Share Rate. This type of effort cost function corresponds to situations where there is an established product that has previously gone through several quality improvements. Consequently, exerting effort to identify and resolve quality issues is more difficult and therefore more costly.

- On average, the product quality was closer to the first best effort levels under the Selective Root Cause Analysis Contract than under the Fixed Share Rate and Partial Cost Allocation Contracts.

- Under the Selective Root Cause Analysis Contract, which is optimized to maximize the manufacturer's expected profit, the quality of the final product (i.e., the failure rate of the final product) is, on average, only 3.1% lower than the product quality in the centralized system. The gap in quality increases as: (i) the unit root cause analysis cost increases, (ii) the convexity of the effort cost function increases, and (iii) initial failure rates of the manufacturer and the supplier increase.

- The Partial Cost Allocation Contract results in overinvestment in quality improvement effort, leading to, on average, 8.45% better product quality than that in the centralized system. This improved quality, however, brings about on average, 37.1% additional cost compared to the centralized system. The overinvestment in product quality increases when: (i) the unit recall cost increases, (ii) the convexity of the effort cost function decreases, and (iii) the initial failure rates of the manufacturer and the supplier increase.

2.5 Asymmetric Information on Supplier Failure Rate

In the previous section, we showed that under complete information, the Selective Root Cause Analysis Contract is a flexible contract format that attains First Best effort levels when the supplier's and the manufacturer's efforts are subject to double moral hazard. In this section, we relax the assumption that the manufacturer has full information about supplier's initial failure rate, and present a mixed model of adverse selection followed by moral hazard on the supplier side. Our goal is to investigate the effectiveness of the Selective Root Cause Analysis Contract format to screen supplier type as well as to induce effort to improve quality.

Consistent with the extant literature (Lim (2001), Baiman et al.(2000, 2001, 2003), Laffont and Martimort (2002)), we make the following assumptions:

B1. We assume that the supplier's process can be of either a low quality process (i.e., high failure) type with probability α or high quality process (i.e., low failure) type with probability $(1-\alpha)$, where $0 < \alpha < 1$. Furthermore, the high failure type supplier has an initial failure rate of $\lambda_{s_H}^0$ while the low failure type supplier has an initial failure rate of $\lambda_{s_L}^0$, where $\lambda_{s_H}^0 > \lambda_{s_L}^0$. At the time of contracting, the supplier knows her type, while the manufacturer knows that the supplier can have failure rates $\lambda_{s_H}^0$ or $\lambda_{s_L}^0$ with probability α and $(1-\alpha)$, respectively.

To make the analysis tractable, and consistent with the existing literature, we assume that the amount of effort he spends on improving the quality of his component is already decided independent of the supplier's effort improvement decision.

B2. To capture the impact of information asymmetry on the supplier side, we assume that the manufacturer has already invested quality improvement effort in his process when he offers a contract to the supplier. We will denote the manufacturer's failure rate by λ_m^0.

Assumption B2 represents cases in which a quality level for the component has already been decided and a limited budget is assigned for it. Thus, the failure rate λ_m^0 in our model represents the manufacturer's final component failure. We also perform a numerical study in which we relax this assumption and we study cases in which we study a menu of contracts where the manufacturer also exerts optimal effort to improve his process quality (please refer to Section 2.6).

B3. We will assume that there are two types of quality improvement efforts that the supplier of type j can exert, i.e., high effort $e_{s_j}^H$ and low effort $e_{s_j}^L$, independent of the supplier type. From the convexity assumption on the effort cost function, it follows that $C_{s_j}(e_{s_j}^H) > C_{s_j}(e_{s_j}^L)$.

Low quality effort e^L corresponds to a marginal improvement in the supplier's component quality, while high quality effort e^H corresponds to a significant improvement in her product quality. We believe that, while simple enough to make our analysis tractable, this assumption captures the dynamics of the effort decision in our setting and its impact on the the optimal menu of contracts. Furthermore, in the real world, the decision on how to improve quality is sometimes limited to two or three options. Thus, assuming two effort levels is also not far from many cases in practice. We also performed a numerical study in which we relax this assumption and we consider continuous effort functions (please refer to Section 2.6).

B4. We assume that the supplier's quality improvement effort is not contractible; therefore, it cannot be specified in a contract.

Hence, in addition to the adverse selection problem stated in assumptions B1 and B2, the manufacturer also faces a moral hazard problem on the supplier side.

In what follows, we model the following sequence of events.

1. *The manufacturer moves first and offers a menu of Selective Root Cause Analysis Contracts to the supplier.*

2. *The supplier j either rejects the menu or accepts one of the contracts from the menu.*

3. *The supplier j exerts effort $e^r_{s_j}$, where $r = L$ and $r = H$ will denote the low and high effort levels, respectively.*

4. *The manufacturer procures the part, manufactures the product, and sells it at price r.*

5. *The final quality of the product and the recall cost is realized and shared according to the contract accepted in step 2.*

As a benchmark case, let us also define the centrally coordinated (First Best) effort level under complete information about supplier quality. In other words, if the supplier's initial failure rate were known, and his effort level was observable, the First Best effort levels would be given by the solution to the following optimization problem defined for each type supplier.

$$\max_{e^C_{s_j}} \Pi^C_j = r - u_m - u_s - \omega(1 - e^{-[\lambda^0_m + \lambda^0_{S_j}(1-e^C_{s_j})]}) - C_s(e^C_{s_j}) \tag{2.2}$$

where $e^{*C}_{s_j}$ denotes the First Best effort level of supplier type j, ($j \in \{H, L\}$). The optimal $e^{*C}_{s_j}$ trade-offs the expected recall cost to the cost of exerting quality improvement effort.

The next section characterizes the optimal menu of contracts. This is followed by a numerical study that investigates the value of implementing a menu relative to the complete information model in terms of its impact on supply chain cost and on final product quality.

2.5.1 Optimal Menu of Selective Root Cause Analysis Contracts

The manufacturer offers a menu of Selective Root Cause Analysis Contracts $\{S_L, S_H\}$ where $S_L = (p_L, R_L, \overline{T}_L)$ denotes the contract designed for the low failure type supplier, and $S_H = (p_H, R_H, \overline{T}_H)$ is the contract designed for the high failure type supplier. Laffont and Martimort (2002) show that in a mixed modeling framework, the revelation principle (Kreps 1990) still applies and therefore one can focus on the menu of contracts which induces truthful revelation of supplier type. After a contract is accepted from the menu, the supplier chooses her investment in quality improvement effort to maximize her profits.

Let $\Pi_{s_j}(S_k)$ denote the profits of supplier type j, where $j = \{L, H\}$ when she chooses Contract S_k from the menu and $k = \{L, H\}$.

$$\Pi_{s_j}(S_k) = p_k - u_s - Z(\lambda^0_{S_j}, R_k, \overline{T}_k, e^r_{s_j}) - C_s(e^r_{s_j}) \tag{2.3}$$

where $Z(\lambda^0_{S_j}, R_k, \overline{T}_k, e^r_{s_j})$ represents the supplier's share of expected recall cost under Contract S_k with parameters $R_k, \overline{T}_k$ when her initial failure rate is $\lambda^0_{S_j}$ and she exerts effort $e^r_{s_j}$. This is given by:

$$\begin{aligned} Z(\lambda^0_{S_j}, R_k, \overline{T}_k, e^r_{s_j},) \quad &= \quad (\omega + c_r)\frac{\lambda^0_{S_j}(1 - e^r_{s_j})}{\lambda^0_m + \lambda^0_{S_j}(1 - e^r_{s_j})}(1 - e^{-[\lambda^0_m + \lambda^0_{S_j}(1-e^r_{s_j})]\overline{T}_k}) \\ &+\omega R_k(e^{-[\lambda^0_m + \lambda^0_{S_j}(1-e^r_{s_j})]\overline{T}_k} - e^{-[\lambda^0_m + \lambda^0_{S_j}(1-e^r_{S_j})]}) \end{aligned} \tag{2.4}$$

To ensure truthful revelation of supplier type, the menu of contracts should be *incentive compatible*. More specifically, given an incentive compatible menu, the supplier of type λ_{s_L} weakly prefers $S_L = (p_L, R_L, \overline{T}_L)$ over $S_H = (p_H, R_H, \overline{T}_H)$, and the supplier of type $\lambda^0_{s_H}$ weakly prefers $S_H = (p_H, R_H, \overline{T}_H)$ over $S_L = (p_L, R_L, \overline{T}_L)$. These requirements are formalized by the following *Incentive Compatibility (I.C.) Constraints.*

$$p_H - u_s - Z(\lambda^0_{S_H}, R_H, \overline{T}_H, e^r_{s_H}) - C_s(e^r_{s_H}) \quad \geq \quad p_L - u_s - Z(\lambda^0_{S_H}, R_L, \overline{T}_L, e^r_{s_H}) - C_s(e^r_{s_H}) \tag{2.5}$$
$$p_L - u_s - Z(\lambda^0_{S_L}, R_L, \overline{T}_L, e^r_{s_L}) - C_s(e^r_{s_L}) \quad \geq \quad p_H - u_s - Z(\lambda^0_{S_L}, R_H, \overline{T}_H, e^r_{s_L}) - C_s(e^r_{s_L}) \tag{2.6}$$

Secondly, for a contract to be accepted, it must yield to each type supplier at least his outside opportunity level, which we normalize to zero. Therefore, the following *Participation Constraints* must be satisfied, respectively, for the high and the low failure type supplier:

$$\Pi_{s_H}(S_H) \quad \geq \quad 0 \tag{2.7}$$
$$\Pi_{s_L}(S_L) \quad \geq \quad 0 \tag{2.8}$$

Lastly, to induce optimal effort in equilibrium, the two moral hazard *Incentive Constraints* must hold depending on the effort level that the manufacturer wants to induce from each supplier type. For instance, if it is optimal to induce low effort level from the high failure rate supplier and high effort level from the low failure rate supplier, then the moral hazard constraints are given by:

$$p_H - u_s - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^H_{s_H}) - C_s(e^H_{s_H}) \quad < \quad p_H - u_s - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^L_{s_H}) - C_s(e^L_{s_H}) \tag{2.9}$$
$$p_L - u_s - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^H_{s_L}) - C_s(e^H_{s_L}) \quad > \quad p_L - u_s - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^L_{s_L}) - C_s(e^L_{s_L}) \tag{2.10}$$

which simplify to:

$$Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^L_{s_H}) - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^H_{s_H}) \quad < \quad C_s(e^H_{s_H}) - C_s(e^L_{s_H}) \tag{2.11}$$
$$Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^L_{s_L}) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^H_{s_L}) \quad > \quad C_s(e^H_{s_L}) - C_s(e^L_{s_L}) \tag{2.12}$$

Besides designing the optimal menu to screen the supplier type, the manufacturer's problem also involves solving for the optimal effort level to induce from each type. Constraints (2.5) to (2.10) characterize the set of incentive feasible menus. Below, we first characterize the manufacturer's optimization problem solely considering the adverse selection issue. Later, we will impose the moral hazard constraints and discuss how to integrate them into the manufacturer's optimal solution.

2.5.2 Separation of Supplier Types

Adding constraints (2.5) and (2.6), we obtain:

$$Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^r_s) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, e^r_s) \leq Z(\lambda^0_{s_H}, R_L, \overline{T}_L, e^r_s) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^r_s) \tag{2.13}$$

Any incentive feasible menu satisfying (2.5) and (2.6) is required to satisfy (2.13). To ensure separation of types, constraint (2.13) will impose *a monotonicity constraint* (Laffont and Martimort (2002)) on contract parameters R_k and $\overline{T}_k$. Furthermore, we require that the Spence-Mirrlees single crossing property holds (Laffont and Martimort, 2002). This property ensures that only the low failure type supplier's incentive compatibility constraint is binding in equilibrium. To establish the single crossing property, we need $\frac{\partial Z}{\partial\lambda\partial R} = Z_{\lambda R} > 0$ and $\frac{\partial Z}{\partial\lambda\partial\overline{T}} = Z_{\lambda\overline{T}} > 0$ (Laffont and Martimort, 2002). For $Z_{\lambda R} > 0$ and $Z_{\lambda\overline{T}} > 0$ to hold, we require that the total failure rate satisfies $\lambda^0_m + \lambda^0_{s_H}(1 - e^L_s) < 1$.

Lemma 1: *A sufficient condition to ensure the separation of supplier type in equilibrium under a menu of Selective Root Cause Analysis contracts is given by* $\lambda^0_m + \lambda^0_{s_H}(1 - e^L_s) < 1$.

Lemma 1 states that when the largest attainable failure rate is bounded from above by 1, then screening of supplier type is possible with a menu of contracts. Note that $\lambda_m^0 + \lambda_{s_H}^0(1 - e_s^L)$ is the failure rate of the final product that results in a recall. It is not unrealistic to assume that this failure rate is less than one, since considering a constant failure rate of recall related failures, the above condition implies that the probability of a failure that results in a recall should be less than 63%, which is much larger than what really occurs in practice. Therefore, this condition is not a restrictive assumption.

The intuition behind this condition becomes clear when it is rewritten as $\lambda_m^0 < 1 - \lambda_{s_H}^0(1 - e_s^L)$, which imposes an upper bound on the manufacturer's failure rate. Now consider an extreme case, where λ_m^0 is very large, much larger than the failure rates of the both supplier types. In this case, the failure would be the manufacturer's fault with probability 1. Under these circumstances, the menu of contracts consists of two contracts that have $\overline{T}_H^*$ and $\overline{T}_L^*$ very close to 1. This implies that R_H^* and R_L^* has almost no impact in separating the supplier type. Thus, both suppliers have incentive to choose the contract with higher price, since under both contracts, with probability 1, the recall will be the manufacturer's fault.

Given the cross partial derivatives $Z_{\lambda R} > 0$ and $Z_{\lambda \overline{T}} > 0$, constraint (2.13) results in monotonicity constraints of $R_H < R_L$ and $\overline{T}_H < \overline{T}_L$ on contract parameters in any incentive compatible menu.

Proposition 6 *In an incentive compatible menu of Selective Root Cause Analysis Contracts, which ensures separation of supplier types, it is sufficient that the high failure rate supplier has a lower fixed share rate ($R_H < R_L$) and a higher root cause analysis threshold ($\overline{T}_H < \overline{T}_L$) than the low failure rate type supplier.*

Next, we rewrite equations (2.5) and (2.6) as follows:

$$\Pi_{s_L}(S_L) \geq \Pi_{s_H}(S_H) + Z(\lambda_{s_H}^0, R_H, \overline{T}_H, e_{s_H}^r) - Z(\lambda_{s_L}^0, R_H, \overline{T}_H, e_{s_L}^r) \tag{2.14}$$

$$\Pi_{s_H}(S_H) \geq \Pi_{s_L}(S_L) + Z(\lambda_{s_L}^0, R_L, \overline{T}_L, e_{s_L}^r) - Z(\lambda_{s_H}^0, R_L, \overline{T}_L, e_{s_L}^r) \tag{2.15}$$

In constraint (2.14), the expression $Z(\lambda_{s_H}^0, R_H, \overline{T}_H, e_{s_H}^r) - Z(\lambda_{s_L}^0, R_H, \overline{T}_H, e_{s_L}^r)$ denotes the *information rent* given to the low failure type supplier due to his ability to mimic the high failure type supplier. The manufacturer's challenge is to determine the least costly way to give up rent to the low failure rate supplier provided by any incentive compatible contract. As will be shown later, since the manufacturer's profits are decreasing in $\Pi_{s_L}(S_L)$ and $\Pi_{s_H}(S_H)$, in equilibrium, the manufacturer sets $\Pi_{s_L}(S_L) = Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^H_s) - Z(\lambda^0_{s_L}, R_H, \overline{T}_H, e^H_s)$ and $\Pi_{s_H}(S_H) = 0$. Also, note that constraint (2.14) implies constraint (2.7), therefore the latter is ignored in characterizing the optimal contract.

2.5.3 Manufacturer's Choice of Optimal Menu

Let Π_m denote the manufacturer's expected profits, then Π_m is given by:

$$\begin{aligned} \Pi_m \;=\; & r - u_m - \alpha p_H - (1-\alpha)p_L - \alpha\Big[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-e^r_{s_H})]}) - Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^r_{s_H})\Big] \\ & -(1-\alpha)\Big[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-e^r_{s_L})]}) - Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^r_{s_L})\Big] \end{aligned} \tag{2.16}$$

The manufacturer solves:

$$\begin{aligned} & \max_{e^r_{s_L},\, e^r_{s_H},\, S_L,\, S_H} \Pi_m \\ \textit{subject to} \;:\; & \\ & \textit{Constraints}\;\; (2.7),\; (2.8),\; (2.11),\; (2.12),\; (2.14),\; \textit{and}\; (2.15) \end{aligned}$$

Notice that one can rewrite

$$\begin{aligned} p_H &= \Pi_{s_H}(S_H) + Z(\lambda^0_{s_H}, R_H, \overline{T}_H, e^r_{s_H}) + C_s(e^r_{s_H}) + u_s \\ p_L &= \Pi_{s_L}(S_L) + Z(\lambda^0_{s_L}, R_L, \overline{T}_L, e^r_{s_L}) + C_s(e^r_{s_L}) + u_s \end{aligned}$$

Substituting p_H and p_L into the manufacturer's objective function, we obtain:

$$\begin{aligned} \Pi_m \;=\; & r - u_m - u_s - \alpha\Pi_{s_H}(S_H) - (1-\alpha)\Pi_{s_L}(S_L) \\ & -\omega\Big[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-e^r_{s_H})]})\Big] - \omega\Big[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-e^r_{s_L})]})\Big] \\ & -\alpha C_s(e^r_{s_H}) - (1-\alpha)C_s(e^r_{s_L}) \end{aligned} \tag{2.17}$$

Since the manufacturer's profits are decreasing in $\Pi_{s_H}(S_H)$ and $\Pi_{s_L}(S_L)$, in equilibrium constraint (2.7) and (2.14) are binding, which results in the following optimization problem for the manufacturer:

$$\max_{e^r_{s_L},\, e^r_{s_H},\, S_L,\, S_H} \Pi_m = r - u_m - u_s - (1-\alpha)\Big[Z(\lambda^0_{S_H}, R_H, \overline{T}_H, e^r_{s_H}) - Z(\lambda^0_{S_L}, R_H, \overline{T}_H, e^r_{s_L})\Big] \quad (2.18)$$
$$-\omega\Big[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{S_H}(1-e^r_{s_H})]})\Big] - \omega\Big[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{S_L}(1-e^r_{s_L})]})\Big]$$
$$-\alpha C_s(e^r_{s_H}) - (1-\alpha)C_s(e^r_{s_L})$$

In the manufacturer's problem stated above, his profits function consists of the following terms: (i) the unit revenue, (ii) the expected information rent given to the low failure type supplier, (iii) the expected total recall cost, and (iv) the expected supplier quality improvement effort cost. The manufacturer's challenge is to choose the optimal supplier effort ($e^r_{s_L}$, $e^r_{s_H}$) for each type as well as to determine ($R^*_H, \overline{T}^*_H$) and (R^*_L ,$\overline{T}^*_L$) such that $R^*_H < R^*_L$ and $\overline{T}^*_H < \overline{T}^*_L$ to satisfy incentive compatibility constraints and thus screen supplier type.

A couple of interesting observations can be made regarding the manufacturer's optimization problem. First, note that the manufacturer's optimization problem is independent of (R_L ,$\overline{T}_L$). While the contract parameters (R_L ,$\overline{T}_L$) do not drive the profits of the manufacturer directly, they have an indirect effect through the moral hazard constraint (2.10), which induces optimal quality improvement effort from the low failure rate supplier. Secondly, note that, if we ignore the expected information rent, the manufacturer's optimization problem is separable into two sub problems each of which solves the centrally coordinated effort level under complete information about each type of supplier.

Below we list our insights regarding the optimal solution to the manufacturer's optimization problem (please refer to the On-Line Appendix for proofs of these remarks).

Remark 1: It is optimal to induce high effort from a high failure rate supplier (i.e., $e^H_{s_H}$) when the information rent given to the low failure rate supplier and the incremental effort cost is less than the savings in expected recall cost incurred at the higher effort level. More specifically, $e^H_{s_H}$ is optimal when,

$$\frac{(1-\alpha)}{\alpha}[Z(\lambda^0_{S_H}, R^*_H, \overline{T}^*_H, e^H_{S_H}) - Z(\lambda^0_{S_L}, R^*_H, \overline{T}^*_H, e^r_{s_L})] + [C_s(e^H_{s_H}) - C_s(e^L_{s_H})] \quad (2.19)$$
$$> \omega[e^{-[\lambda^0_m + \lambda^0_{s_H}(1-e^H_{s_H})]} - e^{-[\lambda^0_m + \lambda^0_{s_H}(1-e^L_{s_H})]}]$$

holds where R_H^* and T_H^* exists and are given by the solution to the following moral hazard constraint:

$$Z(\lambda_{s_H}^0, R_H^*, \overline{T}_H^*, e_{s_H}^L) - Z(\lambda_{s_H}^0, R_H^*, \overline{T}_H^*, e_{s_H}^H) = C_s(e_{s_H}^H) - C_s(e_{s_H}^L) \tag{2.20}$$

Otherwise, $R_H^* = 0$ and $T_H^* = 0$.

Notice that when α is close to zero, the first term in (2.19) approaches infinity and it becomes optimal to induce low effort from high failure rate supplier. When α is close to zero, the likelihood of having a low failure rate supplier $(1-\alpha)$ is close to 1. This translates to a probability of paying the information rent close to one. The information rent increases in the effort exerted by the high failure rate supplier. Therefore, to minimize the expected information rent, the manufacturer sets the effort level of the high failure rate supplier to e^L. To induce low effort from the high failure rate supplier, it is optimal to set $R_H^* = 0$ and $\overline{T}_H^* = 0$.

Remark 2: When it is optimal for the manufacturer to induce low effort from the high failure rate supplier (i.e. $R_H^* = 0$ and $\overline{T}_H^* = 0$), then it is optimal to induce the centrally coordinated (First Best) effort level $(e_{s_L}^{*C})$ from the low failure rate supplier.

Suppose $e_{s_L}^{*C} = e_{s_L}^H$ then the optimal R_L^* and $\overline{T}_L^*$ is chosen to satisfy the following moral hazard constraint.

$$Z(\lambda_{s_L}^0, R_L^*, \overline{T}_L^*, e_{s_L}^L) - Z(\lambda_{s_L}^0, R_L^*, \overline{T}_L^*, e_{s_L}^H) = C_s(e_{s_L}^H) - C_s(e_{s_L}^L) \tag{2.21}$$

One can also show that a solution $(R_L^*, \overline{T}_L^*)$ which satisfies (2.21) exists.

Remark 3: When $R_H^* > 0$ and $\overline{T}_H^* > 0$ (i.e., when it is optimal to induce high effort from the high failure rate supplier) then it is optimal to induce high effort from the low failure rate supplier when the savings in external quality costs dominate the information rent and the incremental effort cost incurred due to high effort. Specifically, high effort is optimal from the low failure rate supplier when:

$$\begin{aligned} & [Z(\lambda_{s_L}^0, R_H^*, \overline{T}_H^*, e_{s_L}^L) - Z(\lambda_{s_L}^0, R_H^*, \overline{T}_H^*, e_{s_L}^H)] + [C_s(e_{s_L}^H) - C_s(e_{s_L}^L)] \\ < \quad & \omega[e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-e_{s_L}^H)]} - e^{-[\lambda_m^0 + \lambda_{s_L}^0(1-e_{s_L}^L)]}] \end{aligned} \tag{2.22}$$

where $(R_H^*, \overline{T}_H^*)$ satisfies (2.20) and $R_L^* > R_H^*$ and $\overline{T}_L^* > \overline{T}_H^*$ satisfies:

$$Z(\lambda_{s_L}^0, R_L^*, \overline{T}_L^*, e_{s_L}^L) - Z(\lambda_{s_L}^0, R_L^*, \overline{T}_L^*, e_{s_L}^H) = C_s(e_{s_L}^H) - C_s(e_{s_L}^L) \tag{2.23}$$

Furthermore, one can also show that $R_L^* > R_H^*$ and $\overline{T}_L^* > \overline{T}_H^*$ exists (please see the On-Line Appendix for a proof).

In summary, we show that even when the manufacturer does not have complete information about the process quality of his supplier, he could design a menu of Selective Root Cause Analysis Contracts to both screen supplier types and at the same time induce supplier effort. To ensure separation of supplier type, in the optimal menu of contracts, the high failure rate supplier is allocated a smaller share of total cost (smaller R) and a smaller root cause analysis threshold ($\overline{T}_H$) than the cost sharing contract designed for the low failure rate supplier (i.e. $R_H < R_L$ and $\overline{T}_H < \overline{T}_L$). Since the manufacturer's problem involves both an adverse selection and a moral hazard issue, we find that if the manufacturer were to induce high effort from the high failure rate supplier, then he would need to allocate higher information rent to the low failure rate supplier. Therefore, the existence of an adverse selection problem adds an information cost to inducing effort in the moral hazard problem. In the Selective Root Cause Analysis Contract, the contract parameters R and T provide the much needed flexibility to the manufacturer to handle this effort inducement versus information rent trade-off in the supply chain.

2.6 Numerical Study for Information Asymmetry Case

In the previous section we presented our analytical results that provide some insights into the properties of the optimal menu of Selective Root Cause Analysis Contracts. In that section we made the following assumptions, which we are relaxing in our numerical study in this section: (i) we assumed that there are two effort levels (high and low); in this section, however, we relax this assumption and we assume that the quality improvement effort is a continuous decision variable and assumes a value between 0 and 1, and (ii) we assumed that only the supplier exerts quality

Table 2.4: Parameter Values for Numerical Analysis

Parameter	Values	Parameter	Values
ω	$10, 100, 1000$	c_r	$0,\ 0.1\omega,\ 0.5\omega,\ \omega$
γ_m	$10, 100, 500$	γ_s	$0.2\gamma_m,\ 0.5\gamma_m,\ \gamma_m,\ 2\gamma_m,\ 5\gamma_m,\ 10\gamma_m$
λ_m^0	$0.05, 0.4, 0.8$	λ_{s1}^0	$0.2\lambda_m^0,\ 0.5\lambda_m^0,\ \lambda_m^0,\ 2\lambda_m^0,\ 5\lambda_m^0$
λ_{s2}^0	$0.2\lambda_{s1}^0,\ 0.8\lambda_{s1}^0$		

improvement effort; in this section, however, we relax this assumption and we assume that the manufacturer can also exert quality improvement effort between 0 and 1.

We investigate the implications of information asymmetry. Specifically, we are investigating the following questions:

1. *Value of Information:* How much does knowing the information about the supplier's failure rate (i.e., supplier type) improve the manufacturer's costs and the quality of the final product? Under what circumstances is the value of this information significant?

2. *Value of Menu of Contracts:* How much does using a menu of Selective Root Cause Analysis Contracts reduce the manufacturer's cost? What is the impact of implementing the menu of contracts on the quality of the final product? Under what circumstances is the impact of the menu of contracts on manufacturer's cost and on final product quality significant?

Our numerical study is based on the combination of the following set of parameter values that result in 5400 different experiments. Please refer to the On-Line Appendix for a detailed description of the method used to calculate the optimal parameter values.

We also assume the same functional format of the effort cost functions $C_s(e_s)$ and $C_m(e_m)$ as in our first numerical analysis.

To establish a basis of comparison to represent the case with no perfect information, we evaluate the manufacturer's total expected cost when he designs a single optimal Selective Root Cause Analysis Contract based on the higher supplier failure rate.[7] We call this contract the *conservative*

[7]Assuming a conservative failure rate ensures that both type of suppliers accept the contract offered by the

case. Furthermore, we refer to the manufacturer's optimal expected cost when he knows the exact supplier's failure rate before contracting as the *perfect information case.* Next, we present the value of knowing the supplier type information measured by the impact of this information on the manufacturer's total expected costs and the final product quality.

2.6.1 Value of Information

To investigate the value of perfect information about the supplier's product quality (i.e., failure rate), for each of our 5400 cases, we compared the manufacturer's cost and product quality under perfect information with those in conservative cases through the following metrics:

$$\begin{aligned} VOI_{cost} &= \frac{\Pi_m^{*Conserv.} - \Pi_m^{*Perfect}}{\Pi_m^{*Perfect}} \times 100\% \\ VOI_{quality} &= \frac{\Lambda_T^{*Conserv.} - \Lambda_T^{*Perfect}}{\Lambda_T^{*Perfect}} \times 100\%, \end{aligned}$$

where VOI_{cost} is the percent decrease in the manufacturer's total cost if he can acquire the supplier's exact failure rate. On the other hand, $VOI_{quality}$ is the percent decrease in the failure rate (i.e., percent increase in quality) of the final product, if the manufacturer acquires information about the supplier's failure rate.

Impact on Costs

Based on our numerical study, we found that knowing the supplier's failure rate information can decrease the manufacturer's cost, on average, by 14.28%. We also observed that, in some cases, the value of information can be as high as 172.22%. The value of information increases as (i) the difference between the failure rates of the high and the low quality supplier increases, (ii) the unit recall cost is high compared to the unit cost of root cause analysis, and (iii) the initial failure rate of the manufacturer is less than the initial failure rate of the supplier.

We also observed that, all else being equal, the value of information has its maximum value when the likelihood of high failure rate supplier, $\alpha = 0.5$ (which represents the maximum unpredictability about the supplier's failure rate.)

manufacturer.

We find that the value of information is particularly valuable to a manufacturer when he has a good production process in place that is characterized by a lower failure rate. When this is the case, the likelihood of the recall cost being incurred due to the supplier's process quality is higher. Consequently, the value of information about supplier quality is higher. Therefore, for a manufacturer with good internal process capabilities, it is critical to know with what type of supplier he is contracting.

Impact on Quality

Although the the manufacturer's expected cost always decreases as he receives perfect information (i.e., VOI_{cost} is always positive), this does not mean that the quality of the final product also improves when perfect information becomes available. We observed that in 1658 out of 5400 cases of our numerical study, the $VOI_{quality}$ was negative, which indicates that the quality of the final product is lower in scenarios with perfect information. Specifically, we found that in the 1658 cases that $VOI_{quality}$ was negative, the average and the minimum of $VOI_{quality}$ were -1.6% and -12%, respectively. The remaining 3742 had positive VOI_{qualit} with an average and a maximum of 2.3% and 89.06%, respectively.

Note that, the optimal contracts under both conservative and perfect information are designed to capture the best tradeoff between effort cost and expected recall cost (and thus to minimize the manufacturer's total expected cost). Minimization of the manufacturer's total expected cost does not necessarily guarantee the improvement in product quality. It, however, is interesting to find cases that perfect information not only improves costs, but also improves quality. This corresponds to cases with negative $VOI_{quality}$. we observed that when: (i) both manufacturer and supplier produce low quality components (i.e., the initial failure rates of the manufacturer and the supplier are high), (ii) the manufacturer produces a better quality component than the supplier (i.e., the supplier's failure rate is higher than the manufacturer's failure rate), and (iii) the unit recall cost is high, then having perfect information about the supplier's quality significantly improves the quality of the final product.

Our numerical results show that the products for which failure can lead to serious safety hazards (i.e., product failure can lead to a high recall cost, e.g., the tire recall experienced by Ford and Firestone), it is critical to know the internal process capabilities of the supplier and its product's failure rate. Therefore, in these cases, the manufacturer can benefit from a long term relationship with its supplier where he acquires a better understanding of the supplier's product and process characteristics.

2.6.2 Value of Menu of Contracts

In the previous section we provided insights into cases in which the value of perfect information about supplier failure rate can reduce the manufacturer's cost significantly. Those cases present an opportunity to use a means such as menu of contracts to capture some of the value of information. In this section, we investigate how much of the value of perfect information can be captured through the optimal menu of Selective Root Cause Analysis Contracts. To measure this, we use the following two metrics:

$$\begin{aligned} VOM_{cost} &= \frac{\Pi_m^{*Conserv.} - \Pi_m^{*Menu}}{\Pi_m^{*Conserv.}} \times 100\% \\ VOM_{quality} &= \frac{\Lambda_T^{*Conserv.} - \Lambda_T^{*Menu}}{\Lambda_T^{*Conserv.}} \times 100\%, \end{aligned}$$

VOM_{cost} is the percent decrease in the manufacturer's total cost if it offers the optimal menu of contracts to the supplier. On the other hand, $VOM_{quality}$ is the percent decrease in the failure rate of the final product under the menu of contracts, if the manufacturer could acquire perfect information about the supplier's failure rate.

For each of 5400 cases of our numerical study, we obtained the optimal menu of contracts, and we calculated the manufacturer's expected cost as well as the quality of the final product.

Impact on Costs

Based on our numerical study, we found that using the optimal menu of contracts can decrease the manufacturer's cost, on average, by 13.18%. Comparing this number with the average VOI_{cost}

(which represents the average value of perfect information), we see that the menu of contracts, on average, captures 92% (= 1 − (14.28% − 13.18%)/14.28%) of the value of perfect information. We also observed that, similar to the value of perfect information, the maximum value of menu (i.e., the maximum VOM_{cost}) was also as high as 172.22%. These observations imply that the optimal menu of Selective Root Cause Analysis Contracts is an efficient way to deal with information asymmetry.

We observe that the same conditions that result in the higher value of information (i.e., conditions discussed in section 2.6.1) also result in higher value for the menu. This is expected, because if the menu captures a large fraction of the value of information, then if the value of information is low, so is the value of the menu.

Impact on Quality

To investigate the impact of implementing the optimal menu of contracts on final product quality, we study the $VOM_{quality}$ for all of our 5400 cases. We observed that implementing the optimal menu of contracts results in higher final product quality compared to that under conservative case. Specifically, we found that $VOM_{quality}$ has an average and a maximum of 12.5% and 81.8%, respectively. The improvement in product quality under the menu of contracts is larger when: (i) the manufacturer has a better initial quality than the supplier, (ii) there is a larger difference in quality of the products of two supplier types, and (iii) the unit recall cost is larger.

In conclusion, our numerical experiments show that implementing a menu of Selective Root Cause Analysis Contracts is particularly valuable for a firm when the product is new to the market (i.e., the initial failure rates are generally high), the manufacturer has relatively better process capabilities than his supplier (i.e., lower initial failure rate of the manufacturer as compared to the supplier) and the supplier is new to the manufacturer, in the sense that the manufacturer has less information about the supplier's process capabilities and faces higher uncertainty about supplier type (a large difference in different type supplier failure rate).

2.7 Conclusion

As design, engineering, and manufacturing activities evolve into the shared responsibility of supply chain members, manufacturers face the challenging task of managing their suppliers' incentives to invest in improving process quality. In this paper, we focus on recall instances, and we introduce two external quality cost sharing contracts to improve final product quality when both the manufacturer's and the supplier's quality improvement effort decisions are subject to moral hazard and when there is information asymmetry between the manufacturer and the supplier regarding the supplier's process quality.

The extant literature has discussed the Fixed Share Rate Contract, which allocates quality costs to supply chain members irrespective of the root cause of the quality problem. In this paper, we focus on understanding how the root cause analysis information should be used in contract design. When both the manufacturer's and the supplier's quality improvement effort decisions are subject to moral hazard, their quality improvement effort levels can be coordinated to achieve First Best effort levels either by implementing a Selective Root Cause Analysis Contract or a Partial Cost Allocation Contract . However, we show that a Selective Root Cause Analysis Contract attains higher total supply chain profits as it incurs lower root cause analysis costs. Interestingly, we find that to coordinate quality improvement effort decisions in a supply chain, it is not always necessary to use the root cause analysis information to allocate quality costs even if this information were perfect and available at no cost. In fact, we find that always allocating the total recall cost to the party who is at fault and has the sole responsibility for the quality problem can lead to overinvestment in quality improvement effort and can be costly to the supply chain. A Selective Root Cause Analysis Contract, which adjusts cost sharing rule to the time of failure, overcomes the overinvestment problem, and attains the First Best effort levels from the supply chain members.

From our first numerical analysis, we find that the Selective Root Cause Analysis Contract consistently performed better than the Fixed Share Rate and the Partial Cost Allocation Contracts in terms of the average and the maximum total supply chain cost. Furthermore, on average, the product quality is closer to the First Best effort levels under the Selective Root Cause Analysis Contract than under the Fixed Share Rate and Partial Cost Allocation Contracts. The Partial Cost Allocation Contract results in overinvestment of quality improvement effort leading to a lower product failure rate than that in the centralized system. This improved quality, however, brings about additional costs compared to a centralized system.

In the last section of the paper, we relax the complete information assumption and introduce a mixed model of adverse selection and moral hazard to investigate the effectiveness of a menu of Selective Root Cause Analysis Contracts to both screen suppler type and induce quality improvement effort. Even when the manufacturer does not have complete information about the process quality of his supplier, we show that one can design a menu of Selective Root Cause Analysis Contracts to both screen supplier type and, at the same time, induce supplier effort. We show that to ensure separation of supplier type, in the optimal menu of contracts, the high failure rate supplier is allocated a smaller share of total cost (smaller R) and the root cause analysis threshold for the high failure rate supplier is lower than that for the cost sharing contract designed for the low failure rate supplier (i.e. $R_H < R_L$ and $\overline{T}_H < \overline{T}_L$). We also characterize an interaction effect between the adverse selection and the moral hazard problems in the sense that, if the manufacturer were to induce high effort, then he would need to allocate higher informational rent to the low failure rate supplier. In the Selective Root Cause Analysis Contract, the contract parameters R and T provided flexibility to the manufacturer to handle the effort inducement versus information rent trade-off in the supply chain.

From our second numerical study, we find that the value of information is particularly higher for a manufacturer who has relatively better production processes in place than the supplier's process. The value of knowing supplier failure rate proved to be higher when the product is new to the market leading to higher initial failure rates. We also find that for the products where failure can lead to serious safety hazards (i.e., cases with high unit recall cost), such as the tire recall experienced by Ford and Firestone, the value of information is higher and therefore it is critical to know the internal process capabilities of the supplier and her product's failure rate. Therefore, in these cases, the manufacturer can benefit from a long term relationship with his supplier through which he could acquire a better understanding of the supplier's product and process characteristics.

Our numerical analysis also shows that by implementing a menu of Selective Root Cause Analysis Contracts, a manufacturer can attain very close to perfect information outcome. Similar to the value of perfect information analysis, we find that implementing a menu of Selective Root Cause Analysis Contracts is particularly important for a firm when the product is new to the market (i.e., the initial failure rates are generally high), the manufacturer has relatively better process capabilities than his supplier (i.e. lower initial failure rate of the manufacturer than the supplier) and the supplier is new to the manufacturer, in the sense that the manufacturer has less information about supplier's process capabilities and faces higher uncertainty about supplier type (a large difference in different type supplier failure rate).

In this paper, we made some assumptions to provide a first cut analysis of the double moral hazard problem in external quality cost management. Our future research will relax these assumptions to develop further understanding of additional complicating factors. For instance, we assumed no interaction between components, i.e., no simultaneous failures. If this assumption were relaxed, we conjecture that the Selective Root Cause Analysis Contract would still be effective in managing the quality improvement efforts of the manufacturer and the supplier. We also assumed that the root cause analysis can perfectly determine the failed component in the product. To enrich the present analysis, one could model an error rate in determining the party at fault. We anticipate that in this case, one could easily determine a critical error rate, above which contracts such as Fixed Share Rate that do not utilize root cause analysis information would be the preferred contract format instead of a coordinating scheme that used imperfect information.

The supply chain we considered consisted of a single manufacturer and a single supplier. A direct extension of this study is to look into a network of suppliers and understand the design of external quality cost sharing contracts with multiple suppliers.

APPENDIX A OF CHAPTER 2

Proofs of Analytical Results

Before we present our proofs, we first list additional notation in the following table, which we will use in this document to simplify exposition of the proofs.

Notations	Sign	Notations	Sign
$G = \frac{\lambda_m^o(1-e_m)}{\lambda_m^o(1-e_m)+\lambda_s^o(1-e_s)}$	$+$	$H = 1 - e^{-[\lambda_m^o(1-e_m)+\lambda_s^o(1-e_s)]}$	$+$
$1 - G = \frac{\lambda_s^o(1-e_s)}{\lambda_m^o(1-e_m)+\lambda_s^o(1-e_s)}$	$+$	$H'_m = -\lambda_m^0 e^{-(\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s))}$	$-$
$K = \frac{\lambda_m^o}{\lambda_m^o(1-e_m^C)+\lambda_s^o(1-e_s^C)}$	$+$	$H'_s = -\lambda_s^0 e^{-(\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s))}$	$-$
$G'_m = -K(1-G)$	$-$	$H''_{mm} = -(\lambda_m^0)^2 e^{-(\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s))}$	$-$
$G'_s = GK\frac{\lambda_s^o}{\lambda_m^o}$	$+$	$H''_{ss} = -(\lambda_s^0)^2 e^{-(\lambda_m^0(1-e_m^c)+\lambda_s^0(1-e_s^c))}$	$-$
$G''_{sm} = -K^2\frac{\lambda_s^o}{\lambda_m^o}(1-2G)$	$+/-$	$H'_{ms} = -\lambda_m^0\lambda_s^0 e^{-(\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s))}$	$-$
$G''_{mm} = -2K^2(1-G)$	$-$	$K'_s = \frac{\lambda_s^o}{\lambda_m^o}K^2$	$+$
$G''_{ss} = 2(\frac{\lambda_s^o}{\lambda_m^o})^2K^2G$	$+$	$K'_m = K^2$	$+$

PROOF OF PROPOSITION 1

This proposition presents a sufficient condition on the second order derivatives of the manufacturer's and the supplier's effort cost functions to ensure concavity of the central planner's profit function.

The central planner's problem is given by:

$$\underset{e_m\ ,\ e_s}{Max}\ \Pi^C = r - u_s - u_m - \omega\Big[1 - e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}\Big] - C_m(e_m) - C_s(e_s) \tag{2.24}$$

To ensure concavity of the central planner's profit function, we investigate the signs of the second order derivatives of the central planner's objective function, which are given by:

$$\frac{\partial^2\Pi^C}{\partial e_m^2} = \omega(\lambda_m^0)^2 e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}\ -\ C''_m(e_m) < 0 \tag{2.25}$$

$$\frac{\partial^2\Pi^C}{\partial e_m^2} = \omega(\lambda_s^0)^2 e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}\ -\ C''_s(e_s) < 0 \tag{2.26}$$

Below, we prove that $C''_m(e_m) > \omega(\lambda_m^0)^2$ and $C''_s(e_s) > \omega(\lambda_s^0)^2$ for $\forall(e_s, e_m) \in [0,1]\times[0,1]$ guarantee concavity of the central planner's profit function and consequently a unique solution to her optimization problem.

Notice that the LHS of inequality (2.25) has its largest value when $e_s = 1$. Therefore, it is sufficient to have:

$$\omega(\lambda_m^0)^2 e^{-\lambda_m^0(1-e_m)} - C''_m(e_m) < 0 \quad \text{for} \quad 0 \le e_m \le 1 \tag{2.27}$$

or equivalently,

$$\omega(\lambda_m^0)^2 e^{-\lambda_m^0(1-e_m)} \; < C''_m(e_m) \quad \text{for} \quad 0 \leq \; e_m \; \leq 1 \tag{2.28}$$

Similarly, the LHS of inequality (2.26) assumes its largest value when $e_m = 1$. Therefore, it is sufficient to have

$$\omega(\lambda_s^0)^2 e^{-\lambda_s^0(1-e_s)} \; - \; C''_s(e_s) < 0 \quad \text{for} \quad 0 \leq \; e_s \leq 1$$

or equivalently,

$$\omega(\lambda_s^0)^2 e^{-\lambda_s^0(1-e_s)} \; < C''_s(e_s) \quad \text{for} \quad 0 \leq \; e_s \; \leq 1$$

Since $e^{-\lambda_s^0(1-e_s)} \leq 1$ and $e^{-\lambda_m^0(1-e_m)} \leq 1$, to ensure concavity of the centralized profits, it is sufficient to have $C''_m(e_m) > \omega(\lambda_m^0)^2$ and $C''_s(e_s) > \omega(\lambda_s^0)^2$. Therefore, to ensure concavity of the central planner's optimization problem, we require that the second order derivatives of the manufacturer's and the supplier's effort cost functions are greater than a threshold defined by the initial failure rates λ_m^0, λ_s^0 and the unit recall cost, ω. When the initial failure rates and/or the unit recall cost increase, the sufficient condition for concavity becomes more restrictive.

PROOF OF PROPOSITION 2

The manufacturer's and the supplier's optimization problems are, respectively, given by:

$$\underset{e_m}{Max} : \Pi_m^N = r - u_m - p_0 - \omega(1 - e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]}) - C_m(e_m)$$

$$\underset{e_s}{Max} : \Pi_s^N = p_0 - u_s - C_s(e_s)$$

Since the effort cost function of the supplier is increasing in e_s (i.e., $C'_s(e_s) > 0$), her profit function is decreasing in effort e_s. Therefore, $e_s^{*N} = 0$.

To show that $e_m^{*N} < e_m^{*C}$, we first evaluate the first order condition of the manufacturer's objective function at $e_m^{*N} = e_m^{*C}$ and $e_s^{*N} = 0$. This results in:

$$\frac{\partial \Pi_C}{\partial e_m^C} = \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0]} - C'_m(e_s^{*C})$$

Since,

$$\frac{\partial \Pi_C}{\partial e_m^C} \,|_{e_m^{*C}, e_s^{*C}} = 0 \qquad \Longrightarrow \qquad \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]} = C'_m(e_m^{*C})$$

and $e_m^{*C} > 0$, then one can rewrite the first order optimality condition of the manufacturer's profit function evaluated at $e_m^{*N} = e_m^{*C}$ and $e_s^{*N} = 0$ as follows:

$$\begin{aligned} \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0]} - \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]} \quad &= \\ \omega\lambda_m^0 e^{-\lambda_m^0(1-e_m^{*C})}\left(e^{-\lambda_s^0} - e^{-\lambda_s^0(1-e_s^{*C})}\right) \quad &< \quad 0 \end{aligned}$$

Negativity of the above expression and concavity of the manufacturer's objective function ensure that $e_m^{*N} < e_m^{*C}$. Therefore, compared with the First Best effort level, the manufacturer underinvests in effort under No Cost Sharing, and the supplier exerts zero effort.

PROOF OF PROPOSITION 3

Before we present the proof of Proposition 3, we present Proposition 3a which ensures supermodularity of the effort game under Selective Root Cause Analysis Contract.

Proposition 3a *Under the Selective Root Cause Analysis Contract, to ensure that the effort game played between the supplier and the manufacturer is supermodular, it is sufficient to have* $\lambda_s^0 = k\lambda_m^0$ *where* $k \in [0.36, 2.73]$

Proof of Proposition 3a

Our goal is to identify two sufficient conditions under the Selective Root Cause Analysis Contract, which ensure non-negative cross partial derivatives of the manufacturer's and the supplier's profit functions with respect to e_s and e_m. This enables us to use the properties of supermodular game theory in characterizing the equilibrium outcome of the effort game played between the manufacturer and the supplier.

Consider the supplier's profit function under the Selective Root Cause Analysis Contract:

$$\Pi_s^S = p - u_s - (\omega + c_r)(\frac{\lambda_s^0(1-e_s^S)}{\Lambda_T})[1 - e^{-\Lambda_T \overline{T}}] - R\omega[e^{-\Lambda_T \overline{T}} - e^{-\Lambda_T}] - C_s(e_s) \qquad (2.29)$$

where $\Lambda_T = \lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)$, $0 \le \overline{T} \le 1$, $0 \le R \le 1$, $0 \le e_m \le 1$ and $0 \le e_s \le 1$.

The cross partial derivative of the supplier's profit function with respect to e_m and e_s is given by:

$$\begin{aligned} \frac{\partial^2 \Pi_s^S}{\partial e_m \partial e_s} &= e^{-\Lambda_T} R\omega\lambda_m^0\lambda_s^0 + \frac{e^{-\Lambda_T\overline{T}}(\omega + c_r)\overline{T}^2\lambda_m^0(1-e_s^S)(\lambda_s^0)^2}{\Lambda_T} & (2.30) \\ &\quad - \frac{(1-e^{-\Lambda_T\overline{T}})(\omega+c_r)\lambda_m^0\lambda_s^0}{\Lambda_T^2}(\frac{(\lambda_s^0)^2(1-e_s)}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0}) & \\ &\quad + \frac{e^{-\Lambda_T\overline{T}}(\omega+c_r)\overline{T}\lambda_m^0\lambda_s^0}{\Lambda_T}(\frac{2(1-e_s^S)\lambda_s^0}{\Lambda_T} + 1) & (2.31) \end{aligned}$$

The first two terms and the last term in (2.30) are non-negative, since $0 \le e_m, e_s \le 1$, $0 \le \overline{T} \le$,and $0 \le R \le 1$. The third term is non-negative when $\frac{(\lambda_s^0)^2(1-e_s)}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \le 0$. The LHS of the inequality has its largest value when $e_s = 0$. Therefore, a sufficient condition for (2.30) to be non-negative is $\frac{(\lambda_s^0)^2}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \le 0$.

Now consider the manufacturer's profit function under the Selective Root Cause Analysis Contract:

$$\Pi_m^S = r - u_m - p - (\omega + c_r)(\frac{\lambda_m^o(1-e_m^S)}{\Lambda_T})[1 - e^{-\Lambda_T\overline{T}}] - (1-R)\omega[e^{-\Lambda_T\overline{T}} - e^{-\Lambda_T}] - C_m(e_m) \qquad (2.32)$$

where $\Lambda_T = \lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)$, $\overline{T} \leq 1$, $0 \leq R \leq 1$, $0 \leq e_m \leq 1$ and $0 \leq e_s \leq 1$. The cross partial derivative of Π_m^S with respect to e_s and e_m is given by:

$$\frac{\partial^2 \Pi_s^S}{\partial e_m \partial e_s} = e^{-\Lambda_T}(1-R)\omega\lambda_m^0\lambda_s^0 + \frac{e^{-\Lambda_T \overline{T}}(\omega + c_r)\overline{T}^2\lambda_s^0(1-e_m^S)(\lambda_m^0)^2}{\Lambda_T} \tag{2.33}$$

$$-\frac{(1-e^{-\Lambda_T\overline{T}})(\omega+c_r)\lambda_m^0\lambda_s^0}{\Lambda_T^2}\left(\frac{(\lambda_m^0)^2(1-e_m)}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0}\right) \tag{2.34}$$

$$+\frac{e^{-\Lambda_T\overline{T}}(\omega+c_r)\overline{T}\lambda_m^0\lambda_s^0}{\Lambda_T}\left(\frac{2(1-e_m^S)\lambda_m^0}{\Lambda_T}+1\right) \tag{2.35}$$

The first two terms and the last term in (2.33) are non-negative since, $0 \leq e_m \leq 1$, $0 \leq \overline{T} \leq$,and $0 \leq R \leq 1$. The third term is non-negative when $\frac{(\lambda_m^0)^2(1-e_m)}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$. The left hand side of the inequality has its largest value when $e_m = 0$. Therefore, a sufficient condition for (2.33) to be non-negative is $\frac{(\lambda_m^0)^2}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$.

Consequently, we find that $\frac{(\lambda_s^0)^2}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \leq 0$ and $\frac{(\lambda_m^0)^2}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$ are two sufficient conditions that ensure positive cross partial derivatives of the manufacturer's and the supplier's objective functions, respectively. Positive cross partial derivatives results in a supermodular effort game, in which effort decisions are strategic complements (i.e., higher effort exerted by one player leads to higher effort exerted by the other player).

If we define $\lambda_s^0 = k\lambda_m^0$, then the above two conditions can be rewritten as $k^2 - 2(k+1) < 0$ (instead of $\frac{(\lambda_s^0)^2}{(\lambda_m^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_m^0} \leq 0$) and $2k^2 + 2k - 1 > 0$ (instead of $\frac{(\lambda_m^0)^2}{(\lambda_s^0)^2} - \frac{2(\lambda_m^0+\lambda_s^0)}{\lambda_s^0} \leq 0$).
Using some algebra, we find that when $k \in [\frac{1}{2}(-1+\sqrt{3}), 1+\sqrt{3}] = [0.36, 2.73]$ the two conditions hold simultaneously. This implies that when the manufacturer's and the supplier's initial failure rates are not drastically different, the supermodularity is satisfied. This ends the proof of Proposition 3.a.

Note that the above condition is a sufficient condition that guarantees the supermodularity of the game. In our numerical study we have observed several cases that violated this condition and the game was still supermodular.

We now return to the proof of Proposition 3. First note that $\frac{\partial^2 \Pi_m^S}{\partial e_m \partial e_s} > 0$ and $\frac{\partial^2 \Pi_m^S}{\partial e_m \partial e_s} > 0$ ensures the supermodularity of the profit functions of the manufacturer and the supplier on lattice [0, 1]×[0, 1]. Theorems 2.2 of Vives (1999) (i.e. Tarski's Fixed Point Theorem) and Theorem 2.5 of Vives (1999) (i.e., Topkis' s Theorem about equilibria in supermodular games) establishes parts (a) and (b).

Part (c) follows from the complementarity of the effort choices, i.e., best response function of the manufacturer (supplier) is increasing in the effort choice of the supplier (manufacturer). Specifically, define a pair of equilibria $(\widetilde{e}_s, \widetilde{e}_m)$ and $(\widehat{e}_s, \widehat{e}_m)$ and let $\widetilde{e}_s \leq \widehat{e}_s$ then, $\widetilde{e}_m = e_m^*(\widetilde{e}_s) \leq e_m^*(\widehat{e}_s) = \widehat{e}_m$.

For Part (d), define a pair of equilibria $(\widetilde{e}_s, \widetilde{e}_m)$ and $(\widehat{e}_s, \widehat{e}_m)$ such that $\widetilde{e}_s \leq \widehat{e}_s$ and $\widetilde{e}_m \leq \widehat{e}_m$. From

$\frac{\partial \Pi_s}{\partial e_m} > 0$ (i.e., supplier's profits are increasing with the manufacturer's quality improvement effort), it follows that $\Pi_s^*(\widetilde{e}_s, \widetilde{e}_m) \leq \Pi_s^*(\widetilde{e}_s, \widehat{e}_m) \leq \Pi_s^*(e_s^*(\widehat{e}_m), \widehat{e}_m) = \Pi_s^*(\widehat{e}_s, \widehat{e}_m)$. A similar argument can be made for the manufacturer's profits under the equilibria $(\widetilde{e}_s, \widetilde{e}_m)$ and $(\widehat{e}_s, \widehat{e}_m)$. Consequently, both the manufacturer and the supplier attain higher profits at the equilibrium in which both show high effort. This completes the proof of Proposition 3.

In what follows, we focus on this equilibrium outcome, which is the most preferred equilibrium (the Pareto optimal outcome) for both parties.

PROOF OF PROPOSITION 4

Under the Selective Root Cause Analysis Contract, the manufacturer's and the supplier's optimization problems are given by:

$$\begin{aligned}
\underset{e_m}{Max} \ &: \ \Pi_m^S = r - u_m - p - (\omega + c_r)G[1 - e^{-\Lambda_T \overline{T}}] - \omega(1-R)[e^{-\Lambda_T \overline{T}} - e^{-\Lambda_T}] - C_m(e_m) \\
\underset{e_s}{Max} \ &: \ \Pi_s^S = p - u_s - (\omega + c_r)(1-G)[1 - e^{-\Lambda_T \overline{T}}] - R\omega[e^{-\Lambda_T \overline{T}} - e^{-\Lambda_T}] - C_s(e_s)
\end{aligned}$$

where $\Lambda_T = \lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)$ and $G = \frac{\lambda_m^0(1-e_m)}{\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)}$.

(i) Coordinating Contract $(\overline{T}^*, R^*)$ *for the Asymmetric Game When* $c_r > 0$*:*

The coordinating values of $\overline{T}^*$ and R^* are given by the simultaneous solution of the following equations which follow from $\frac{\partial \Pi_m^S}{\partial e_m}(\overline{T}^*, R^*) \mid_{e_m^{*C}, e_s^{*C}} = \frac{\partial \Pi^C}{\partial e_m} \mid_{e_m^{*C}, e_s^{*C}} = 0$, and $\frac{\partial \Pi_s^S}{\partial e_s}(\overline{T}^*, R^*) \mid_{e_m^{*C}, e_s^{*C}} = \frac{\partial \Pi^C}{\partial e_s} \mid_{e_m^{*C}, e_s^{*C}} = 0$

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial e_m} &= -\Big[(\omega + c_r)\frac{G_m'^{*C}}{\lambda_m^0}(1 - e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}e^{-\Lambda_T^{*C}\overline{T}}[-(\omega + c_r)G^{*C} + (1-R)]\Big] - \frac{C_m'(e_m^{*C})}{\lambda_m^0} = 0 \\
\frac{\partial \Pi_s^s}{\partial e_s} &= -\Big[-(\omega + c_r)\frac{G_s'^{*C}}{\lambda_s^0}(1 - e^{-\Lambda_T^{*C}\bar{T}}) + \overline{T}e^{-\Lambda_T^{*C}\overline{T}}[-(\omega + c_r)(1 - G^{*C}) + R]\Big] - \frac{C_s'(e_s^{*C})}{\lambda_s^0} = 0
\end{aligned}$$

Unfortunately, the complexity of the above equations do not allow us to derive closed-form solutions for R^* and $\overline{T}^*$. However, we can show that there exists a unique R^* and $\overline{T}^*$ that solves the above first order conditions (FOCs) at the first best effort level. To show this, we add the two FOCs and obtain:

$$\left(\frac{G_s'^{*C}}{\lambda_s^0} - \frac{G_m'^{*C}}{\lambda_m^0}\right)(\omega + c_r)(1 - e^{-\Lambda_T^{*C}\overline{T}}) - (\omega + c_r)(-\overline{T}e^{-\Lambda_T^{*C}\overline{T}}) - \omega(\overline{T}e^{-\Lambda_T^{*C}\overline{T}} - e^{-\Lambda_T^{*C}\bar{T}}) - \frac{C_m'(e_m^{*C})}{\lambda_m^0} - \frac{C_s'(e_s^{*C})}{\lambda_s^0} = 0$$

From the FOCs of the first best case, and $G_s' = GK\frac{\lambda_m^0}{\lambda s}$ and $G_m' = -(1-G)K$ (where $K = \frac{\lambda_m^0}{\Lambda_T^0}$), the above equation simplifies to:

$$\frac{\omega + c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}} = \Big[c_r\overline{T} + \frac{(\omega + c_r)}{\Lambda_T^{*C}}\Big]e^{-\Lambda_T^{*C}\overline{T}}$$

Note that the LHS is positive and is independent of $\overline{T}$ and the RHS is a continuous convex decreasing function of $\overline{T}$. Hence, to show that a unique $\overline{T}$ exists, we show that the RHS expression has a value greater (smaller) than the LHS when $T = 0$ (when$T = 1$) When $T = 0$, the RHS expression equals to $\frac{\omega+c_r}{\Lambda_T^{*C}}$, which is greater than $\frac{\omega+c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}}$, the value of the left hand side. When $T = 1$, we need to show that $\frac{\omega+c_r}{\Lambda_T^{*C}} - \omega e^{-\Lambda_T^{*C}} > \Big[c_r + \frac{(\omega+c_r)}{\Lambda_T^{*C}}\Big]e^{-\Lambda_T^{*C}}$, which simplifies to showing $(\omega + c_r)\frac{1}{\Lambda_T^{*C}} > (\omega+c_r)\frac{1}{\Lambda_T^{*C}}[\Lambda_T^{*C}+1]e^{-\Lambda_T^{*C}}$, which is equivalent to showing $e^{\Lambda_T^{*C}} > 1+\Lambda_T^{*C}$. The inequality $e^{\Lambda_T^{*C}} > 1+\Lambda_T^{*C}$ is satisfied from $e^{\Lambda_T^{*C}} = \sum_{k=0}^{\infty}\frac{(\Lambda_T^{*C})^k}{k!}$.

(ii) Coordinating Contract $(\overline{T}^*, R^*)$ *for the Asymmetric Game When* $c_r = 0$*:*

We evaluate the first order conditions of the manufacturer's and the supplier's profit functions at the first best level of effort. Consequently, we obtain:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial e_m} &= -\omega\Big[G_m^{\prime *C}(1-e^{-\Lambda_T^{*C}\overline{T}})\overline{+T}\lambda_m^0 e^{-\Lambda_T^{*C}\overline{T}}\omega[G^{*C}-(1-R)]-(1-R)\lambda_m^0 e^{-\Lambda_T^{*C}}\Big] - C_m^{'}(e_m^{*C}) = 0\\
\frac{\partial \Pi_s^T}{\partial e_s} &= -\omega\Big[-G_s^{\prime *C}(1-e^{-\Lambda_T^{*C}\overline{T}})+\overline{T}\lambda_s^0 e^{-\Lambda_T^{*C}\overline{T}}[(1-G^{*C})-R]-R\omega\lambda_s^0 e^{-\Lambda_T^{*C}}\Big] - C_s^{'}(e_s^{*C}) = 0
\end{aligned}$$

where $C_m^{'}(e_m^{*C}) = \omega\lambda_m^0 e^{-\lambda_T^{*C}}$, $C_s^{'}(e_s^{*C}) = \omega\lambda_s^0 e^{-\Lambda_T^{*C}}$ (please refer to the first best case presented in the main body of the paper). One can write:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial e_m} &= -\omega\Big[G_m^{\prime *C}(1-e^{-\Lambda_T^{*C}\overline{T}})+\overline{T}\lambda_m^0 e^{-\Lambda_T^{*C}\overline{T}}[-G^{*C}+(1-R)]-(1-R)\lambda_m^0 e^{-\Lambda_T^{*C}}\Big] - \omega\lambda_m^0 e^{-\Lambda_T^{*C}} = 0\\
\frac{\partial \Pi_s^S}{\partial e_s} &= -\omega\Big[-G_s^{\prime *C}(1-e^{-\Lambda_T^{*C}\overline{T}})+\overline{T}\lambda_s^0 e^{-\Lambda_T^{*C}\overline{T}}[G^{*C}-(1-R)]-R\lambda_s^0 e^{-\Lambda_T^{*C}}\Big] - \omega\lambda_s^0 e^{-\lambda_T^{*C}} = 0
\end{aligned}$$

Let $(1 - R) = G^{*C}$,then the above equations are simplified as:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial e_m} &= -\omega\Big[G_m^{\prime *C}(1-e^{-\Lambda_T^{*C}\overline{T}})-(1-R)\lambda_m^0 e^{-\Lambda_T^{*C}}\Big] - \omega\lambda_m^0 e^{-\lambda_T^{*C}} = 0\\
\frac{\partial \Pi_s^S}{\partial e_s} &= -\omega\Big[-G_s^{\prime *C}(1-e^{-\Lambda_T^{*C}\overline{T}})-R\lambda_s^0 e^{-\lambda_T^{*C}}\Big] - \omega\lambda_s^0 e^{-\Lambda_T^{*C}} = 0
\end{aligned}$$

Using $G_m^{\prime} = -K(1-G)$, $G_s^{'} = GK\frac{\lambda_s^0}{\lambda_m^0}$ and $(1-R) = G^{*C}$, we can further simply the above equations into:

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial e_m} &= \omega(1-G^{*C})\lambda_m^0\Big[\frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\lambda_T^{*C}} - e^{-\Lambda_T^{*C}}\Big] = 0\\
\frac{\partial \Pi_s^S}{\partial e_s} &= \omega G^{*C}\lambda_s^0\Big[\frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\lambda_T^{*C}} - e^{-\Lambda_T^{*C}}\Big] = 0
\end{aligned}$$

Solving for $\overline{T}$ from $\frac{(1-e^{-\Lambda_T^{*C}\overline{T}})}{\Lambda_T^{*C}} - e^{-\Lambda_T^{*C}} = 0$ we obtain $\overline{T} = -\frac{Ln(1-\Lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}}$.

(iii) Coordinating Contract in the Symmetric Game When $c_r > 0$*:*

We parameterize $c_r = k\omega$ where $k > 0$ and derive the first order conditions in the symmetric game as follows. Note that in the symmetric game $G^{*C} = \frac{1}{2}$.

$$\begin{aligned}
\frac{\partial \Pi_m^S}{\partial e_m} &= (1+k)\frac{1}{2\Lambda_T}(1-e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}e^{-\lambda_T^{*C}\overline{T}}\Big[\frac{(1+k)}{2} - (1-R)\Big] - Re^{-\lambda_T^{*C}} = 0 \\
\frac{\partial \Pi_s^S}{\partial e_s} &= (1+k)\frac{1}{2\Lambda_T}(1-e^{-\Lambda_T^{*C}\overline{T}}) + \overline{T}e^{-\lambda_T^{*C}\overline{T}}\Big[\frac{(1+k)}{2} - R)\Big] - (1-R)e^{-\Lambda_T^{*C}} = 0
\end{aligned}$$

Since the first expression of both equations are the same, we can write :

$$\begin{aligned}
\overline{T}e^{-\Lambda_T^{*C}\overline{T}}\Big[\frac{(1+k)}{2} - (1-R)\Big] - Re^{-\Lambda_T^{*C}} &= \overline{T}e^{-\lambda_T^{*C}\overline{T}}\Big[\frac{(1+k)}{2} - R\Big] - (1-R)e^{-\lambda_T^{*C}} \\
-\overline{T}e^{-\Lambda_T^{*C}\overline{T}}(1-2R) &= -(1-2R)e^{-\Lambda_T^{*C}} \\
(1-2R)(e^{-\Lambda_T^{*C}} - \overline{T}e^{-\Lambda_T^{*C}\overline{T}}) &= 0
\end{aligned}$$

Hence, when $R^* = (1-R^*) = \frac{1}{2}$ then $\overline{T}^*$ can be calculated from the FOC as follows: $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}}$.

PROOF OF COROLLARY 1

We show that (i) $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} > 0$, and (ii) $\overline{T}^* = 0$ leads to underinvestment in effort by the manufacturer and the supplier. This implies that the Fixed Share Rate Contract, which is a special case of the Selective Root Cause Analysis Contract when $\overline{T}^* = 0$, cannot achieve the first best effort level from the supply chain partners.

(i) From $\lim_{e_s \to 1} C_s'(e_s) = \infty$ and $\lim_{e_m \to 1} C_m'(e_m) = \infty$ and $\lambda_m^0 > 0$ and $\lambda_s^0 > 0$, it follows that $\Lambda_T^{*C} > 0$ and $1 - \lambda_T^{*C}e^{-\Lambda_T^{*C}} < 1$. Consequently, $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} > 0$.

(ii) When $\overline{T}^* = 0$, we obtain the Fixed Share Rate Contract as the special case of Selective Root Cause Analysis Contract. In this case, the manufacturer's and the supplier's optimization problems are given by:

$$\begin{aligned}
&\underset{e_m}{Max} \ : \ \Pi_m^S = r - u_m - p - \omega(1-R)[e^{-\Lambda_T\overline{T}} - e^{-\Lambda_T}] - C_m(e_m) \\
&\underset{e_s}{Max} \ : \ \Pi_s^S = p - u_s - \omega R[e^{-\Lambda_T\overline{T}} - e^{-\Lambda_T}] - C_s(e_s)
\end{aligned}$$

The first order conditions of the manufacturer's and the supplier's optimization problems are, respectively, given by:

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial e_m} &= \omega(1-R)\lambda_m^0 e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]} - C_m'(e_m) = 0 \\
\frac{\partial \Pi_s}{\partial e_s} &= \omega R\lambda_s^0 e^{-[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]} - C_s'(e_s) = 0
\end{aligned}$$

Since $C_m'(e_m^{*C}) = \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]}$ and $C_s'(e_s^{*C}) = \omega\lambda_s^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]}$ and $C_s'(e_s) > 0$, $C_s'(e_s) > 0$, $C_m''(e_m) < 0$ and $C_s''(e_s) < 0$, it follows that the left hand side of the FOCs evaluated at

(e_m^{*C}, e_s^{*C}) are negative. Therefore $e_s^{*F} < e_s^{*C}$ and $e_m^{*F} < e_m^{*C}$, where F denotes the Fixed Share Rate Contract.

PROOF OF COROLLARY 2

We show that (i) $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} < 1$ and (ii) $\overline{T}^* = 1$ leads to overinvestment in effort by the manufacturer and the supplier. This implies that the total cost allocation contract, which is a special case of the Selective Root Cause Analysis Contract when $\overline{T}^* = 1$, cannot achieve the first best effort levels.

(i) Note that $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} < 1$ when $Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}}) > -\Lambda_T^{*C}$. This condition can be restated as $1-\lambda_T^{*C}e^{-\Lambda_T^{*C}} > e^{-\Lambda_T^{*C}}$. Notice that when $\frac{1+\lambda_T^{*C}}{e^{\Lambda_T^{*C}}} < 1$ then $\overline{T}^* = -\frac{Ln(1-\lambda_T^{*C}e^{-\Lambda_T^{*C}})}{\Lambda_T^{*C}} < 1$. Since $e^{\Lambda_T^{*C}} = 1+\Lambda_T^{*C}+\frac{(\Lambda_T^{*C})^2}{2!}+\cdots$, it follows that $\frac{1+\lambda_T^{*C}}{e^{\Lambda_T^{*C}}} < 1$. Consequently, the optimal root cause analysis threshold time that attains First Best effort level is less than 1. Therefore, sharing the external quality costs based always on root cause analysis information (where $R = 0$) cannot attain the centrally coordinated (First Best) quality.

(ii) When $\overline{T}^* = 1$, the manufacturer's and the supplier's optimization problems are given by:

$$\begin{aligned} \underset{e_m}{Max} &: \Pi_m^S = r - u_m - p - (\omega + c_r)G[1-e^{-\Lambda_T}] - C_m(e_m) \\ \underset{e_s}{Max} &: \Pi_s^S = p - u_s - (\omega + c_r)(1-G)[1-e^{-\Lambda_T}] - C_s(e_s) \end{aligned}$$

where $G = \frac{\lambda_m^0(1-e_m)}{\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)}$. The respective first order conditions evaluated at the first best effort levels are given by:

$$\begin{aligned} \frac{\partial \Pi_m}{\partial e_m} &= -(\omega + c_r)(G'_m(1-e^{-\Lambda_T^{*C}}) - G\lambda_m^0 e^{-\Lambda_T^{*C}}) - C'_m(e_m^{*C}) \\ \frac{\partial \Pi_s}{\partial e_s} &= -(\omega + c_r)(-G'_s(1-e^{-\Lambda_T^{*C}}) - (1-G)\lambda_s^0 e^{-\Lambda_T^{*C}}) - C'_s(e_s^{*C}) \end{aligned}$$

where $G'_m = \frac{\partial G}{\partial e_m}$ and $G'_s = \frac{\partial G}{\partial e_s}$.

Let $c_r = 0$ (since the optimal effort is increasing in $(\omega + c_r)$ the overinvestment result still holds when $c_r > 0$). ¿From $C'_m(e_m^{*C}) = \omega\lambda_m^0 e^{-[\lambda_m^0(1-e_m^{*C})+\lambda_s^0(1-e_s^{*C})]}$, the FOC of the manufacturer's optimization problem evaluated at the first best effort level can be restated as $\frac{\partial \Pi_m}{\partial e_m} = -\omega(G'_m(1-e^{-\Lambda_T^{*C}})) + \omega(1-G)\lambda_m^0 e^{-\Lambda_T^{*C}}$. Since $G'_m < 0$, it follows that $\frac{\partial \Pi_m}{\partial e_m}\mid_{e_m^{*C}, e_s^{*C}} > 0$. Therefore, when $\overline{T}^* = 1$, the manufacturer overinvests in quality improvement effort relative to the First Best effort level. A similar reasoning is used to prove the supplier's overinvestment in effort result.

PROOF OF PROPOSITION 5

The manufacturer's and the supplier's objective functions under the Partial Cost Allocation Contract are given by:

$$\begin{aligned}
\underset{e_m}{Max} \ &: \ \Pi_m^P = r - u_m - p - (\omega + c_r)((1 - R_m)\frac{\lambda_m^o(1 - e_m^P)}{\Lambda_T} + (1 - R_s)\frac{\lambda_s^o(1 - e_s)}{\Lambda_T})(1 - e^{-\Lambda_T}) - C_m(e_m) \\
\underset{e_s}{Max} \ &: \ \Pi_s^P = p - u_s - (\omega + c_r)(R_m\frac{\lambda_m^o(1 - e_m)}{\Lambda_T} + R_s\frac{\lambda_s^o(1 - e_s)}{\Lambda_T})(1 - e^{-\Lambda_T}) - C_s(e_s)
\end{aligned}$$

The supermodularity of the symmetric effort game under partial cost allocation ensures the existence of at least one Nash equilibrium. Furthermore, similar to the Selective Root Cause Analysis Contract, one can easily show that the equilibria are rankable and both the supplier and the manufacturer attain higher profits in the equilibrium where they both show high effort. Therefore, in what follows, we focus on this most preferred (Pareto optimal) equilibrium point and discuss the effort coordinating contract.

Next, we show the supermodularity result for the symmetric effort game under Partial Cost Allocation Contract. This is followed by the derivation of the effort coordinating contract parameters.

We restate the manufacturer's and the supplier's optimization problems as follows. Let $c_r = k\omega$ and $k \geq 0$, then:

$$\begin{aligned}
\underset{e_m}{Max} \ : \ \Pi_m &= r - u_m - p - \omega(1 + k)[(1 - R_m)G + (1 - R_s)(1 - G)]H - C_m(e_m) \\
\underset{es}{Max} \ : \ \Pi_s &= p - u_s - \omega(1 + k)[R_mG + R_s(1 - G)]H - C_s(e_s)
\end{aligned}$$

where R_m is the supplier's share of total recall cost when the manufacturer is at fault and R_s is the supplier's share of total recall cost when the supplier is at fault. The first order derivatives of the manufacturer's and the supplier's objective functions are, respectively, given by:

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial e_m} &= -\omega(1 + k)\big[((1 - R_m)G_m^{'} - (1 - R_s)G_m^{'})H + [(1 - R_m)G + (1 - R_s)(1 - G)]H_m^{'}\big] - C_m^{'}(e_m) \\
\frac{\partial \Pi_s}{\partial e_s} &= -\omega(1 + k)\big[[R_mG_s^{'} - R_sG_s^{'}]H + [(R_mG + R_s(1 - G)]H_s^{'}\big] - C_s^{'}(e_s)
\end{aligned}$$

The cross partial derivatives of the manufacturer's and the supplier's objective functions are are respectively given by:

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial e_m \partial e_s} &= -\omega(1 + k)\Big[((1 - R_m)G_{ms}^{'} - (1 - R_s)G_{ms}^{'})H + ((1 - R_m)G_m^{'} - (1 - R_s)G_m^{'})H_s^{'} \\
&\quad +((1 - R_m)G_s^{'} - (1 - R_s)G_s^{'})H_m^{'} + [(1 - R_m)G + (1 - R_s)(1 - G)]H_{ms}^{'}\Big] \\
\frac{\partial \Pi_m}{\partial e_s \partial e_m} &= -\omega(1 + k)\Big[[(R_m)G_{sm}^{'} - (R_s)G_{sm}^{'}]H + [(R_m)G_s^{'} - (R_s)G_s^{'}]H_m^{'} \\
&\quad +[(R_m)G_m^{'} - (R_s)G_m^{'}]H_s^{'} + [(R_m)G + (R_s)(1 - G)]H_{sm}^{'}\Big]
\end{aligned}$$

Note that in the symmetric effort game where the supplier and the manufacturer have identical initial process failure rates and effort cost functions, $G_{ms}^{'} = 0$ and $G_m^{'} = -G_s^{'}$ and $H_s^{'} = H_m^{'}$. Consequently, the above expressions simplify to:

$$\begin{aligned}\frac{\partial\Pi_m}{\partial e_m \partial e_s} &= -\omega(1+k)[(1-R_m)G+(1-R_s)(1-G)]H_{ms}^{'}\\ \frac{\partial\Pi_m}{\partial e_s \partial e_m} &= -\omega(1+k)[(1-R_m)G+(1-R_s)(1-G)]H_{ms}^{'}\end{aligned}$$

Since $H_{ms}^{'} < 0$, it follows that $\frac{\partial\Pi_m}{\partial e_m \partial e_s} \geq 0$ and $\frac{\partial\Pi_m}{\partial e_s \partial e_m} \geq 0$, which ensures the supermodularity of the symmetric effort game under Partial Cost Allocation Contract.

To analyze the effort coordinating contract under partial cost allocation scheme, consider the first order derivatives of the manufacturer's and the supplier's objective functions that can be simplified as follows:

$$\begin{aligned}\frac{\partial\Pi_m}{\partial e_m} &= -\omega(1+k)\Big[\Delta G_m^{'}H+(\Delta GH_m^{'}+(1-R_s)H_m^{'})\Big]-C_m^{'}(e_m)\\ \frac{\partial\Pi_s}{\partial e_s} &= -\omega(1+k)\Big[(-\Delta G_s^{'})H+(R_sH_s^{'}-\Delta GH_s^{'})\Big]-C_s^{'}(e_s)\end{aligned}$$

where $\Delta = R_s - R_m$. We evaluate $\frac{\partial\Pi_m}{\partial e_m}$ and $\frac{\partial\Pi_s}{\partial e_s}$ at the first best effort levels (e_s^{*C}, e_m^{*C}). Then, we solve for R_m^* and Δ^* that satisfy $\frac{\partial\Pi_m}{\partial e_m}(e_s^{*C}, e_m^{*C}, R_m^*, \Delta^*) = \frac{\partial\Pi^C}{\partial e_m^C}(e_s^{*C}, e_m^{*C})$ and $\frac{\partial\Pi_s^P}{\partial e_s}(e_s^{*C}, e_m^{*C}, R_s^*, \Delta^*) = \frac{\partial\Pi^C}{\partial e_s^C}(e_s^{*C}, e_m^{*C})$ simultaneously to obtain

$$\begin{aligned}R_m^* &= \frac{A^*D^*-B^*\lambda_m^0 e^{-\Lambda_T^{*C}}+A^*\lambda_s^0 e^{-\Lambda_T^{*C}}}{B^*C^*+A^*D^*}\\ \Delta^* &= R_s^*-R_m^* \text{ and } \Delta^* = \frac{R_sC^*-\lambda_m^0 e^{-\Lambda_T^{*C}}}{A^*}\end{aligned}$$

where $A^* = -(1+\frac{c_r}{\omega})(G_m^{'}H - GH_m^{'})$, $B^* = (1+\frac{c_r}{\omega})(G_s^{'}H - GH_s^{'})$, $C^* = (1+\frac{c_r}{\omega})H_m^{'}$ and $D^* = (1+\frac{c_r}{\omega})H_m^{'}$ are evaluated at $(e_m^{*C}\ ,\ e_s^{*C})$.

PROOF OF COROLLARY 3

Under the selective root cause analysis, at the First Best effort level, the likelihood of performing a root cause analysis is given by $(1-e^{-\Lambda_T\overline{T}^*})$, where $\Lambda_T = \Lambda_T = \lambda_m^0(1-e_m^C)+\lambda_s^0(1-e_s^C)$ and $\overline{T}^* < 1$. The expected root cause analysis cost of the supply chain is given by $c_r(1-e^{-\Lambda_T\overline{T}^*})$.

Under the partial cost allocation scheme, at the first best effort level, the root cause analysis is performed for all failures. Consequently, the expected root cause analysis cost of the supply chain is given by $c_r(1-e^{-\Lambda_T})$.

Since $\overline{T}^* < 1$, at the first best effort level, the expected root cause analysis cost under Selective Root Cause Analysis Contract is less than the expected root cause analysis cost under Partial Cost Allocation Contract. As the threshold time $\overline{T}^*$ gets smaller, the cost difference between the two contracts gets larger.

PROOF OF PROPOSITION 6

We start the proof by showing a condition which ensures separation of supplier type under the Spence-Mirrlees single crossing property (lemma 1). Next, for the Selective Root Cause Analysis menu of contracts, we prove the monotonicity constraints on the contract parameters (R_H , T_H) and (R_L , T_L)

Spence-Mirrlees Single Crossing Property

To ensure that Spence-Mirrlees single crossing property holds, we require that $\frac{\partial Z}{\partial \lambda_{s_j} \partial R} = Z_{\lambda_{s_j} R} > 0$ and $\frac{\partial Z}{\partial \lambda_{s_j} \partial \overline{T}} = Z_{\lambda_{s_j} \overline{T}} > 0$ hold (Laffont and Martimort, 2002). To this end, we investigate the sign of $\frac{\partial Z}{\partial \lambda_{s_j} \partial R} = Z_{\lambda_{s_j} R}$ and $\frac{\partial Z}{\partial \lambda_{s_j} \partial \overline{T}} = Z_{\lambda_{s_j} \overline{T}}$ where $Z(\lambda_{S_j}, R_k, \overline{T}_k, e^r_{s_j})$ is given by:

$$\begin{aligned} Z(\lambda_{S_j}, R_k, \overline{T}_k, e^r_{s_j}) &= w\frac{\lambda_{s_j}(1-e^r_{s_j})}{\lambda_m + \lambda_{s_j}(1-e^r_{s_j})}(1 - e^{-(\lambda_m + \lambda_{s_j}(1-e^r_{s_j}))\overline{T}_k}) \\ &\quad + wR_k(e^{-(\lambda_m + \lambda_{s_j}(1-e^r_{s_j}))\overline{T}_k} - e^{-(\lambda_m + \lambda_{s_j}(1-e^r_{s_j}))}) \end{aligned}$$

We calculate $\frac{\partial Z}{\partial \lambda_{s_j} \partial R} = Z_{\lambda_{s_j} R}$ and $\frac{\partial Z}{\partial \lambda_{s_j} \partial \overline{T}} = Z_{\lambda_{s_j} \overline{T}}$ as follows:

$$\begin{aligned} Z_{\lambda_{s_j} R} &= e^{-\overline{T}\lambda_T} w(1-e^r_{s_j})(e^{-(1-\overline{T})\lambda_T} - \overline{T}) \\ Z_{\lambda_{s_j} \overline{T}} &= e^{-\overline{T}\lambda_T} w(1-e^r_{s_j})[(1-R)(1-\overline{T}(1-e^r_{s_j})\lambda_{s_j}) + \overline{T}R\lambda_m] \end{aligned}$$

Now consider $Z_{\lambda_{s_j} R}$. Note that $Z_{\lambda_{s_j} R} > 0$ if $(e^{-(1-\overline{T})\lambda_T} - \overline{T}) > 0$. $(e^{-(1-\overline{T})\lambda_T} - \overline{T}) > 0$ holds if $\lambda_T < -\frac{Ln(\overline{T})}{(1-\overline{T})}$. Notice that $-\frac{Ln(\overline{T})}{(1-\overline{T})}$ is decreasing in $\overline{T}$, $\underset{\overline{T}\to 0}{lim} - \frac{Ln(\overline{T})}{(1-\overline{T})} = \infty$ and $\underset{\overline{T}\to 1}{lim} - \frac{Ln(\overline{T})}{(1-\overline{T})} = 1$. Hence the most restrictive bound on λ_T is 1. Therefore, $Z_{\lambda_{s_j} R} > 0$ for $0 \leq \overline{T} \leq 1$ if $\lambda_T = \lambda_m + \lambda_s(1-e^r_{s_j}) < 1$ for $e^r_{s_j} \in \{e^L_{s_j}, e^H_{s_j}\}$. Hence, it is sufficient that $\lambda_T = \lambda_m + \lambda_s(1-e^L_{s_j}) < 1$ holds, which ensures $Z_{\lambda_{s_j} R} > 0$.

Now consider $Z_{\lambda_{s_j} \overline{T}} > 0$. From $0 \leq \overline{T} \leq 1$, $Z_{\lambda_{s_j} R} > 0$ $\big($i.e., $\lambda_T = \lambda_m + \lambda_s(1-e^L_{s_j}) < 1\big)$ and $(1-e^r_{s_j})\lambda_{s_j} < 1$, it follows that $(1 - \overline{T}(1-e^r_{s_j})\lambda_{s_j}) > 0$. Consequently, $Z_{\lambda_{s_j} \overline{T}} > 0$. Therefore, $\lambda_m + \lambda_s(1-e^L_{s_j}) < 1$ is sufficient to guarantee that both $Z_{\lambda_{s_j} R} > 0$ and $Z_{\lambda_{s_j} \overline{T}} > 0$ hold.

Monotonicity of (R_H , T_H) and (R_L , T_L)

To ensure truthful revelation of supplier type, the menu of contracts should be *incentive compatible*. More specifically, given an incentive compatible menu, the supplier of type λ_{s_L} weakly prefers $S_L = (p_L, R_L, \overline{T}_L)$ over $S_H = (p_H, R_H, \overline{T}_H)$ and the supplier of type $\lambda^0_{s_H}$ weakly prefers $S_H = (p_H, R_H, \overline{T}_H)$ over $S_L = (p_L, R_L, \overline{T}_L)$. These requirements are formalized by the following *Incentive Compatibility (I.C.) Constraints.*

$$p_H - u_s - Z(\lambda^0_{S_H}, R_H, \overline{T}_H, e^r_{s_H}) - C_s(e^r_{s_H}) > p_L - u_s - Z(\lambda^0_{S_H}, R_L, \overline{T}_L, e^r_{s_H}) - C_s(e^r_{s_H}) \quad (2.5)$$

$$p_L - u_s - Z(\lambda^0_{S_L}, R_L, \overline{T}_L, e^r_{s_L}) - C_s(e^r_{s_L}) > p_H - u_s - Z(\lambda^0_{S_L}, R_H, \overline{T}_H, e^r_{s_L}) - C_s(e^r_{s_L}) \quad (2.6)$$

By adding (2.5) and (2.6) we obtain:

$$Z(\lambda^0_{S_H}, R_H, \overline{T}_H, e^r_{s_j}) - Z(\lambda^0_{S_L}, R_H, \overline{T}_H, e^r_{s_j}) < Z(\lambda^0_{S_H}, R_L, \overline{T}_L, e^r_{s_j}) - Z(\lambda^0_{S_L}, R_L, \overline{T}_L, e^r_{s_j}) \quad (2.36)$$

$Z_{\lambda_{s_j}R} > 0$, $Z_{\lambda_{s_j}\overline{T}} > 0$and $Z_{\lambda_{s_j}R\overline{T}} > 0$ imply that the expression $Z(\lambda^0_{S_H}, R, \overline{T}, e^r_{s_j}) - Z(\lambda^0_{S_L}, R, \overline{T}, e^r_{s_j})$ is increasing with R and T. Therefore to satisfy (2.36), it is sufficient to have $R_L > R_H$ and $\overline{T}_L > \overline{T}_H$.

Note that $Z_{\lambda_{s_j}R\overline{T}} = e^{-\lambda_T}w(1-e_{s_j})(1-e^{(1-\overline{T})\lambda_T}\overline{T}).Z_{\lambda_{s_j}R\overline{T}}$ is non-negative if $(1-e^{(1-\overline{T})\lambda_T}\overline{T}) \geq 0$. Furthermore, $(1-e^{(1-\overline{T})\lambda_T}\overline{T}) \geq 0$ if $e^{(1-\overline{T})\lambda_T} \leq \frac{1}{\overline{T}}$. Notice that $0 \leq \overline{T} \leq 1$ and both sides of the inequality are decreasing in $\overline{T}$ with the LHS assuming values in the range $[1, e^{(1-\overline{T})\lambda_T}]$ and the RHS assuming values in the range $[1,\infty)$. Consequently, $(1-e^{(1-\overline{T})\lambda_T}\overline{T}) > 0$ and $Z_{\lambda_{s_j}R\overline{T}} > 0$.

PROOF OF REMARK 1

We first characterize the condition under which it is optimal to induce high effort from the high failure rate supplier (i.e., $e^H_{s_H}$). Fixing the low failure rate supplier's effort level at $e^r_{s_L}$, we compare the manufacturer's profits when the high failure rate supplier exerts high effort ($\widehat{\Pi}_m$) to his profits when the high failure rate supplier exerts low effort ($\widetilde{\Pi}_m$). This is formalized as follows (from (2.18)):

$$\begin{aligned}
\widehat{\Pi}_m &= r - u_m - u_s - (1-\alpha)[Z(\lambda^0_{S_H}, R_H, \overline{T}_H, e^H_{s_H}) - Z(\lambda^0_{S_L}, R_H, \overline{T}_H, e^r_{s_L})] \\
&\quad -\omega[\alpha(1-e^{-[\lambda^0_m+\lambda^0_{S_H}(1-e^H_{s_H})]})] - \omega[(1-\alpha)(1-e^{-[\lambda^0_m+\lambda^0_{S_L}(1-e^r_{s_L})]})] \\
&\quad -\alpha C_s(e^H_{s_H}) - (1-\alpha)C_s(e^r_{s_L})
\end{aligned}$$

$$\begin{aligned}
\widetilde{\Pi}_m &= r - u_m - u_s \qquad (2.37) \\
&\quad -\omega[\alpha(1-e^{-[\lambda^0_m+\lambda^0_{S_H}(1-e^L_{s_H})]})] - \omega[(1-\alpha)(1-e^{-[\lambda^0_m+\lambda^0_{S_L}(1-e^r_{s_L})]})] \\
&\quad -\alpha C_s(e^L_{s_H}) - (1-\alpha)C_s(e^r_{s_L})
\end{aligned}$$

The condition given below directly follows from simplification of the inequality $\widehat{\Pi}_m > \widetilde{\Pi}_m$.

$$\begin{aligned}
&\frac{(1-\alpha)}{\alpha}[Z(\lambda^0_{S_H}, R^*_H, \overline{T}^*_H, e^H_{S_H}) - Z(\lambda^0_{S_L}, R^*_H, \overline{T}^*_H, e^r_{s_L})] + [C_s(e^H_{s_H}) - C_s(e^L_{s_H})] \qquad (2.38) \\
> \quad &\omega[e^{-[\lambda^0_m+\lambda^0_{s_H}(1-e^H_{s_H})]} - e^{-[\lambda^0_m+\lambda^0_{s_H}(1-e^L_{s_H})]}]
\end{aligned}$$

Note that a solution $R^*_H > 0$ and $T^*_H > 0$ to (2.20) exists.

$$(2.20) \qquad Z(\lambda^0_{S_H}, R^*_H, \overline{T}^*_H, e^L_{s_H}) - Z(\lambda^0_{S_H}, R^*_H, \overline{T^*}_H, e^H_{s_H}) = C_s(e^H_{s_H}) - C_s(e^L_{s_H})$$

The proof follows from (i) $\frac{\partial Z}{\partial e_{s_j}} < 0$, (ii) $\frac{\partial Z}{\partial e_{s_j}\partial R} < 0$ and $\frac{\partial Z}{\partial e_{s_j}\partial T} < 0$, and (iii) $C'_s(e^k_{s_j}) > 0$ which implies $C_s(e^H_{s_H}) - C_s(e^L_{s_H}) > 0$.

Condition (i) implies that Z is decreasing in the supplier's effort. Conditions (ii) and (iii) imply that as R_H and T_H increase, the difference $Z(\lambda^0_{S_H}, R^*_H, \overline{T^*}_H, e^L_{s_H}) - Z(\lambda^0_{S_H}, R^*_H, \overline{T^*}_H, e^H_{s_H})$ increases. Furthermore, since $Z(\lambda^0_{S_H}, 0, 0, e^L_{s_H}) = 0$, $Z(\lambda^0_{S_H}, 0, 0, e^H_{s_H}) = 0$ and $C_s(e^H_{s_H}) - C_s(e^L_{s_H})$ is positive, there exists $R^*_H > 0$ and $\overline{T}^*_H > 0$ that solves (2.20).

Note that $\frac{\partial Z}{\partial e_{s_j}} < 0$ because

$$\frac{\partial Z}{\partial e_{s_j}} = -\frac{(1-e^{-\lambda_T \overline{T}})(c+\omega)\lambda^0_{s_j}\lambda^0_m}{\lambda^2_T} - \frac{e^{-\lambda_T \overline{T}}(c+\omega)e_{s_j}\overline{T}\lambda^{02}_{s_j}}{\lambda_T} - R\omega\lambda^0_{s_j}(e^{-\lambda_T} - e^{-\lambda_T \overline{T}}\overline{T})$$

and $e^{-\lambda_T} - e^{-\lambda_T \overline{T}}\overline{T} > 0$ for $0 \le \overline{T} \le 1$, which implies that all of the terms in $\frac{\partial Z}{\partial e_{s_j}}$ expression are negative.

Furthermore, $Z_{e_{s_j}R} = -e^{-\overline{T}\lambda_T} w(e^{-(1-\overline{T})\lambda_T} - \overline{T})\lambda_T$ and $Z_{e_{s_j}R} = Z_{e_{s_j}\overline{T}} = e^{-\overline{T}\lambda_T} w\lambda[-(1-R) + \overline{T}(-R\lambda^0_m + (1-R)(1-e_{s_j})\lambda^0_s)]$. From $0 \le \overline{T} \le 1$ and $e^{-(1-\overline{T})\lambda_T} - \overline{T} > 0$, it follows that $Z_{e_{s_j}R} < 0$.From $0 \le \overline{T} \le 1$, $0 \le R \le 1$ and $\lambda_m(1-e_m) + (1-e_s)\lambda_s < 1$, it follows that $[-(1-R) + \overline{T}(-R\lambda^0_m + (1-R)(1-e_{s_j})\lambda^0_s)] < 0$.Consequently, $Z_{e_{s_j}\overline{T}} < 0$.

PROOF OF REMARK 2

Notice that when $R^*_H = 0$ and $\overline{T}^*_H = 0$, the information rent is zero. Consequently, the manufacturer chooses $e^r_{s_L}$ to minimize the following objective function: $\Pi_m = r - u_s - u_m - (1-\alpha)[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-e^r_{s_L})]}) + C_s(e^r_{s_L})] - \alpha[\omega(1 - e^{-[\lambda^0_m + \lambda^0_{s_H}]}) + C_s(e^L_{s_H})]$. Since $\frac{\partial \Pi_m}{\partial e^r_{s_L}} = (1-\alpha)\frac{\partial \Pi^C_j}{\partial e^r_{s_L}}$, the manufacturer chooses the centrally coordinated first best effort level as the optimal effort for the low failure rate supplier.

Suppose $e^{*C}_{s_L} = e^H_{s_L}$ then the optimal R^*_L and $\overline{T}^*_L$ is chosen to satisfy the following moral hazard constraint.

$$Z(\lambda^0_{S_L}, R^*_L, \overline{T}^*_L, e^L_{s_L}) - Z(\lambda^0_{S_L}, R^*_L, \overline{T}^*_L, e^H_{s_L}) = C_s(e^H_{s_L}) - C_s(e^L_{s_L}) \qquad (2.21)$$

R^*_L and T^*_L exists because (i) $\frac{\partial Z}{\partial e_{s_j}} < 0$, (ii) $\frac{\partial Z}{\partial e_{s_j}\partial R} < 0$ and $\frac{\partial Z}{\partial e_{s_j}\partial T} < 0$, and (iii) $C'_s(e^k_{s_j}) > 0$ which implies $C_s(e^H_{s_L}) - C_s(e^L_{s_L}) > 0$.

Condition (i) implies that Z is decreasing in the supplier's effort. Conditions (ii) and (iii) imply that as R_L and T_L increase, the difference $Z(\lambda^0_{S_L}, R^*_L, \overline{T^*}_L, e^L_{s_L}) - Z(\lambda^0_{S_H}, R^*_L, \overline{T^*}_L, e^H_{s_L})$ increases. Furthermore, since $Z(\lambda^0_{S_L}, 0, 0, e^L_{s_L}) = 0$, $Z(\lambda^0_{S_L}, 0, 0, e^H_{s_L}) = 0$ and $C_s(e^H_{s_L}) - C_s(e^L_{s_L})$ is positive, then there exists $R^*_L > 0$ and $\overline{T}^*_L > 0$ that solves (2.21).

PROOF OF REMARK 3

When $R_H^* > 0$ and $\overline{T}_H^* > 0$ (i.e., when it is optimal to induce high effort from the high failure rate supplier), then it is optimal to induce high effort from the low failure rate supplier when the savings in external quality costs dominate the information rent and the incremental effort cost incurred due to high effort. Specifically, high effort is optimal from the low failure rate supplier when:

$$\begin{aligned} & [Z(\lambda^0_{s_L}, R_H^*, \overline{T}_H^*, e^L_{s_L}) - Z(\lambda^0_{s_L}, R_H^*, \overline{T}_H^*, e^H_{s_L})] + [C_s(e^H_{s_L}) - C_s(e^L_{s_L})] \\ < \quad & \omega[e^{-[\lambda^0_m + \lambda^0_{s_L}(1-e^H_{s_L})]} - e^{-[\lambda^0_m + \lambda^0_{s_L}(1-e^L_{s_L})]}] \end{aligned} \tag{2.22}$$

To prove the above inequality, we fix the effort of the high failure rate supplier to $e^H_{s_H}$. We define $\widehat{\Pi}_m$ as the manufacturer's profits when the low failure rate supplier shows high effort, and $\widetilde{\Pi}_m$ as the manufacturer's profits when the low failure rate supplier shows low effort. Specifically,

$$\begin{aligned} \widehat{\Pi}_m \quad = \quad & r - u_m - u_s - (1-\alpha)[Z(\lambda^0_{S_H}, R_H, \overline{T}_H, e^H_{s_H}) - Z(\lambda^0_{S_L}, R_H, \overline{T}_H, e^H_{s_L})] \\ & -\omega[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{S_H}(1-e^H_{s_H})]})] - \omega[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{S_L}(1-e^H_{s_L})]})] \\ & -\alpha C_s(e^H_{s_H}) - (1-\alpha) C_s(e^H_{s_L}) \end{aligned} \tag{2.39}$$

$$\begin{aligned} \widetilde{\Pi}_m \quad = \quad & r - u_m - u_s \\ & -\omega[\alpha(1 - e^{-[\lambda^0_m + \lambda^0_{S_H}(1-e^H_{s_H})]})] - \omega[(1-\alpha)(1 - e^{-[\lambda^0_m + \lambda^0_{S_L}(1-e^L_{s_L})]})] \\ & -\alpha C_s(e^H_{s_H}) - (1-\alpha) C_s(e^L_{s_L}) \end{aligned} \tag{2.40}$$

The condition in (2.22) directly follows from simplification of the inequality $\widehat{\Pi}_m > \widetilde{\Pi}_m$.

Based on arguments presented in Remark 1 and Remark 2, one can show that $(R_H^*, \overline{T}_H^*)$ and $(R_L^*, \overline{T}_L^*)$ exist. Furthermore, $\frac{\partial Z}{\partial e_{s_j} \partial \lambda_j} < 0$ (i.e., the impact of effort on the expected recall cost is larger at higher initial failure rates) implies that $Z(\lambda^0_{S_H}, R_H^*, \overline{T}_H^*, e^L_{s_H}) - Z(\lambda^0_{S_H}, R_H^*, \overline{T}_H^*, e^H_{s_H}) > Z(\lambda^0_{S_L}, R_H^*, \overline{T}_H^*, e^L_{s_H}) - Z(\lambda^0_{S_L}, R_H^*, \overline{T}_H^*, e^H_{s_H})$. From $\frac{\partial Z}{\partial e_{s_j} \partial R} < 0$ and $\frac{\partial Z}{\partial e_{s_j} \partial T} < 0$, it follows that $Z(\lambda^0_{S_L}, R_H^*, \overline{T}_H^*, e^L_{s_H}) - Z(\lambda^0_{S_L}, R_H^*, \overline{T}_H^*, e^H_{s_H}) < Z(\lambda^0_{S_L}, R_L^*, \overline{T}_L^*, e^L_{s_H}) - Z(\lambda^0_{S_L}, R_L^*, \overline{T}_L^*, e^H_{s_H})$ if $R_L^* > R_H^*$ and $\overline{T}_L^* > \overline{T}_H^*$. Therefore, $Z(\lambda^0_{S_L}, R_L^*, \overline{T}_L^*, e^L_{s_H}) - Z(\lambda^0_{S_L}, R_L^*, \overline{T}_L^*, e^H_{s_H}) = C_s(e^H_{s_L}) - C_s(e^L_{s_L})$ is satisfied when $R_L^* > R_H^*$ and $\overline{T}_L^* > \overline{T}_H^*$.

Note that $\frac{\partial Z}{\partial e_{s_j} \partial \lambda_j}$ is given by:

$$\begin{aligned} \frac{\partial Z}{\partial e_{s_j} \partial \lambda_j} \quad = \quad & \frac{-2(1 - e^{-\lambda_T \overline{T}})\omega(1-e_{s_j})^2 \lambda^2_{s_j}}{\lambda^3_T} - \frac{(1 - e^{-\lambda_T \overline{T}})\omega}{\lambda_T} - \frac{3e^{-\lambda_T \overline{T}}\omega(1-e_{s_j})\overline{T}\lambda^0_{s_j}}{\lambda_T} \\ & + R\omega(-e^{-\lambda_T}(1-(1-e_{s_j})\lambda^0_{s_j}) - e^{-\lambda_T \overline{T}}(1-(1-e_{s_j})\lambda^0_{s_j}\overline{T})) \end{aligned}$$

¿From lemma 1 (i.e., $\lambda^0_m + (1-e_{s_j})\lambda^0_{s_j} < 1$) and $0 \le \overline{T} \le 1$, it follows that $\frac{\partial Z}{\partial e_{s_j} \partial \lambda_j} < 0$.

APPENDIX B OF CHAPTER 2

An Algorithm for Obtaining the Optimal Menu of Selective Root Cause Analysis Contracts

In this appendix we present our algorithm for obtaining the optimal menu of contracts. This algorithm is for general cases where both the manufacturer and the supplier can exert quality improvement efforts, which assume continuous values in the range of [0,1]. To calculate the optimal ($\overline{T_L^*}$, R_L^*) and ($\overline{T_H^*}$, R_H^*), we use the following sequence of steps:

Step 1: For a given $\overline{T}_H$ and R_H and manufacturer effort e_m, for each supplier type, we calculate the optimal effort levels and the optimal expected costs, which are, respective, denoted by $E_s\big[cost|\overline{T}_H,\, R_H, \lambda_{high}\big]$ and $E_s\big[cost|\overline{T}_H,\, R_H, \lambda_{low}\big]$ for high and low failure rate suppliers.

Step 2: We set the price of the high failure rate supplier to $p_H = E_s\big[cost|\overline{T}_H,\, R_H, \lambda_{high}\big]$, which makes her indifferent between accepting or rejecting the contract. We calculate the information rent (π) of the low failure rate supplier by $\pi = E_s\big[cost|\overline{T}_H, R_H, \lambda_{high}\big] - E_s\big[cost|\overline{T}_H, R_H, \lambda_{low}\big]$.

Step 3: We calculate $E_m\big[cost|\overline{T}_H,\, R_H, p_H, \lambda_{high}\big]$, the manufacturer's expected cost given that the supplier is the high failure rate supplier.

Step 4: We solve the following optimization problem to determine the optimal contract parameters for the low failure rate supplier, i.e. ($\overline{T_L^*}$, R_L^*). Notice from (2.41) that we leave the low failure rate supplier indifferent between accepting or rejecting the contract.

$$\underset{(\overline{T}_L, R_L)}{\mathbf{Min}}\ E_m\Big[cost|\overline{T}_L, R_L, p_2, \lambda_{low}\Big]$$

subject to:

$$p_L \quad = \quad \pi + E_s\Big[cost|\overline{T}_L, R_L, \lambda_{low}\Big] \tag{2.41}$$

Step 5: With the probability α, the supplier has a high failure rate and with probability $(1-\alpha)$, the supplier has a low failure rate. We can calculate the expected manufacturer's cost to optimize the manufacturer's profits by solving:

$$\begin{aligned} &\underset{e_m}{Min} E_m\Big[cost|\overline{T}_L, R_L, p_L, \overline{T}_H, R_H, p_H\Big] \\ &= \alpha E_m\Big[cost|\overline{T}_H, R_H, p_H, \lambda_{high}\Big] + (1-\alpha) E_m\Big[cost|\overline{T}_L, R_L, p_L, \lambda_{low}\Big] \end{aligned}$$

Step 6: We change $\overline{T}_H$ and R_H (between 0 and 1) and repeat steps 1 to 5 until we obtain ($\overline{T_L^*}, R_L^*$) and ($\overline{T_H^*}, R_H^*$) that minimizes $E_m^*\big[cost|\overline{T}_L, R_L, p_L, \overline{T}_H, R_H, p_H\big]$.

Chapter 3

External Quality Cost Sharing Contracts and Pricing Strategies in the Monopoly Market

ABSTRACT

In this chapter, we investigate the equilibrium behavior of a decentralized supply chain with the quality cost-sharing contracts in the monopoly market. The implementation of the quality cost-sharing strategy will improve the product quality, and the improved product quality will result in more sales and profits. Meanwhile, the sale price will affect the market demand and the profit, too. How to design the quality cost-sharing contract and how to set up the sale price influences the total profit of this supply chain.

Here we consider the quality cost-sharing contracts conducted in five different ways: centrally coordinated, no share, fixed share rate, total cost allocation, and selective root cause analysis. We systematically compare these contracts between one manufacturer and one supplier in a monopoly market under the following two scenarios, (i) one-stage decision process; (ii) two-stage decision process.

We find that the optimal quality improvement effort levels depend on the failure information, effort cost information, and consumer preferences. Neither sale prices nor the market competition influence the optimal quality improvement effort. The contract with the selective root cause analysis achieves the maximum market share and profit when compared with the other contracts in the decentralized environment.

3.1 Introduction

How to manage product quality has been an important topic in corporate strategies in order to strengthen competitiveness, to expand market share, and to maximize profits. Companies have implemented quality control processes, such as TQM, Six Sigma, and SPC, on their own production or service processes. However, because of the economies of scale and their technological ability, the companies may outsource their parts, component design, and production to other members in their supply chains. Both the automotive and computer industries are excellent practitioners to incorporate the outsourcing strategy to strengthen competitiveness in the market.

As companies outsource more product designs and manufacturing activities to other members in the supply chain, how to improve the quality of the final product has become a challenge beyond the boundaries of its in-house process capabilities. In addition to on-site quality improvement, manufacturers might also ask suppliers to take the responsibility of quality management and to share the external quality costs, like warranty costs, recall expenses, and other related costs.

In 2004, automakers in North America spent $12 billion to fix vehicle quality problems, which amounted to approximately $400 $700 per car sold in the U.S.. They spent an average of 250 days to fix a quality problem, and incurred $1 million for each day of product recall. Who should pay for these huge expenses? Traditionally, manufacturers pay for all quality problems like warranty claims and recalls, even though the product failures are due to suppliers' mistakes. However, recently, manufacturers have started to ask their suppliers to take the responsibility for product quality, e.g. Ford-Firestone's tire recall in 2000. Without any pre-determined contract, the negotiation, even a lawsuit, will take a long time and many resources to settle the case. Therefore, what kind of quality cost-sharing contract do they need? How should they share those recall and/or warranty costs in order to minimize the total expected quality cost?

There are a number of papers discussing the effects of incentives for quality cost-sharing between members in supply chains. (Corbett and Decroix 2001, Lim 2001, Reyniers and Tapiero 1995, Baiman, et al. 2000, 2001, and 2003, Balachandran and Radhakrishnan 2005) These authors design contracts that act as quality control tools to push suppliers to improve their component quality. Penalties will be enforced if there are failure detections during the incoming appraisals before sale or warranty claims after sale. With the penalties, the both players will choose the optimal effort levels to improve the product quality in order to reduce the potential failures. The optimal effort is driven by the trade-offbetween the marginal effort cost and the marginal repair cost. However, the market size and the sale price are set as constants in these papers. If the quality cost-sharing contract is successfully implemented in a supply chain, the benefits will include not only warranty cost reduction and quality improvement, but also increased product competitiveness and expanded market share. The extra profits will result from the cost reduction and the market expansion.

In this paper, we will focus on how the manufacturer could set up the pricing strategy and quality cost-sharing contracts in order to maximize the total profits. We will expand on the contract designs developed by Chao, Iravani and Savaskan (2007), but we will focus on the warranty cost instead of the recall cost. The share of quality costs can be determined by either a fixed share rate or by the root cause analysis, or both. The contracts with a fixed share rate and total cost allocation will result in under and over-investment, respectively, by comparing with the centrally coordinated environment. Also, as the root cause analysis cost increases, the failure analysis becomes less attractive in terms of profitability. In this study, we also show that a contract with the selective root cause analysis will eliminate the inefficiency, due to a fixed share rate and the total cost allocation.

The objective function is to maximize the profits which depend on the expected quality costs, market demand, and the sale price. The expected quality costs include the potential warranty cost and the effort cost, which are determined by the effort levels. We model the consumer demand with a multinomial Logit model, where the sale price and the final product quality will determine the consumers' purchase utility. Then, we compare the optimal effort levels and the optimal sale prices under different contracts in the monopoly market. We show that the optimal effort levels can be obtained with the closed form solutions and easily compared among different contracts. Moreover, the optimal effort depends on the market demographics, but not on the number of competitors or the sale price. A pure monopoly product hardly exists today, and we regard the product with very small scale competition and very limited substitutes as the monopoly products. There are some examples, like F-35 joint strike fighter in the defense industry and Apple iPhone in the electronic industry. When the manufacturers introduces a new product into the market, they can enforce the quality cost-sharing contracts to coordinate the quality improvement effort, to set up the sale prices, and to maximize the profits.

In the following section, we briefly survey the literature about quality costing-sharing and pricing strategies. In Section 3.3, we introduce the models and the assumptions about quality cost-sharing contracts and market situations. Then, Section 3.4 will compare the different contracts in terms of the effort level, sale price, and profits in a monopoly market. We present the managerial insights and our conclusions in Section 3.5.

3.2 Literature

The main focus of this paper is on modeling external quality cost-sharing contracts between a manufacturer and a supplier. In addition to the share rate and/or time threshold, the manufacturer will also decide the potential sale price in the monopoly market, in order to maximize the total profit on the supply chain. This paper contributes to several streams of research, and we will review them below.

A group of papers have discussed the design of quality cost-sharing contracts. Reyniers and Tapiero (1995a and 1995b), Lim (2001) and Baiman el al. (2000) used the game theoretic setup to characterize the Nash equilibrium between the supplier's choice of component quality improvement and the manufacturer's choice of inspection strategy. Both the manufacturer and the supplier share the expected cost together with a fixed share rate and they try to minimize the total expected cost with the given prices. Balahandran and Radhakrishnan (2005) considered a double moral hazard situation concerning quality investment effort. They focused on the best use of incoming inspection information to achieve first best effort levels from the supply chain members. In our paper, we will relax the incoming part inspection, and the manufacturer will also exert the quality improvement effort. Both parties will contribute their effort to increase the profits on the supply chain. Corbett and Decroix (2001) have discussed a shared saving contract for the reduction of indirect materials consumption. Both parties work together to put the equilibrium effort to achieve the goal of the consumption reduction.

Baiman et al. (2001) investigated that product defects happened due to non-separable and separable failures. If the failure was separable, the manufacturer could perfectly identify the cause that led to the product failure. The responsibility could be clarified, and the individual who caused this failure should pay the cost of that external quality failure. If the failure was non-separable, both the manufacturer and the supplier would share the cost of the external failure together. With this failure analysis information, both parties exert proper effort levels to ensure quality improvement and external quality cost reduction. We will treat the information to identify the responsibility as the root cause analysis. In this paper, we will discuss how to incorporate the root cause analysis information into the external quality cost sharing contract.

Under the different contract setting, there are many observations about the final equilibrium. Iyer et al. (2005) studied how the buyer could design a menu of contracts to suppliers, in order to minimize the expected cost by assessing the supplier's capability and allocating some internal resource to help the supplier. They found that the optimal resource commitment depends on the interactions with supplier's capability. If the buyer resource and supplier capability were substitutes, then the buyer would put more effort in the second best equilibrium (over-investment) and the information rent would be driven down; the opposite would be true if they were complements.

Chao et al. (2007) compared the cost sharing contracts with first best, no sharing, a fixed share rate, total cost allocation, partial cost allocation and the selective root cause analysis. The adoption of the root cause analysis is certainly beneficial to the reduction of product failure opportunity. However, too much information may result in over-investment and the cost of the root cause analysis would vary the profitability of the cost-sharing contract. By implementation of selective root cause analysis, the supply chain could achieve the lower expected cost and the better product quality in the decentralized supply chain. In this paper, we continue the setup in Chao et al. (2007), but focus on the pricing strategy and the influence from the sale price.

In this chapter, the total profit will be determined by the unit profit and market share where the market share will be defined as a multinomial Logit model. The Logit model is easy to use and good for predicting the market share. It estimates the impact of different product characteristics, such as price, quality, band recognition, physical dimensions, promotions, etc. on the consumer purchase behaviors.

Anderson and Palma (1992) presented an approach to describe the demand of heterogeneous consumers. They used the multinomial Logit (MNL) model as an analytical tool into market analysis, and used it to address the question of market and optimal product diversity. The Logit model can also provide a convenient representation of the degree of heterogeneity of consumer tastes. They showed the sale price as the sum of the cost and the value creation, where the value creation could be calculated by the ratio of the horizontal differentiation to the probability of a consumer non-choosing.

Besanko et al. (1998) showed an empirical study of Logit brand choice where the price could be determined endogenously from the equilibrium results of Nash competition among the manufacturer and the retailer. The value creation and market share for the product could also be obtained endogenously. The market share leader must also be the value-creation leader. They validated their findings in two product categories: yogurt and catsup. They also found that the bias in the price coefficients is due to consumer heterogeneity, rather than to price endogeneity, even though there was no consumer heterogeneity in band preferences, price, and other marketing mix variables. If the price endogeneity was ignored, Berry(1994) showed the estimation methods can be severely misleading by applying Monte Carlo methods. Draganska and Jain (2004) also showed the price endogeneity for the simultaneous estimation of structure demand based on a likelihood-based method.

Bernstein and Federgruen (2004) studied the equilibrium model for industries with price and service decisions within the price competition only, simultaneous price and service-level competition, and two-stage competition (service level then price). Retailers optimize their own sale prices, fill rates, and base stock levels, in order to maximize the final profits, which are determined by the unit profits and the potential shortage costs. They showed the existence of a unique Nash equilibrium of infinite-horizon stationary strategies, and in a reduced game. In the simultaneous game or two-stage game with generalized MNL or linear demand functions, each retailer's optimal service level is completely independent from the characteristics of the other competitors, and the optimal service level is obtained by equalizing the incremental operational costs and the incremental retail price value. That is, the retailers could choose their own service levels on the basis of their own characteristics only.

3.3 Basic Model Formulation

Consider a manufacturer produces a product that consists of two components, one from his own process, and the other from his supplier at a unit purchase price p. The unit price, p, is negotiated between the manufacturer and supplier.There are unit production costs, c_m and c_s, associated with the production processes on the manufacturer's and supplier's sites, respectively. The product is sold under a warranty and the unit revenue (sale price) v is collected by the manufacturer. Without losing the generality, we normalize the warranty period into 1. During the warranty period, the product may fail to perform its function if any one of those two components fails. These failure events are identically and independently distributed. The manufacturer will take responsibility to restore the broken component to function normally as a new one instantaneously, and the repair cost, ω, will be incurred. To simplify the exposition, we assume the repair cost per warranty claim, ω, is independent of the cause of the product failure. Here the repair cost, ω, does not only include the repair cost for the broken component, but also the labor cost, logistics cost, goodwill cost, and other expenses related to this warranty claim. Thus, ω is a constant in dependent of the failure resulted from the manufacturer's component or the supplier's component. The quality cost-sharing contract is designed so that the manufacturer and the supplier share this quality cost and exert a certain effort to improve the quality of their own components.

3.3.1 Failure Mechanism and Effort Cost Function

We assume the number of failures happening during the warranty period is a Poisson distribution, and the initial failure rates of the components are common knowledge to both parties, and are denoted as λ_m^0 and λ_s^0 for the manufacturer's and the supplier's components, respectively. The failure rates are time homogeneous during the warranty period and they are independent of each other. Once the broken component has been fixed, it performs as well as a new one, and the failure rate for that component is the same as the original one. In the quality cost-sharing contract, we assume that the manufacturer and the supplier will exert some effort to improve the product quality. We use e_m and e_s to denote the percentage of quality improvement effort exerted by the manufacturer and the supplier, respectively, where $0 \leq e_m, e_s < 1$. Under the effort level e_m (e_s) the manufacturer (supplier) can reduce the failure rate from λ_m^0 (λ_s^0) to $\lambda_m^0[1-e_m]$ ($\lambda_s^0[1-e_s]$). And the expected total repair cost can be reduced from $\omega[\lambda_m^0+\lambda_s^0]$ to $\omega[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)]$ during the warranty period.

In addition to the reduced repair cost, the total quality cost also includes the quality improvement cost. The effort of e_m and e_s will cost the manufacturer and the supplier $C_m(e_m)$ and $C_s(e_s)$, respectively. We consider $C_m(e_m)$ and $C_s(e_s)$ to be twice continuously differentiable on [0,1), and increasing convex in effort. The assumptions of $C_m^{'}(0) = 0$, $C_s^{'}(0) = 0$, $\lim\limits_{e_m \to 1} C_m^{'}(e_m) = \infty$, and $\lim\limits_{e_s \to 1} C_s^{'}(e_s) = \infty$ ensure that there is an interior solution in the effort game. In this chapter, we will use $C_m(e_m) = \gamma_m[-Ln(1-e_m)-e_m]$ and $C_s(e_s) = \gamma_s[-Ln(1-e_s)-e_s]$ as the effort cost functions for the manufacturer and the supplier, respectively. The parameter γ_m and γ_s are the convexity of the cost functions. The larger the values of γ_m and γ_s are, the faster the effort costs increase with the effort exerted by the manufacturer and the supplier.

3.3.2 Quality Cost Sharing Contracts

Traditionally, the manufacturer would pay all quality costs for the warranty claim and the quality improvement (no share contract). By implementing the quality cost-sharing contract between the supplier and the manufacturer, this total expected quality cost will shrink, and the final product quality will be enhanced.

In this paper, we will discuss five different quality cost sharing contracts:

- Centrally coordinated supply chain (C): There is a central planner who optimizes the effort levels for the manufacturer and the supplier, in order to maximize the total profits on the supply chain instead of individual profits.

- No share (X): There is no cost sharing scheme between the manufacturer and the supplier. The manufacturer will improve his own component and pay all the quality costs.

- Fixed share rate (F): Once a warranty claim happens, the manufacturer and the supplier will share the repair cost with a certain share rate.

- Total cost allocation (T): Once a warranty claim happens, a root cause analysis will be conducted to identify the cause of the failure. The repair cost and the analysis cost will be assigned to the party who is liable for this warranty claim.

- Selective root cause analysis (S): Once a warranty claim happens before a threshold time, $\overline{T}$ ($\overline{T} \leq$ warranty period), both parties share the costs as in Contract T. If the warranty claim happens after $\overline{T}$, but still during the warranty period, they will share the cost as in Contract F. That is, Contract F and T are the special cases of Contract S with $\overline{T}$=0 and $\overline{T}$=1.

For each contract, except contract C, the manufacturer will figure out how to set up the optimal effort, sale price, and contract parameters in order to maximize his profit according to the different scenarios and contracts. Meanwhile, the supplier will make the decision on how much effort to exert in order to improve his component quality in order to minimize the expected quality cost.

3.3.3 Market Share

In this paper, we will systematically study different contracts under different decision processes. We would like to analyze the equilibrium states for the different contracts under the following three scenarios:

(i) One-stage decision : The manufacturer will choose the optimal effort levels and the sale price together when the manufacturer designs the quality cost-sharing contract in the monopoly market, in order to maximize total profits in the monopoly market.

(ii) Two-stage decision : We assume the manufacturer will design the quality cost-sharing contract first in order to minimize the expected quality cost per unit and the optimal product quality levels will be determined. Then the manufacturer will determine the sale price at the second stage in order to maximize the total profits within a monopoly market.

We will investigate how the quality cost-sharing contracts influence the manufacturer's profits in a monopoly market. The manufacturer can set up a high sale price in order to increase the profits; however, the higher price will result in less demand. Also, the improved quality due to the quality cost-sharing contracts will increase the sales volume. We need a demand function composed of the product quality and sale price, in order to help us calculate the final profits.

First, we assume the market size, N, is a constant. And the utility, V_1, of a consumer purchasing this product from the manufacturer with the price v and the quality $\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)$ can be set as:

$$V_1 = a_1 - a_q^M[\lambda_m^0(1-e_m)+\lambda_s^0(1-e_s)] - a_v^M v = (a_1 - a_q^M\lambda_m^0 - a_q^M\lambda_s^0) + a_q^M(\lambda_m^0 e_m + \lambda_s^0 e_s) - a_v^M v$$

where a_1 is the attribute coefficient for the consumer preference to the manufacturer, and a_q^M and a_v^M are the attribute coefficients for the consumer preference for the product quality and sale price in the monopoly market, respectively. The utility level associated with non-purchase is denoted as V_0.

Since the utility of purchasing this product from this manufacturer is not the same for all individuals, we assume an individual's utility from purchasing this product is given by:

$$U_1 = V_1 + \mu\epsilon_1,$$

ϵ_1 is a random variable with zero mean and unit variance and μ is a positive parameter that represents the horizontal differentiation in the market. By assuming ϵ_1 is identically, independently Gumbel distributed, the manufacturer's market share can be expressed by the multinomial logit model.

$$\text{Market share in the monopoly marker } (M_M) = \frac{W_{M1}}{W_{M1} + W_{M0}}$$

where

$$\begin{aligned} W_{M1} &= e^{\frac{a_1^M + a_q^M(\lambda_m^0 e_m + \lambda_s^0 e_s) - a_v^M v}{\mu}} \;; \quad a_1^M = a_1 - a_q^M \lambda_m^0 - a_q^M \lambda_s^0 \;; \\ W_{M0} &= e^{\frac{V_0}{\mu}}. \end{aligned}$$

3.4 Analysis for Quality Cost-Sharing Contracts

In this section, we would like to discuss the differences between the one- or two-stage decision process in a monopoly market. We will discuss how the manufacturer and the supplier behave under these five different quality cost-sharing contracts in the monopoly market.

3.4.1 Contracts in the centrally coordinated supply chain (Contract C)

First, let's study the quality cost-sharing contract in a centrally coordinated supply chain. Since there is a central planner to optimize the total costs or profits within the supply chain, we can relax the constraints of the players' individual rationality.

The total profit for one-stage decision:

$$\Pi_{(i)}^C = \max_{v, e_m, e_s} \quad N\Big\{v - c_m - c_s - \omega[\lambda_m^0(1 - e_m) + \lambda_s^0(1 - e_s)] - C_m(e_m) - C_s(e_s)\Big\}M_M \quad (3.1)$$

The total profit for two-stage decision:

$$\Pi^C_{(ii)} = \max_{v} \quad N\Big\{v - c_m - \omega[\lambda^0_m(1-e_m) + \lambda^0_s(1-e_s)] - C_m(e_m) - C_s(e_s)\Big\}M_M \tag{3.2}$$

$$s.t. \quad e_m, e_s = arg \min_{e_m,e_s}\Big\{c_s + \omega[\lambda^0_m(1-e_m) + \lambda^0_s(1-e_s)] + C_m(e_m) + C_s(e_s)\Big\} \tag{3.3}$$

$\Pi^C_{(i)}$ and $\Pi^C_{(ii)}$ are strictly concave in e_m, e_s and v. Then we can find the first order condition and obtain the optimal solutions for the effort levels and the sale prices for different scenarios.

Theorem 1. *For the centrally coordinated supply chain in the monopoly market, there is a unique equilibrium existing in this game, where the optimal effort levels in the game can be found by solving the following equations:*

$$w = \frac{1}{\lambda^0_m}\frac{\partial C_m(e_m)}{\partial e_m} - \frac{a^M_q}{a^M_v} \quad and \quad w = \frac{1}{\lambda^0_s}\frac{\partial C_s(e_s)}{\partial e_s} - \frac{a^M_q}{a^M_v} \text{ for one-stage decision} \tag{3.4}$$

and

$$w = \frac{1}{\lambda^0_m}\frac{\partial C_m(e_m)}{\partial e_m} \quad and \quad w = \frac{1}{\lambda^0_s}\frac{\partial C_s(e_s)}{\partial e_s} \text{ for two-stage decision.} \tag{3.5}$$

Proof: Listed in the Appendix.

From Theorem 1, there is an unique optimal solution of the effort levels and the sale prices for one- and two-stage in the centrally coordinated supply chain. The optimal effort for the manufacturer and the supplier could be obtained by equation (3.4) and (3.5), respectively. We can tell the optimal effort levels in the one-stage game are higher than those in the two-stage game. In the one-stage game, the central planner only balances the decrease of the potential repair cost and the increase of the effort cost. Thus, the optimal effort levels depend only on the repair cost and the initial failure rate (as shown in equation (3.5)). In Scenario (ii), in addition to the repair cost and the effort cost, the central planner also considers the change of the market share which is influenced by the product quality. Since the objective is to maximize the total profits along this supply chain, in addition to the cost reduction, how to increase the profits by expanding the market share will be another important task to tackle. By improving the product quality, the supply chain can generate the extra profits due to the reduction of the repair cost and the increase of the market share in the two-stage decision. Thus, the both parties will exert more effort to reach the new equilibrium where the marginal cost, due to the effort cost, is equal to the marginal profits, due to the reduction of expected repair cost and the increase of market demand. Thus, the optimal effort levels in one stage decision are higher than those in two-stage decision. The extra effort levels depend on the market characteristic: the ratio of the customer's quality preference to the price preference. The more preference for the quality consumers preform, the higher optimal effort level it is.

3.4.2 No sharing within the supply chain (Contract X)

In this section, we will find the most prevailing situation in the industry where the manufacturer will take all responsibility about the warranty claims. Since the supplier needs not to pay any repair cost, the supplier will exert no effort to improve his component quality. Thus, the manufacturer will optimize his own effort level and sale price.

The manufacturer's profit for one-stage decision:

$$\Pi^X_{(i)} = \max_{v,e_m} \quad N\Big\{v - c_m - p - \omega[\lambda^0_m(1-e_m) + \lambda^0_s] - C_m(e_m)\Big\}M_M,$$

The manufacturer's profit for two stage decision:

$$\begin{aligned} \Pi^X_{(ii)} = \quad & \max_{v} \quad N\Big\{v - c_m - p - \omega[\lambda^0_m(1-e_m) + \lambda^0_s] - C_m(e_m)\Big\}M_M \\ & s.t. \quad e_m = arg\ \min_{e_m}\Big\{c_s + \omega[\lambda^0_m(1-e_m) + \lambda^0_s] + C_m(e_m)\Big\}. \end{aligned}$$

Theorem 2. *For a supply chain without quality cost-sharing in the monopoly market, there is a unique equilibrium existing in this game, where the optimal effort level in the game can be found by solving the following equations*

$$w = \frac{1}{\lambda^0_m}\frac{\partial C_m(e_m)}{\partial e_m} - \frac{a^M_q}{a^M_v} \quad and \quad e_s = 0 \ for\ one\text{-}stage\ decision,$$

and

$$w = \frac{1}{\lambda^0_m}\frac{\partial C_m(e_m)}{\partial e_m} \quad and \quad e_s = 0 \ for\ two\text{-}stage\ decision.$$

Proof: The proof is very similar to the proof for Theorem 1.

3.4.3 Quality cost-sharing contract with the selective root cause analysis (Contract S)

If the warranty claim happens before time $\overline{T}$, the root cause analysis is conducted in order to find why it happened, and who is responsible for this warranty claim. If the warranty claim happens after $\overline{T}$, the supplier will share with the repair cost with the portion R.

We denote $\Lambda = \lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)$

The manufacturer profit for the one-stage decision:

$$\Pi^S_{(i)} = \max_{v,e_m,R,\overline{T}} N\Big\{v - c_m - p - (\omega + c_r)\lambda_m^0(1-e_m)\overline{T} - \omega(1-R)(1-\overline{T})\Lambda - C_m(e_m)\Big\}M_M \quad (3.6)$$

$$s.t. \quad e_s = arg\ \min_{e_s}\Big\{c_s + (\omega + c_r)\lambda_s^0(1-e_s)\overline{T} + R\omega(1-\overline{T})\Lambda + C_s(e_s)\Big\} \quad (3.7)$$

$$p - c_s - (\omega + c_r)\lambda_s^0(1-e_s)\overline{T} - R\omega(1-\overline{T})\Lambda - C_s(e_s) \geq 0. \quad (3.8)$$

The manufacturer profit for the two-stage decision:

$$\Pi^S_{(ii)} = \max_{v} N\Big\{v - c_m - p - (\omega + c_r)\lambda_m^0(1-e_m)\overline{T} - \omega(1-R)(1-\overline{T})\Lambda - C_m(e_m)\Big\}M_M \quad (3.9)$$

$$s.t. \quad e_m, R, \overline{T} = arg\ \min_{e_m,R,\overline{T}}\Big\{c_m + p + (\omega + c_r)\lambda_m^0(1-e_m)\overline{T} + \omega(1-R)(1-\overline{T})\Lambda + C_m(e_m)\Big\} \quad (3.10)$$

$$e_s = arg\ \min_{e_s}\Big\{c_s + (\omega + c_r)\lambda_s^0(1-e_s)\overline{T} + R\omega(1-\overline{T})\Lambda + C_s(e_s)\Big\} \quad (3.11)$$

$$p - c_s - (\omega + c_r)\lambda_s^0(1-e_s)\overline{T} - R\omega(1-\overline{T})\Lambda - C_s(e_s) \geq 0. \quad (3.12)$$

where (3.8) and (3.12) are the individual rationality constraints to make sure the supplier will accept this contract.

Theorem 3. *For the contract with the selective root cause analysis in the monopoly market, there is a unique equilibrium existing in this game, where the optimal effort levels in the game can be found by solving the following equations:*

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - (\omega + 2c_r)\overline{T} - \frac{a_q^M}{a_v^M}, \quad \textit{for for one-stage decision,} \quad (3.13)$$

and

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - (\omega + 2c_r)\overline{T}, \qquad \textit{for two-stage decision,} \quad (3.14)$$

where

$$\frac{\partial C_s(e_s)}{\partial e_s} = R\omega\lambda_s^0 + (\omega + c_r - R\omega)\overline{T}\lambda_s^0 \quad \textit{for both one- and two-stage decision.}$$

and the equilibrium is unique in Scenario (ii) only when

$$\frac{\partial^2 C_m(e_m)}{\partial e_m^2} + \frac{a_q^M \lambda_m^0}{\mu}\Big(\frac{a_q^M \lambda_m^0}{a_v^M} - 1\Big) > 0. \tag{3.15}$$

Proof: Listed in the Appendix.

With the different combinations of R and $\overline{T}$, the optimal effort levels will vary from case to case. As we know, it will result in *under-investment* when $\overline{T}$=0 (Contract F) and *over-investment* when when $\overline{T}$=1 (Contract T). Thus, by carefully choosing the right R and $\overline{T}$, the optimal effort levels can be equal to the first best effort levels.

Theorem 4. *When R=0.5 and $\overline{T} = \frac{\omega}{\omega+2C_r}$, the optimal manufacturer's effort level for Contract S will be equal to first best effort levels.*

Proof: This proof is straightforward by comparing the results between Theorem 1 and Theorem 5, so we ignore it here.

3.4.4 Comparisons among different contracts

In this section, we would like to compare quality improvement effort levels, sale prices and profits among different quality cost-sharing contracts.

Quality Improvement Effort Levels

From Theorem 1-3, we can find that the optimal effort levels in one-stage decision are not less than those in two-stage decision. The cost of the extra effort can be compensated by the extra profit from the extra demand, due to the better quality. Thus, if the market information is available, it is better to make quality improvement and price decisions based on the information of the component failure rate, effort cost functions, and the consumer preferences for price and quality at the same time.

Before we compare the quality improvement effort levels among different contracts, we would like to show the optimal effort levels for Contract F and Contract T. Contract F and Contract T are the special cases for Contract S with $\overline{T}$=1 and $\overline{T}$=0. Thus, we can get the optimal effort levels for those two contracts in Corollary 1.

Corollary 1. *For the quality cost-sharing contract with a fixed share rate in a monopoly market, the optimal effort levels can be found by solving the following equations:*

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - \frac{a_q^M}{a_v^M}, \quad \frac{\partial C_s(e_s)}{\partial e_s} = R\omega\lambda_s^0 \quad \textit{for one-stage decision} \tag{3.16}$$

and

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s}, \quad \frac{\partial C_s(e_s)}{\partial e_s} = R\omega\lambda_s^0 \qquad \textit{for two-stage decision.} \tag{3.17}$$

For the quality cost-sharing contract with total cost allocation in a monopoly market, the optimal effort levels can be found by solving the following equations:

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} - \frac{a_q^M}{a_v^M} - c_r \quad \textit{and} \quad \omega = \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - c_r \textit{ for one-stage decision} \tag{3.18}$$

and

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} - c_r \quad \textit{and} \quad \omega = \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - c_r \textit{ for two-stage decision.} \tag{3.19}$$

By comparing with equation (3.4), (3.5), and (3.16), (3.17), we can find the *under-investment* where the optimal effort levels for Contract F will be lower than the optimal effort levels in Contract C. In Contract F, one party will take the advantage from the effort exerted by the other party. Thus, each party will exert less effort in Contract F than in Contract C. From equations (3.16) and (3.17), we find that those effort is *substitute* each other, and the optimal effort is located at the second best equilibrium position, where the sum of the total marginal effort costs from both is equal to the repair cost.

On the other hand, the optimal effort levels in Contract T are higher than those in Contract C. The extra effort is used to compensate the root cause analysis cost. The higher the warrant cost and the analysis cost are, the higher the manufacturer and the supplier's optimal effort levels are. Thus, we can observe the over-investment in Contract T. Also the manufacturer and the supplier's effect are *complimentary.*

Next, we would like to compare the differences on the optimal effort levels among contracts. Here we define the notations e_j^{k*} as the optimal effort level at the equilibrium point where $k = \{C, X, F, T, S\}$, and $j = \{m, s\}$ represent the contract type and the player.

Corollary 2. *Among the different contracts under the same decision process, the comparisons of optimal effort levels are,*

$$e_m^{T*} \geq e_m^{C*} = e_m^{X*} \geq e_m^{F*}, \ \ \textit{and} \quad e_m^{T*} \geq e_m^{S*} \geq e_m^{F*}$$

and

$$e_s^{T*} \geq e_s^{C*} \geq e_s^{F*} \geq e_s^{X*} = 0, \ \ \textit{and} \quad e_s^{T*} \geq e_s^{S*} \geq e_s^{F*}.$$

Contract T will result in the best quality product, because the extra analysis cost for each failure will force both players to exert more effort to minimize the potential failure opportunity. That is why there will be over-investment in Contract T by comparing with Contract C. On the other hand, under-investment will be observed in Contract F because of moral hazard. From Corollary 2, we can find $e_m^{T*} \geq e_m^{S*} \geq e_m^{F*}$. For Contract S, by assigning different $\overline{T}$ and R, the optimal effort, e_m^S, will be different. The value of e_m^S can be manipulated between e_m^{T*} and e_m^{F*} by adjusting the contract parameters, $\overline{T}$ and R. Since the first best effort levels are also between e_m^{T*} and e_m^{F*}, we can find the proper $\overline{T}$ and R, to get $e_m^S = e_m^{C*}$ as shown in Theorem 6. That means the first best effort levels can happen in Contract S. (Chao, Iravani, and Savaskan 2006)

The analysis cost (c_r) will determine the rankings of the effort levels. When the analysis cost is extremely low, the implementation of root cause analysis will result in the first best effort levels in Contracts T and S. Then we can get $e_m^{T*} = e_m^{S*} = e_m^{C*} = e_m^{X*}$ and $e_s^{T*} = e_s^{S*}$. As the root cause analysis cost increases, the difference among different contracts will be noticeable. Figure 3.1 shows the effort level comparison among the different contracts.[1]

[1] $N = 1$, $c_m = 500$, $c_s = 300$, $\lambda_m^0 = 0.5$, $\lambda_s^0 = 0.3$, $\omega = 500$, $\gamma_m = 100$, $\gamma_s = 80$, $a_q^M = 1$, $a_v^M = 1$, $a_1 = 1300$,

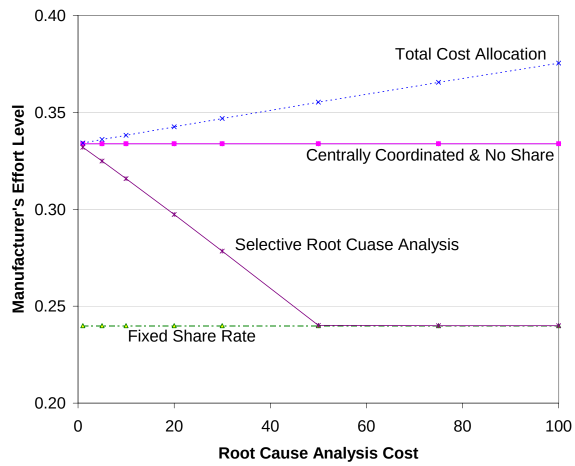


Figure 3.1: Manufacturer's Effort Levels under Quality Cost Sharing Contracts

Sale Prices

Before we compare the sale prices for different contracts, we need to decide the purchase price for the supplier's component. The purchase prices should meet the individual rationality constraints in order to ask the supplier to accept these contracts. In our study, we assume that the purchase price is equal to the supplier's expected cost based on the contract parameters. That is, we take the equal signs in those individual rationality constraints. Then we can get the purchase price for the

$V_0 = 150$ and $\mu = 50$.

supplier's component, and we can get the optimal sale prices for the different contracts as[2]:

$$
\begin{aligned}
v^{C*} &= c_m + c_s + \omega\Lambda^C + C_m(e_m^{C*}) + C_s(e_s^{C*}) + \frac{\mu}{a_v^M}\frac{1}{1-M_M^C} \\
v^{X*} &= c_m + c_s + \omega\Lambda^X + C_m(e_m^{X*}) + \frac{\mu}{a_v^M}\frac{1}{1-M_M^X} \\
v^{F*} &= c_m + c_s + \omega\Lambda^F + C_m(e_m^{F*}) + C_s(e_s^{F*}) + \frac{\mu}{a_v^M}\frac{1}{1-M_M^F} \\
v^{T*} &= c_m + c_s + (\omega + c_r)\Lambda^T + C_m(e_m^{T*}) + C_s(e_s^{T*}) + \frac{\mu}{a_v^M}\frac{1}{1-M_M^T} \\
v^{S*} &= c_m + c_s + \omega\Lambda^S + c_r\Lambda^S\overline{T} + C_m(e_m^{S*}) + C_s(e_s^{S*}) + \frac{\mu}{a_v^M}\frac{1}{1-M_M^S} \\
\textit{where} \quad & \Lambda^j = \lambda_m^0(1-e_m^{j*}) + \lambda_s^0(1-e_s^{j*}), \quad j \in \{C, X, F, T, S\}.
\end{aligned}
$$

Since the market share is a function of the sale price, the sale prices is obtained endogenously from the above equations as Anderson and Palma (1992) and Besanko et al. (1998) discuss. The sale prices can be divided into two parts: one is the expected cost and the other is the markup (profit). The expected cost includes the production cost ($c_m + c_s$), warranty cost ($\omega\Lambda$), analysis cost ($c_r\Lambda$) and the effort cost ($C_m(e_m) + C_s(e_s)$). The sale price should be greater than the expected cost; otherwise, the manufacturer incurs loss from the sale of the product. The markup part (value creation) depends on the market share. The market share leader will also be the profit leader as Besanko et al. discuss.

Corollary 3. *Under the same contract, the sale price in the one-stage decision process will be higher than that in the two-stage decision process. That is,* $v_{(i)}^{j*} \geq v_{(ii)}^{j*}$ where $j \in \{C, X, F, T, S\}$.

Since the optimal effect levels in two-stage decision are lower than those in one-stage decision, then quality cost (warranty cost and effort cost) in two-stage decision will be lower. However, the inferior product quality will result in the smaller market share and less profits. Thus, the lower quality cost and the smaller market share will result in the lower sale price in the two-stage decision.

[2]The sale prices for Contract C, X and S can be obtained from from the proof of Theorem 1-3. The sale prices for Contract F and T can be obtained as we get it for Contract S.

Corollary 4. *The relationship among the sale prices of the different contracts can be described as*

$$v^{X*} \geq v^{F*} \geq v^{C*}, \qquad v^{T*} \geq v^{S*} \geq v^{C*}.$$

The expected cost for Contract C will be lowest among all contracts, because of the effective coordination between the manufacturer and the supplier on the quality improvement. Then we can know the lowest sale price will happen in Contract C. If there is no root cause analysis, Contract X has a highest sale price, because only the manufacturer put the effort on the quality improvement, and the expected warranty cost remains high. If there is a root cause analysis, the sale price for Contract T will be higher than that for Contract S, because of the cost inefficiency due to the extra root cause analysis cost. The analysis cost, c_r, plays a very important role here to determine which contract's sale price is the highest. If the analysis cost is very high, Contract T will spend too much cost on the root cause analysis, and we will get $v^{T*} \geq v^{X*} \geq v^{F*} \geq v^{S*} \geq v^{C*}$. However, if the analysis cost is negligent, Contract T and Contract S will behave like Contract C. Then we can get $v^{X*} \geq v^{F*} \geq v^{T*} \geq v^{S*} \geq v^{C*}$. Figure 3.2 shows the sale price comparison among different contracts.

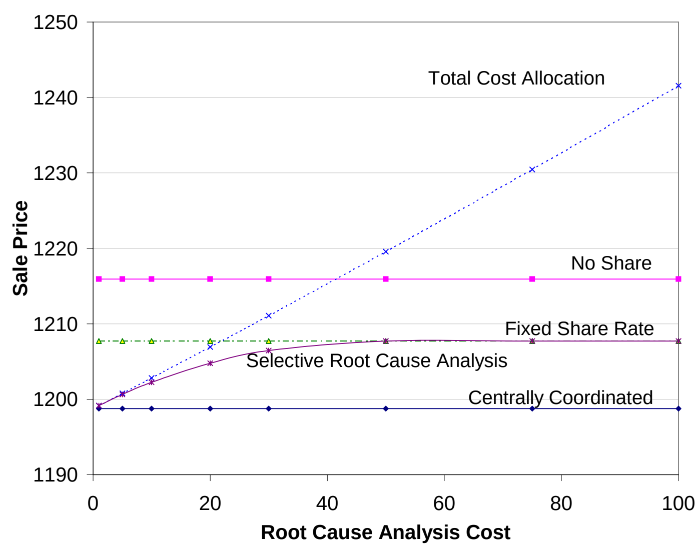


Figure 3.2: Sale Prices under Quality Cost Sharing Contracts

Total Profits and Market Share

So the sale prices will include all expected costs and the markup (that is $\frac{\mu}{a_v^M}\frac{1}{1-M_M}$). Then we can get the general total profit function as:

$$\Pi^j = N\frac{\mu}{a_v^M}\frac{M_M^j}{1-M_M^j} \qquad j \in \{C, X, F, T, S\}. \tag{3.20}$$

Thus, the final profit of a contract depends on the market share which is determined by the sale price and the product quality. From the Corollaries 1 and 3, we can know the quality and price differences among different contracts. Can we say that a contract with better quality will have more profit? In other words, can the extra quality improvement cost be compensated by extra profits? Since the general profit function (3.20) depends on the market share only, we can compare contracts based on their utility values which determine the market share.

Theorem 5. *When comparing two different contracts in a monopoly market, the contract j is more profitable than the contract k when*

$$a_q^M[\lambda_m^0(e_m^{j*} - e_m^{k*}) + \lambda_s^0(e_s^{j*} - e_s^{k*})] \geq a_v^M(v^{j*} - v^{k*}) \tag{3.21}$$

where $j, k \in \{C, X, F, T, S\}$.

Proof: listed in Appendix.

Theorem 5 shows that the superiority (in term of profits) of a contract depends on the change of the utility function value; that is, the contract with the higher customers purchase utility value will bring more profit. If the increase of the utility value, due to quality improvement, can compensate for the decrease of the utility value, due to price changes, extra profits will be generated. Thus, if the market is very sensitive to the quality, the contract with the best quality will be the most profitable, and Contract T might be the good choice. That is, the extra quality will be free in the quality sensitive market where the extra quality improvement can generate extra market share, and the effort cost can be compensated by those extra profits. Otherwise, the best quality might not guarantee the highest profits in the price sensitive market. Figure 3.3 shows the comparison of market shares and profits among different contracts under different root cause analysis costs.

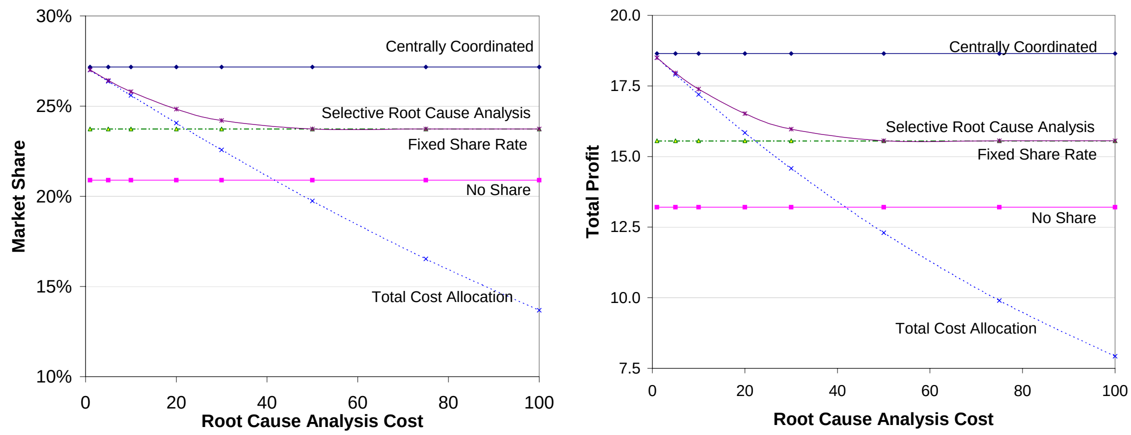


Figure 3.3: Market Share (left) and Total Profits (right) under Quality Cost Sharing Contracts.

Figure 3.3 shows that Contract S performs best in the decentralized environment. When the analysis cost is low, Contract S behaves like Contract T. When the analysis cost is high, it behaves like Contract F. Contract S can lessen the inefficiency due to the over- or under-investment.

Pricing Strategy and Profits

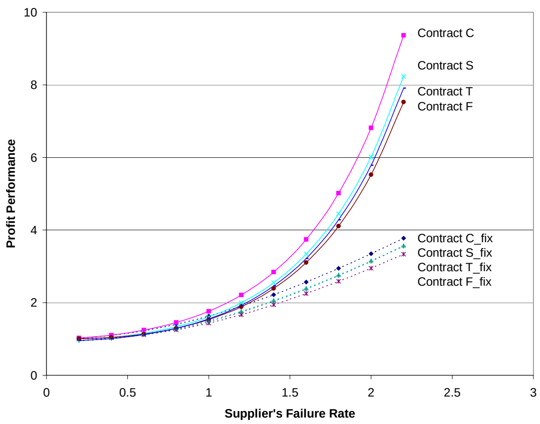
If the sale price is given and fixed, the profit maximization problem will be the same with the cost minimization problem. Adopting Contract S in the decentralized environment is the best choice as Chao, Iravani and Savaskan (2007) studied. However, the better quality and lower sale price can attract more customers. Thus, by choosing the “proper” effort levels and sale prices, the supply chain can maximize profits. Since we know the optimal effort levels for the different contracts are independent from the sale price, it will be very interesting to figure out whether the manufacturer should lower the sale price in order to stimulate more demands and increase profits. Also under which situations does the manufacturer take the pricing strategy into consideration for higher profits?

Here we will discuss two pricing strategies: fixed pricing strategy and the optimal pricing strategy. The sale price for the fixed pricing strategy is regarded as the sale price in the Contract X. That is, that sale price is optimal for no sharing case and that price will be apply to the other contracts under the fixed pricing strategy. The optimal pricing strategy is to obtain the sale price, like the other contract parameters, from the profit maximization problem.

We define the profit performance of Contract j as the ratio of Contract j's profit to Contract X's profit, $\frac{\Pi^j}{\Pi^X}, j = C, S, T, F$. The higher value of this ratio is, the more profit Contract j will generate. We find all ratios will increase as the customer preference to the sale price and the supplier's failure rate increase, especially for those contracts with the optimal pricing strategy. When the supplier's failure rate is high, the product quality will be improved more significantly by implementing the cost-sharing contract. Since the market share is defined as the Logit model with the product quality, the market share and the profit will increase exponentially as the product quality is improved (as shown in Figure 3.4 *left*). The optimal price is the tradeoff between the unit profit decrease and the market demand increase, in order to obtain the potential maximum profits. Thus, the benefit of the cost-sharing contract with the optimal pricing strategy for the higher supplier's failure rate is more attractive.

When customers are very price sensitive, the profit difference between the fixed and optimal pricing strategies will also become more significant. Since customers care about the price more than the quality, the better quality cannot create the extra demand more effectively, and the supply chain cannot increase more profits, either. To adjust the sale price will be the more effective way to increase the demand. Especially by adopting the optimal pricing strategy, the benefit will become noteworthy as the customer preference to the sale price increases.

On the contrary, when the horizontal differentiation in the market and the customer preference to the manufacturer decrease, the profit performance becomes better. Higher consumer heterogeneity (that is, more horizontal differentiation) will lead to a tougher market. The profit performance difference among the different contracts and different pricing strategies under more intense horizontal differentiation are very insignificant as shown in Figure 3.4 *right*. The way to get more demand (or profit) is not an easy job if the consumer preference disperses widely. On the other hand, if the consumer heterogeneity is small, there is the opportunity to get more profit by adjusting the sale price. Thus, the optimal pricing strategy will become more attractive under small market horizontal



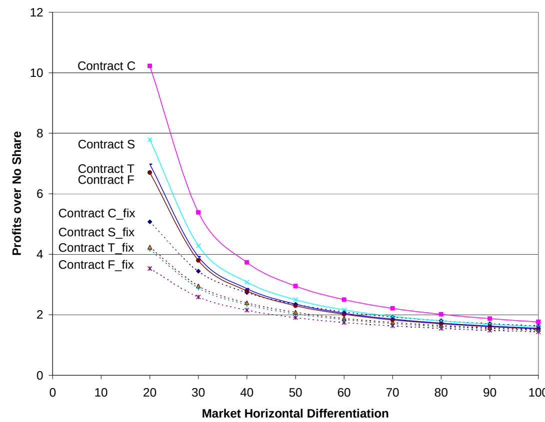


Figure 3.4: Profit Performance, $\frac{\Pi^j}{\Pi^X}, j = C, S, T, F$, for each Contract under the Influence of the Supplier's Failure Rate (*left*) and Market Horizontal Differentiation (*right*), where the contracts with _fix extension in the figure are the ones with fixed pricing strategy.

differentiation.

When consumers prefer the manufacturer's product, the optimal pricing strategy will become less attractive. On the contrary, if the consumers preference to the manufacturer decreases, the price will play a more important role to increase the consumer purchase utility values. The higher the utility value is, the higher demand will be. Therefore, if the consumer brand loyalty is not obvious, the price should be optimized according to the contract type.

3.5 Summary & Managerial Insights

In this section, we will highlight the insights for cost-sharing contracts and the pricing strategy. In particular, we will interpret anecdotal and empirical observation about the manufacturer-supplier relationship. In the decentralized environment, each player will seek his own maximum profit, and avoid the loss even if the system does not obtain maximum profits. What benefits can the supply chain obtain by implementing the cost-sharing contracts? When can these contracts generate attractive extra profits? What pricing strategy should the supply chain adopt in order to maximize profits? And when?

Optimal Effort Levels

As we show in Theorem 1-3 and Corollary 1, the optimal effort levels can be expressed with closed-form equations. The optimal effort levels are determined by the effort cost function, initial failure rate, analysis cost, and the market demography. The manufacturer will consider how to reduce the expected cost, composed of the warranty cost and the effort cost, and how to find the tradeoff between the quality improvement and the extra demand. The most important thing is that they are independent from the sale prices. The manufacturer needs not to care about the price war. The optimal effort level will bring up the maximum profit for a given sale price.

Quality vs. Profits

This question has puzzled companies for a long time, "Is the quality free? " The better product quality will consume more resources (money, time, spaces, labors, ...etc.) to analyze, discuss, plan, test, and be realized. If we focus on the costs of these resources, the quality is not free. The higher quality ones seek, the higher cost they will spend. However, the better quality will induce more demands and generate more revenues. The extra money due to extra demands could compensate the extra expense on the quality improvement. In the end, the company can generate extra profits and own the better quality product. Also there is no "loss" but "gain" for the extra quality improvement. How can the manufacturer force the supplier to enhance his component quality? He can behave like Japanese manufacturers that send the technology teams to their suppliers' worksite, to share the specification and production information (Iyer, Schwarz, and Zenious 2005), or to acquire the suppliers' production facilities. How can the manufacturer minimize the total expected cost, coordinate with the supplier in terms of quality improvement, and also maintain the autonomy at the same time? We need a contract (or mechanism) that regulates the players' behaviors. Contract F and T are commonly discussed in the literature (Lim 2000, Baiman et al. 2003, 2001, 2000). There are some inefficiencies and less profits in those contracts. The inefficiency results from the share rate (under-investment) and the root cause analysis (over-investment). We can combine the features together (like Contract S) and the inefficiency could be minimized. By implementing the cost-sharing contract with the selective root cause analysis (Contract S), the supply chain will avoid the inefficiency, enhance the product quality, and generate more profits.

Price Sensitivity vs. Pricing Strategy

When customers are very price sensitive, the profit difference between the fixed and optimal pricing strategies will become more significant. It is because the optimal prices can optimize the maximum profits under the different quality levels, since the quality levels are different from one contract to the other. However, if the manufacturer adopts the optimal pricing strategy, should the final optimal price be lower than the original fixed price? A lower sale price will increase the demand, but it will also result in the lower unit profit. On the other hand, a higher sale price will increase the unit profit, but it will decrease the demand. The price is composed of the unit profit (price minus the expected cost) and the markup. The unit profit is a linear increasing function of the price, but the markup is a quadratic decreasing function of the price. With the better product quality, both the unit profit and the markup will be increased. If the manufacturer increases the price, the unit profit will increase, but the markup will decrease more. The final profit with the increasing price will not be the maximum. Thus, the optimal price will be lower than the original fixed price, and the final profit with the optimal price will be more than one with the fixed price. Will the better quality and lower cost result in the lower price? This sounds like a contradiction. However, in the real world, let's look at the product life cycle (initiation-growth-mature-decline). Because of the bottleneck reduction, and extra capacity, the manufacturer can focus on the market expansion. At this moment, the production cost and the constraints will be lessened, and the manufacturer can seek for the market growth. The best way to expand the market share is to lower the sale price. CPUs, blue-ray players, and MP3 players are good example of better quality, but lower price.

Market Diversity vs. Pricing

If the market horizontal differentiation is very small, it is more profitable to adopt the optimal sale prices. When the market horizontal differentiation is small, the consumer preferences are much more uniform, and the competition will be more severe. The market share and the profits will be very sensitive to the change of the sale price and the product failure rate. In addition to the reduction of the failure rate, the optimization of the sale price will help the supply chain maximize the profits. On the contrary, if the market horizontal differentiation is big, the consumers will be inert to the change of the quality or price. That is, the consumers have the wide variety of preferences on the products and the change of product quality, and the sale price will not drastically influence the market share and the profits. It will not be so attractive to introduce the optimal pricing strategy when the horizontal differentiation is huge.

Supplier's Failure Rate vs. Pricing

The higher the supplier's initial failure rate is, the higher benefits that the supply chain can generate with the optimal pricing strategy. Chao et al. 2007 showed that the expected cost could be reduced with the implementation of the cost-sharing contract in the supply chain, especially for the supplier with higher failure rates. Thus, the worse the supplier's component quality is, the more necessary the cost-sharing contract with optimal pricing strategy is. The introduction of the cost-sharing contract can help the manufacturer to assure that both players will exert the effort to improve the final product quality, and the adoption of the optimal pricing strategy will help them maximize the profits and the market share.

APPENDIX

First, we show the first derivative of market share, (M_M), with respect to price (v) and effort (e_m, e_s) are

$$\frac{\partial M_M}{\partial v} = -\frac{a_v^M}{\mu} M_M(1-M_M), \tag{3.22}$$

$$\frac{\partial M_M}{\partial e_m} = \frac{a_q^M \lambda_m^0}{\mu} M_M(1-M_M), \tag{3.23}$$

$$\frac{\partial M_M}{\partial e_s} = \frac{a_q^M \lambda_s^0}{\mu} M_M(1-M_M). \tag{3.24}$$

Proof of Theorem 1 Contract within centrally coordinated supply chain in a monopoly market

Two-stage decision process:

Take the first order condition for equation (3.3), we can get equation (3.5). Since $C_m(e_m)$ and $C_s(e_s)$ are strictly increasing convex functions, there is a optimal solution to minimize the costs. Thus, there is one equilibrium at the first stage of Scenario (i).

Since the effort levels have been determined at the first stage, next, we take the first order condition on equation (3.2) with respect to he sale price, v. We also assume $K^C = c_m + c_s + \omega[\lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)] + C_m(e_m) + C_s(e_s)$ and K^C is a constant because we can get the optimal effort levels from the first stage. Then

$$NM_M + N\Big\{v - K^C\Big\}\frac{\partial M_M}{\partial v} = 0$$

$$NM_M = N\Big\{v - K^C\Big\}\frac{a_v^M}{\mu} M_M(1-M_M)$$

where $\frac{\partial M_M}{\partial v} = -\frac{a_v^M}{\mu} M_M(1-M_M)$

Then

$$\frac{1}{1-M_M} = \frac{a_v^M}{\mu}\Big\{v - K^C\Big\}$$

$$e^{\frac{a_1' + a_q^M \lambda_m^0 e_m + a_q^M \lambda_s^0 e_s - a_v^M v - V_0}{\mu}} + 1 = \frac{a_v^M}{\mu}\Big\{v - K^C\Big\}$$

$$\frac{1}{v-K^C} e^{\frac{a_1' + a_q^M \lambda_m^0 e_m + a_q^M \lambda_s^0 e_s - a_v^M v - V_0}{\mu}} = \frac{a_v^M}{\mu} - \frac{1}{v-K^C} \tag{3.25}$$

The right hand side of equation (3.25) is a decreasing convex function from the $+\infty$ to 0 within $[K^C, \infty]$. And the right hand side is an increasing concave function from the $-\infty$ to $\frac{a_v^M}{\mu}$ within $[K^C, \infty]$. Thus, there will be an optimal v to validate equation (3.25). And that solution is unique. This concludes the proof for two-stage decision process.

One-stage decision process:

Take the first order condition on equation (3.1) with respect to price (v) and effort (e_m, e_s).

$$\begin{aligned}
\text{Assuming } K^C &= c_m + c_s + \omega[\lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)] + C_m(e_m) + C_s(e_s) \\
\frac{\partial \Pi^C_{(i)}}{\partial v} &= NM_M + N(v-K^C)\frac{\partial M_M}{\partial v} = 0 \\
\frac{\partial \Pi^C_{(i)}}{\partial e_m} &= NM_M\Big\{\omega\lambda_m^0 - \frac{\partial C_m(e_m)}{\partial e_m}\Big\} + N(v-K^C)\frac{\partial M_M}{\partial e_m} = 0 \\
\frac{\partial \Pi^C_{(i)}}{\partial e_s} &= NM_M\Big\{\omega\lambda_s^0 - \frac{\partial C_s(e_s)}{\partial e_s}\Big\} + N(v-K^C)\frac{\partial M_M}{\partial e_s} = 0
\end{aligned}$$

Plug (3.22), (3.23) and (3.24) into the above three equations, respectively. Then we can get,

$$\begin{aligned}
NM_M &= \frac{a_v^M}{\mu}N(v-K^C)M_M(1-M_M) \\
NM_M\Big\{\frac{\partial C_m(e_m)}{\partial e_m} - \omega\lambda_m^0\Big\} &= \frac{a_q^M\lambda_m^0}{\mu}N(v-K^C)M_M(1-M_M) \\
NM_M\Big\{\frac{\partial C_s(e_s)}{\partial e_s} - \omega\lambda_s^0\Big\} &= \frac{a_q^M\lambda_s^0}{\mu}N(v-K^C)M_M(1-M_M)
\end{aligned}$$

Then dividing by NM_M at both sides for all three equations,

$$\frac{\mu}{a_v^M} = (v-K^C)(1-M_M) \tag{3.26}$$

$$\frac{\mu}{a_q^M\lambda_m^0}\Big\{\frac{\partial C_m(e_m)}{\partial e_m} - \omega\lambda_m^0\Big\} = (v-K^C)(1-M_M) \tag{3.27}$$

$$\frac{\mu}{a_q^M\lambda_s^0}\Big\{\frac{\partial C_s(e_s)}{\partial e_s} - \omega\lambda_s^0\Big\} = (v-K^C)(1-M_M) \tag{3.28}$$

The left hand sides of above three equations are the same. Then, we can get

$$\begin{aligned}
\frac{\mu}{a_q^M\lambda_m^0}\Big\{\frac{\partial C_m(e_m)}{\partial e_m} - \omega\lambda_m^0\Big\} = \frac{\mu}{a_v^M} \qquad &\Rightarrow \qquad \omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} - \frac{a_q^M}{a_v^M} \\
\frac{\mu}{a_q^M\lambda_s^0}\Big\{\frac{\partial C_s(e_s)}{\partial e_s} - \omega\lambda_s^0\Big\} = \frac{\mu}{a_v^M} \qquad &\Rightarrow \qquad \omega = \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - \frac{a_q^M}{a_v^M}
\end{aligned}$$

Thus, the optimal effort levels can be determined by the warranty cost, the initial failure rates, and the market sensitivities of the price and the quality. Once we find the optimal solutions for the effort level, we can use the same method for the two-stage decision process to prove there is an optimal solution for the price. This concludes the proof for one-stage decision process. **QED**

Proof of Theorem 3 Contract with selective root cause analysis in a monopoly market

Two-stage decision process :

Take the first order condition for the equation (3.10) and (3.11) with respect to e_m and e_s, respectively. We can get

$$\frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} = (\omega + c_r)\overline{T} + (1-R)\omega(1-\overline{T}) = (\omega + c_r)\overline{T} - (1-R)\omega\overline{T} + (1-R)\omega \tag{3.29}$$

$$\frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} = (\omega + c_r)\overline{T} + R\omega(1-\overline{T}) = (\omega + c_r)\overline{T} - R\omega\overline{T} + R\omega \tag{3.30}$$

Thus,

$$\omega = \frac{1}{\lambda_m(1-R)}\frac{\partial C_m(e_m)}{\partial e_m} = \frac{1}{\lambda_m R}\frac{\partial C_s(e_s)}{\partial e_s} \tag{3.31}$$

(3.29)+(3.30)

$$\omega = \frac{1}{\lambda_m}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s}\frac{\partial C_s(e_s)}{\partial e_s} - (\omega + 2c_r)\overline{T} \tag{3.32}$$

Now, we would like to prove the optimal solution is unique for Scenario (i). Redefine the manufacturer's cost function (equation 3.10) as the profit function:

$$\pi_m^S = \underset{e_m,R,\overline{T}}{Max}\Big\{ - p - (\omega + c_r)\lambda_m^0(1-e_m)\overline{T} - \omega(1-R)(1-\overline{T})\Lambda - C_m(e_m)\Big\} \tag{3.33}$$

First, taking the derivative with respect to $\overline{T}$ and R for equation (3.29) and (3.30). We get

$$\frac{\partial^2 C_m(e_m)}{\partial e_m \partial \overline{T}} = \lambda_m^0(c_r + \omega R) \tag{3.34}$$

$$\frac{\partial^2 C_m(e_m)}{\partial e_m \partial R} = \lambda_m^0(\omega\overline{T} - \omega) \tag{3.35}$$

$$\frac{\partial^2 C_s(e_s)}{\partial e_s \partial \overline{T}} = \lambda_s^0(\omega + c_r - \omega R) \tag{3.36}$$

$$\frac{\partial^2 C_s(e_s)}{\partial e_s \partial R} = \lambda_s^0(-\omega\overline{T} + \omega) \tag{3.37}$$

From equation(3.30), we also get

$$R\omega(1-\overline{T}) = \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - (\omega + c_r)\overline{T} \tag{3.38}$$

Now taking the first derivative of equation (3.33)

$$\begin{aligned}
\frac{\partial \pi_m^S}{\partial e_m} &= \lambda_m^0(\omega + c_r)\overline{T} + \omega\lambda_m^0(1-R)(1-\overline{T}) - \frac{\partial C_m(e_m)}{\partial e_m} \\
&= \lambda_m^0\overline{T}(\omega + 2c_r) + \lambda_m^0\omega - \frac{\lambda_m^0}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - \frac{\partial C_m(e_m)}{\partial e_m}
\end{aligned}$$

Then, we can get (by plugging equation (3.34)-(3.37))

$$\begin{aligned}
-\frac{\partial^2 \pi_m^S}{\partial e_m^2} &= \frac{\partial^2 C_m(e_m)}{\partial e_m^2} > 0 \\
\frac{\partial^2 \pi_m^S}{\partial e_m \partial \overline{T}} &= \lambda_m^0(\omega + 2c_r) - \frac{\lambda_m^0}{\lambda_s^0}\frac{\partial^2 C_s(e_s)}{\partial e_s \partial \overline{T}} - \frac{\partial^2 C_m(e_m)}{\partial e_m \partial \overline{T}} \\
&= \lambda_m^0(\omega + 2c_r - \omega - c_r + R\omega - c_r - R\omega) = 0 \\
\frac{\partial^2 \pi_m^S}{\partial e_m \partial R} &= -\frac{\lambda_m^0}{\lambda_s^0}\frac{\partial^2 C_s(e_s)}{\partial e_s \partial R} - \frac{\partial^2 C_m(e_m)}{\partial e_m \partial R} \\
&= -\lambda_m^0(-\omega\overline{T} + \omega + \omega\overline{T} - \omega) = 0
\end{aligned}$$

Thus, $-\frac{\partial^2 \pi_m^S}{\partial e_m^2} > \frac{\partial^2 \pi_m^S}{\partial e_m \partial \overline{T}} + \frac{\partial^2 \pi_m^S}{\partial e_m \partial R}$. According to Milgrom and Robert (1990), there will be a unique optimal solution of $\{e_m, e_s, R, \overline{T}\}$ for Scenario (i).

Since there is a unique solution for improvement effort at the first stage. We can apply the same proof method in Theorem 1 in order to prove there is a unique solution for sale price at the second stage.

One-stage decision process: Take the equation (3.7) with respect to e_s, we can get equation (3.30).

Take the equation (3.6) with respect to e_m and v, we can get the following equations by assuming $K^S = c_m + p + (\omega + c_r)\lambda_m^0(1-e_m)\overline{T} + \omega(1-R)[\lambda_m^0(1-e_m) + \lambda_s^0(1-e_s)](1-\overline{T}) + C_m(e_m)$.

$$\frac{\partial \Pi_{(i)}^S}{\partial v} = NM_M + N(v-K^S)\frac{\partial M_M}{\partial v} = 0 \tag{3.39}$$

$$\frac{\partial \Pi_{(i)}^S}{\partial e_m} = NM_M\Big\{(\omega+c_r)\lambda_m^0\overline{T} + \omega\lambda_m^0(1-R)(1-\overline{T}) - \frac{\partial C_m(e_m)}{\partial e_m}\Big\} + N(v-K^S)\frac{\partial M_M}{\partial e_m} = 0 \tag{3.40}$$

$$\frac{\partial \Pi_{(i)}^S}{\partial \overline{T}} = NM_M\frac{\partial (v-K^S)}{\partial \overline{T}} + N(v-K^S)\frac{\partial M_M}{\partial \overline{T}} = 0 \tag{3.41}$$

$$\frac{\partial \Pi_{(i)}^S}{\partial R} = NM_M\frac{\partial (v-K^S)}{\partial R} + N(v-K^S)\frac{\partial M_M}{\partial R} = 0 \tag{3.42}$$

Plug (3.22) and (3.23) into the equations (3.39) and (3.40), respectively. Then we can get,

$$\begin{aligned} NM_M &= \frac{a_v^M}{\mu}N(v-K^S)M_M(1-M_M) \\ NM_M\Big\{\frac{\partial C_m(e_m)}{\partial e_m} - (\omega+c_r)\lambda_m^0\overline{T} - \omega\lambda_m(1-R)^0(1-\overline{T})\Big\} &= \frac{a_q^M\lambda_m^0}{\mu}N(v-K^S)M_M(1-M_M) \end{aligned}$$

Then

$$\frac{\mu}{a_v^M} = (v-K^S)(1-M_M) \tag{3.43}$$

$$\frac{\mu}{a_q^M\lambda_m^0}\Big\{\frac{\partial C_m(e_m)}{\partial e_m} - (\omega+c_r)\lambda_m^0\overline{T} - \omega\lambda_m^0(1-R)(1-\overline{T})\Big\} = (v-K^S)(1-M_M) \tag{3.44}$$

The left hand sides of the above two equations should be the same. Then, we can get

$$\frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} = (\omega+c_r)\overline{T} + \omega(1-R)(1-\overline{T}) + \frac{a_q^M}{a_v^M} \tag{3.45}$$

(3.30)+(3.45)

$$\frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} = (\omega+2c_r)\overline{T} + \omega + \frac{a_q^M}{a_v^M}$$

So

$$\omega = \frac{1}{\lambda_m^0}\frac{\partial C_m(e_m)}{\partial e_m} + \frac{1}{\lambda_s^0}\frac{\partial C_s(e_s)}{\partial e_s} - (\omega+2c_r)\overline{T} - \frac{a_q^M}{a_v^M}$$

Now, we would like to prove the uniqueness of the optimal solution.

$$\begin{aligned} \frac{\partial^2 \Pi_m^S}{\partial e_m^2} &= -NM_M\frac{\partial^2 C_m(e_m)}{\partial e_m^2} \\ &\quad +2N\Big\{(\omega+c_r)\lambda_m^0\overline{T} + \omega\lambda_m^0(1-R)(1-\overline{T}) - \frac{\partial C_m(e_m)}{\partial e_m}\Big\}\frac{a_q^M\lambda_m^0}{\mu}M_M(1-M_M) \\ &\quad +N(v-K^S)\frac{a_q^M\lambda_m^0}{\mu}(1-2M_M)\frac{a_q^M\lambda_m^0}{\mu}M_M(1-M_M) \\ &= -NM_M\frac{\partial^2 C_m(e_m)}{\partial e_m^2} - 2N\frac{a_q^M\lambda_m^0}{\mu}\frac{a_q^M\lambda_m^0}{a_v^M}M_M(1-M_M) + N\frac{a_q^M\lambda_m^0}{\mu}\frac{a_q^M\lambda_m^0}{a_v^M}M_M(1-2M_M) \\ &= -NM_M\frac{\partial^2 C_m(e_m)}{\partial e_m^2} - NM_M\frac{a_q^M\lambda_m^0}{\mu}\frac{a_q^M\lambda_m^0}{a_v^M} < 0 \end{aligned} \tag{3.46}$$

where $(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}=-\frac{a_q^M\lambda_m^0}{a_v^M}$ from equation (3.45), and $\frac{\mu}{a_v^M}=(v-K^S)(1-M_M)$ from equation (3.43).

$$\begin{aligned}
\frac{\partial^2\Pi_m^S}{\partial e_m\partial v} &= N\Big\{(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}\Big\}(-\frac{a_v^M}{\mu})M_M(1-M_M)\\
&\quad +N\frac{a_q^M\lambda_m^0}{\mu}M_M(1-M_M)-N(v-K^S)\frac{a_q^M\lambda_m^0}{\mu}(1-2M_M)\frac{a_v^M}{\mu}M(1-M_M)\\
&= 2N\frac{a_q^M\lambda_m^0}{\mu}M_M(1-M_M)-N\frac{a_q^M\lambda_m^0}{\mu}M_M(1-2M_M)\\
&= NM_M\frac{a_q^M\lambda_m^0}{\mu}>0
\end{aligned}\tag{3.47}$$

where $(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}=-\frac{a_q^M\lambda_m^0}{a_v^M}$ from equation (3.45), and $\frac{\mu}{a_v^M}=(v-K^S)(1-M_M)$ from equation (3.43).

$$\begin{aligned}
\frac{\partial^2\Pi_m^S}{\partial e_m\partial\overline{T}} &= NM_M\Big[(\omega+c_r)\lambda_m^0-\omega\lambda_m^0(1-R)-\frac{\partial^2 C_m(e_m)}{\partial e_m\partial\overline{T}}\Big]\\
&\quad +N\Big\{(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}\Big\}\frac{\partial M_M}{\partial\overline{T}}\\
&\quad +N\frac{a_q^M\lambda_m^0}{\mu}M_M(1-M_M)\frac{\partial(v-K^S)}{\partial\overline{T}}+N(v-K^S)\frac{a_q^M\lambda_m^0}{\mu}(1-2M_M)\frac{\partial M_M}{\partial\overline{T}}
\end{aligned}$$

Because $(\omega+c_r)\lambda_m^0-\omega\lambda_m^0(1-R)-\frac{\partial^2 C_m(e_m)}{\partial e_m\partial\overline{T}}=0$ from equation (3.34), $(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}=-\frac{a_q^M\lambda_m^0}{a_v^M}$ from equation (3.45), $\frac{\mu}{a_v^M}=(v-K^S)(1-M_M)$ from equation (3.43), and $(v-K^S)\frac{\partial M_M}{\partial\overline{T}}=-M_M\frac{\partial(v-K^s)}{\partial\overline{T}}$ from (3.41), we can get

$$\begin{aligned}
\frac{\partial^2\Pi_m^S}{\partial e_m\partial\overline{T}} &= 0-N(1-M_M)\frac{a_q^M\lambda_m^0}{\mu}(v-K^S)\frac{\partial M_M}{\partial\overline{T}}-N\frac{a_q^M\lambda_m^0}{\mu}(1-M_M)(v-K^S)\frac{\partial M_M}{\partial\overline{T}}\\
&\quad +N\frac{a_q^M\lambda_m^0}{\mu}(1-2M_M)(v-K^S)\frac{\partial M_M}{\partial\overline{T}}\\
&= -N\frac{a_q^M\lambda_m^0}{\mu}(v-K^S)\frac{\partial M_M}{\partial\overline{T}}=0
\end{aligned}\tag{3.48}$$

$$\begin{aligned}
\frac{\partial^2\Pi_m^S}{\partial e_m\partial R} &= NM_M\Big[-\omega\lambda_m^0(1-\overline{T})-\frac{\partial^2 C_m(e_m)}{\partial e_m\partial R}\Big]\\
&\quad +N\Big\{(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}\Big\}\frac{\partial M_M}{\partial R}\\
&\quad +N\frac{a_q^M\lambda_m^0}{\mu}M_M(1-M_M)\frac{\partial(v-K^S)}{\partial R}+N(v-K^S)\frac{a_q^M\lambda_m^0}{\mu}(1-2M_M)\frac{\partial M_M}{\partial R}
\end{aligned}$$

Because $-\omega\lambda_m^0(1-\overline{T})-\frac{\partial^2 C_m(e_m)}{\partial e_m\partial R}=0$ from equation (3.35), $M_M\frac{\partial(v-K^S)}{\partial R}=-(v-K^S)\frac{\partial M_M}{\partial R}$ from equation (3.42), $(\omega+c_r)\lambda_m^0\overline{T}+\omega\lambda_m^0(1-R)(1-\overline{T})-\frac{\partial C_m(e_m)}{\partial e_m}=-\frac{a_q^M\lambda_m^0}{\mu}$ from equation (3.45), and $\frac{\mu}{a_v^M}=(v-K^S)(1-$

M_M) from equation (3.39), we have

$$\begin{aligned}
\frac{\partial^2 \Pi_m^S}{\partial e_m \partial R} &= 0 - N\frac{a_q^M \lambda_m^0}{\mu}(1-M_M)(v-K^S)\frac{\partial M_M}{\partial R} - N\frac{a_q^M \lambda_m^0}{\mu}(1-M_M)(v-K^S)\frac{\partial M_M}{\partial R} \\
&\quad + N\frac{a_q^M \lambda_m^0}{\mu}(1-2M_M)(v-K^S)\frac{\partial M_M}{\partial R} \\
&= -N\frac{a_q^M \lambda_m^0}{\mu}(v-K^S)\frac{\partial M_M}{\partial R} = 0
\end{aligned} \tag{3.49}$$

According to (3.47), (3.48), and (3.49), we will know this is a supermodular game. And we would like to prove the equilibrium is unique. If we want to prove the uniqueness of the optimal solution, we need to show $-\frac{\partial^2 \Pi_i^S}{\partial e_m^2} > \frac{\partial^2 \Pi_i^S}{\partial e_m \partial R} + \frac{\partial^2 \Pi_i^S}{\partial e_m \partial v} + \frac{\partial^2 \Pi_i^S}{\partial e_m \partial \overline{T}}$. We can get

$$\begin{aligned}
& -\frac{\partial^2 \Pi_i^S}{\partial e_m^2} - \frac{\partial^2 \Pi_i^S}{\partial e_m \partial v} - \frac{\partial^2 \Pi_i^S}{\partial e_m \partial R} - \frac{\partial^2 \Pi_i^S}{\partial e_m \partial \overline{T}} \\
&= NM_M\frac{\partial^2 C_m(e_m)}{\partial e_m^2} + NM_M\frac{a_q^M \lambda_m^0}{\mu}\frac{a_q^M \lambda_m^0}{a_v^M} - NM_M\frac{a_q^M \lambda_m^0}{\mu} - 0 - 0 \\
&= NM_M\Big\{\frac{\partial^2 C_m(e_m)}{\partial e_m^2} + \frac{a_q^M \lambda_m^0}{\mu}\Big(\frac{a_q^M \lambda_m^0}{a_v^M} - 1\Big)\Big\}
\end{aligned}$$

Thus, when $\frac{\partial^2 C_m(e_m)}{\partial e_m^2} + \frac{a_q^M \lambda_m^0}{\mu}\Big(\frac{a_q^M \lambda_m^0}{a_v^M} - 1\Big) > 0$, the optimal solution is unique. This concludes the proof for Scenario (ii). **QED**

Proof of Theorem 5

$$\frac{\Pi^j}{\Pi^k} = \frac{\frac{M_M^j}{1-M_M^j}}{\frac{M_M^k}{1-M_M^k}} = \frac{\frac{W_{M1}^j}{W_{M0}}}{\frac{W_{M1}^k}{W_{M0}}} = \frac{W_{M1}^j}{W_{M1}^k} = e^{\frac{a_q^M[\lambda_m^0(e_m^j - e_m^k) + \lambda_s^0(e_s^j - e_s^k)] - a_v^M(v^j - v^k)}{\mu}} \tag{3.50}$$

where $j, k \in \{C, N, F, T, S\}$

If the contract j is more profitable than the contract k, that is $\frac{\Pi^j}{\Pi^k} \geq 1$, the power of the exponential function , $\frac{a_q^M[\lambda_m^0(e_m^j - e_m^k) + \lambda_s^0(e_s^j - e_s^k)] - a_v^M(v^j - v^k)}{\mu}$, must be grater than or equal to 0. That is

$$a_q^M[\lambda_m^0(e_m^j - e_m^k) + \lambda_s^0(e_s^j - e_s^k)] \geq a_v^M(v^j - v^k)$$

And the contract j and k have their own unique solutions for the effort levels and sale price as we proved in the Theorem 1-3. This concludes the proof for Theorem 5. **QED**

Chapter 4

External Quality Cost Sharing Contracts and Market Competitions

ABSTRACT

In this chapter, we will investigate the equilibrium behavior of many decentralized supply chains with or without the quality cost-sharing contracts in the competitive market. The implementation of the quality cost-sharing strategy will improve the product quality and the improved product quality can result in more sales and profits, but incur extra improvement costs. Meanwhile, the sale price will affect the market demand and the profit, too. How to design the quality cost-sharing contract and how to set up the sale price will influence the total profit of supply chains.

Here we consider the quality cost-sharing contracts conducted in five different ways: centrally coordinated, no share, fixed share rate, total cost allocation, and selective root cause analysis. We systematically compare these contracts under the one-stage decision for one manufacturer and one supplier in a monopoly market.

We found the optimal quality improvement effort levels depend on the failure information, effort cost information, and consumer preferences. Neither sale prices nor the market competition will influence on the optimal quality improvement effort. The contract with the selective root cause analysis will achieve the maximum market share and profit by comparing with the other contracts in the decentralized environment. We also found that the supply chain with the worse product quality, or the lower effort cost constant will benefit more by implementing the quality cost-sharing contracts in the duopoly market.

4.1 Introduction

How to manage product quality has been an important topic in corporate strategies in order to strengthen competitiveness, to expand market share, and to maximize profits. Companies have implemented quality control processes, such as TQM, Six Sigma, and SPC, on their own production or service processes. However, because of the economies of scale and their technological ability, the companies may outsource their parts, component design, and production to other members in their supply chains. Both the automotive and computer industries are excellent practitioners to incorporate the outsourcing strategy to strengthen competitiveness in the market.

As companies outsource more product designs and manufacturing activities to other members in the supply chain, how to improve the quality of the final product has become a challenge beyond the boundaries of its in-house process capabilities. In addition to on-site quality improvement, manufacturers might also ask suppliers to take the responsibility of quality management and to share the external quality costs, like warranty costs, recall expenses, and other related costs.

In 2004, automakers in North America spent \$12 billion to fix vehicle quality problems, which amounted to approximately \$400 \$700 per car sold in the U.S.. They spent an average of 250 days to fix a quality problem, and incurred \$1 million for each day of product recall. Who should pay for these huge expenses? Traditionally, manufacturers pay for all quality problems like warranty claims and recalls, even though the product failures are due to suppliers' mistakes. However, recently, manufacturers have started to ask their suppliers to take the responsibility for product quality, e.g. Ford-Firestone's tire recall in 2000. Without any pre-determined contract, the negotiation, even a lawsuit, will take a long time and many resources to settle the case. Therefore, what kind of quality cost-sharing contract do they need? How should they share those recall and/or warranty costs in order to minimize the total expected quality cost?

There are a number of papers discussing the effects of incentives for quality cost-sharing between members in supply chains. (Corbett and Decroix 2001, Lim 2001, Reyniers and Tapiero 1995, Baiman, et al. 2000, 2001, and 2003, Balachandran and Radhakrishnan 2005) These authors design contracts that act as quality control tools to push suppliers to improve their component quality. Penalties will be enforced if there are failure detections during the incoming appraisals before sale or warranty claims after sale. With the penalties, the both players will choose the optimal effort levels to improve the product quality in order to reduce the potential failures. The final results will be based on the equilibrium between the marginal effort cost and the marginal repair cost. However, the market size and the sale price are set as constants in these papers. If the quality cost-sharing contract is successfully implemented in a supply chain, the benefits will not only include warranty cost reduction and quality improvement, but also increased product competitiveness and expanded market share. The extra profits will result from the cost reduction and the market expansion.

In this paper, we will focus on how the manufacturer could set up the pricing strategy and quality cost-sharing contracts in order to maximize the total profits. We will expand on the contract designs developed by Chao, Iravani and Savaskan (2007), but we will focus on the warranty cost instead of the recall cost. The share of quality costs can be determined by either a fixed share rate or by the root cause analysis, or both. The contracts with a fixed share rate and total cost allocation will result in under and over-investment, respectively, by comparing with the centrally coordinated environment. Also, as the root cause analysis cost increases, the failure analysis becomes less attractive in terms of profitability. In this study, we also show that a contract with the selective root cause analysis will eliminate the inefficiency, due to a fixed share rate and the total cost allocation.

The objective function is to maximize the profits which depend on the expected quality costs, market demand, and the sale price. The expected quality costs include the potential warranty cost and the effort cost which are determined by the effort levels. We model the consumer demand with a multinomial Logit function, where the sale price and the final product quality will determine the consumers' purchase utility. Then, we compare the optimal effort levels and the optimal sale prices under different contracts in the competitive market. We show that the optimal effort levels can be obtained with the closed form solutions and easily compared among different contracts. Moreover, the optimal effort depends on the market demographics but not on the number of competitors or the sale price.

In the following section, we briefly survey the literature about quality costing-sharing and market competition. In Section 4.3, we introduce the models and the assumptions about quality cost-sharing contracts and market situations. Then, Section 4.4 will compare the different contracts in terms of the effort level and profits and discuss the parameter sensitivity. We present our conclusions in Section 4.5.

4.2 Literature

The main focus of this paper is on modeling external quality cost-sharing contracts between a manufacturer and a supplier. In addition to the share rate and/or time threshold, the manufacturer will also decide the potential sale price in the monopoly market, in order to maximize the total profit on the supply chain. This paper contributes to several streams of research, and we will review them below.

A group of papers have discussed the design of quality cost-sharing contracts. Reyniers and Tapiero (1995a and 1995b), Lim (2001) and Baiman el al. (2000) used the game theoretic setup to characterize the Nash equilibrium between the supplier's choice of component quality improvement and the manufacturer's choice of inspection strategy. Both the manufacturer and the supplier share the expected cost together with a fixed share rate and they try to minimize the total expected cost with the given prices. Balahandran and Radhakrishnan (2005) considered a double moral hazard situation concerning quality investment effort. They focused on the best use of incoming inspection information to achieve first best effort levels from the supply chain members. In our paper, we will relax the incoming part inspection, and the manufacturer will also exert the quality improvement effort. Both parties will contribute their effort to increase the profits on the supply chain. Corbett and Decroix (2001) have discussed a shared saving contract for the reduction of indirect materials consumption. Both parties work together to put the equilibrium effort to achieve the goal of the consumption reduction.

Baiman et al. (2001) investigated that product defects happened due to non-separable and separable failures. If the failure was separable, the manufacturer could perfectly identify the cause that led to the product failure. The responsibility could be clarified, and the individual who caused this failure should pay the cost of that external quality failure. If the failure was non-separable, both the manufacturer and the supplier would share the cost of the external failure together. With this failure analysis information, both parties exert proper effort levels to ensure quality improvement and external quality cost reduction. We will treat the information to identify the responsibility as the root cause analysis. In this paper, we will discuss how to incorporate the root cause analysis information into the external quality cost sharing contract.

Under the different contract setting, there are many observations about the final equilibrium. Iyer et al. (2005) studied how the buyer could design a menu of contracts to suppliers, in order to minimize the expected cost by assessing the supplier's capability and allocating some internal resource to help the supplier. They found that the optimal resource commitment depends on the interactions with supplier's capability. If the buyer resource and supplier capability were substitutes, then the buyer would put more effort in the second best equilibrium (over-investment) and the information rent would be driven down; the opposite would be true if they were complements.

Chao et al. (2007) compared the cost sharing contracts with first best, no sharing, a fixed share rate, total cost allocation, partial cost allocation and the selective root cause analysis. The adoption of the root cause analysis is certainly beneficial to the reduction of product failure opportunity. However, too much information may result in over-investment and the cost of the root cause analysis would vary the profitability of the cost-sharing contract. By implementation of selective root cause analysis, the supply chain could achieve the lower expected cost and the better product quality in the decentralized supply chain. In this paper, we continue the setup in Chao et al. (2007), but focus on the pricing strategy and the influence from the sale price.

In this paper, the total profit will be determined by the unit profit and market share where the market share will be defined as a multinomial Logit model. The Logit model is easy to use and good for predicting the market share. It estimates the impact of different product characteristics, such as price, quality, band recognition, physical dimensions, promotions, etc. on the consumer purchase behaviors.

Anderson and Palma (1992) presented an approach to describe the demand of heterogeneous consumers. They used the multinomial Logit (MNL) model as an analytical tool into market analysis, and used it to address the question of market and optimal product diversity. The Logit model can also provide a convenient representation of the degree of heterogeneity of consumer tastes. They showed the sale price as the sum of the cost and the value creation, where the value creation could be calculated by the ratio of the horizontal differentiation to the probability of a consumer non-choosing.

Besanko et al. (1998) showed an empirical study of Logit brand choice where the price could be determined endogenously from the equilibrium results of Nash competition among the manufacturer and the retailer. The value creation and market share for the product could also be obtained endogenously. The market share leader must also be the value-creation leader. They validated their findings in two product categories: yogurt and catsup. They also found that the bias in the price coefficients is due to consumer heterogeneity, rather than to price endogeneity, even though there was no consumer heterogeneity in band preferences, price, and other marketing mix variables. If the price endogeneity was ignored, Berry(1994) showed the estimation methods can be severely misleading by applying Monte Carlo methods. Draganska and Jain (2004) also showed the price endogeneity for the simultaneous estimation of structure demand based on a likelihood-based method.

Bernstein and Federgruen (2004) studied the equilibrium model for industries with price and service decisions within the price competition only, simultaneous price and service-level competition, and two-stage competition (service level then price). Retailers optimize their own sale prices, fill rates, and base stock levels, in order to maximize the final profits, which are determined by the unit profits and the potential shortage costs. They showed the existence of a unique Nash equilibrium of infinite-horizon stationary strategies, and in a reduced game. In the simultaneous game or two-stage game with generalized MNL or linear demand functions, each retailer's optimal service level is completely independent from the characteristics of the other competitors, and the optimal service level is obtained by equalizing the incremental operational costs and the incremental retail price value. That is, the retailers could choose their own service levels on the basis of their own characteristics only.

4.3 Basic Model Formulation

Consider there are n manufacturers in the competitive market and manufacturer i produces a product that consists of two components, one from his own process, and the other from his supplier with a unit purchase price p_i. The unit price, p_i, will be negotiated between the manufacturer and the supplier, and it is independent from the potential quality improvement and the final sale price. There are unit production costs, c_{im} and c_{is}, associated with the production processes on the manufacturer's and supplier's sites, respectively. The product will be sold under warranty and the unit revenue (sale price), v_i, will be collected by Manufacturer i. Without losing generality, we normalize the warranty period into 1 for all products. During the warranty period, the product may fail to perform its function if any one of those two components fails. The manufacturer will take responsibility to restore the broken product to the same quality as a new one instantaneously, and the repair cost will be incurred. To simplify the exposition, we assume the repair cost per warranty claim, ω_i, is independent of the cause of the product failure. Here the repair cost, ω_i, does not include only the repair cost for the broken component, but also the labor cost, logistics cost, goodwill cost, and other expenses related to this warranty claim. Thus, we assume ω_i as a constant no matter the failure resulted from the manufacturer's component or the supplier's component. The quality cost-sharing contract will be designed to ask the manufacturer and the supplier to share this quality cost and to ask both players to exert a certain effort to improve the quality of their own components.

4.3.1 Failure Mechanism and Effort Cost

We assume the number of failures happening during the warranty period is a Poisson distribution, and the initial failure rates for components are common knowledge to both parties, and are denoted by λ_{im}^0 and λ_{is}^0 for Manufacturer i's and Supplier i's components, respectively. The failure rates are time homogeneous during the warranty period and they are independent of each other. Once the broken component has been fixed, it performs as well as a new one, and the failure rate for that component will be the same as the original one. In the quality cost-sharing contract, we assume Manufacturer i and his supplier will exert some effort to improve the product quality (failure rate). We use e_{im} and e_{is} to denote the percentage of quality improvement effort exerted by Manufacturer i and Supplier i, respectively, where $0 \leq e_{im}, e_{is} \leq 1$. Under the effort level e_{im} (e_{is}) the Manufacturer(Supplier) i can reduce the failure rate from λ_{im}^0 (λ_{is}^0) to $\lambda_{im}^0[1-e_{im}]$ ($\lambda_{is}^0[1-e_{is}]$). And the expected total repair cost can be reduced from $\omega[\lambda_{im}^0+\lambda_{is}^0]$ to $\omega[\lambda_{im}^0(1-e_{im})+\lambda_{is}^0(1-e_{is})]$ during the warranty period.

In addition to the reduced repair cost, the total quality cost will also include the quality improvement cost. The effort of e_{im} and e_{is} will cost Manufacturer i and Supplier i $C_{im}(e_{im})$ and $C_{is}(e_{is})$, respectively. We consider $C_{im}(e_{im})$ and $C_{is}(e_{is})$ to be twice continuously differentiable on [0,1), and increasing convex in effort. The assumptions of $C_{im}^{'}(0)=0$, $C_{is}^{'}(0)=0$, $\underset{e_{im}\to 1}{lim} C_{im}^{'}(e_{im})=\infty$, and $\underset{e_{is}\to 1}{lim} C_{is}^{'}(e_{is})=\infty$ ensure that there is an interior solution in the effort game. In this paper, we will use $C_{im}(e_{im})=\gamma_{im}[-Ln(1-e_{im})-e_{im}]$ and $C_{is}(e_{is})=\gamma_{is}[-Ln(1-e_{is})-e_{is}]$ as the effort cost functions for Manufacturer i and his supplier, respectively. The parameter γ_{im} and γ_{is} are the convexity of the cost functions. The larger values γ_{im} and γ_{is} are, the faster the effort costs increase with the effort exerted by Manufacturer i and Supplier i.

4.3.2 Quality Cost Sharing Contracts

Traditionally, the manufacturer will pay all quality costs for the warranty claim and the quality improvement (no share). By implementing the quality cost-sharing contract between the supplier and the manufacturer, this total expected quality cost will shrink, and the final product quality will be enhanced.

In this paper, we will discuss five different contracts:

- Centrally coordinated supply chain (C): There is a central planner who optimizes the effort levels for the manufacturer and the supplier, in order to maximize the total profits on the supply chain.

- No share (X): There is no cost sharing scheme between the manufacturer and the supplier. The manufacturer will improve his own component and pay all the quality costs. The supplier will not put any effort since there is no contract to ask the supplier to share the warranty cost.

- Fixed share rate (F): Once a warranty claim happens, the manufacturer and the supplier will share the total quality cost with a fixed ratio.

- Total cost allocation (T): Once a warranty claim happens, a root cause analysis will be conducted to identify the cause of the failure. The quality cost and the analysis cost will be assigned to the party who is liable for this warranty claim.

- Selective root cause analysis (S): Once a warranty claim happens before a threshold time $\overline{T}$ ($\overline{T} \leq$ warranty period), both parties share the costs like in Contract T. If the warranty claim happen after $\overline{T}$, but still during the warranty period, they will share the cost with a fixed share rate like in Contract F. That is, Contract T (where $\overline{T}$=1) and Contract F (where $\overline{T}$=0) are the special cases of Contract S.

Under each contract, the supplier i will make the decision on how much effort to exert in order to improve his component quality in order to minimize the expected quality cost. Meanwhile, the manufacturer i will figure out how to set up the optimal effort, sale price and contract parameters in order to maximize his profit according to the different scenarios and contract parameters.

4.3.3 Market Competition

In this paper, we will systematically analyze the equilibrium states for the different contracts under the market competition.

In the competitive market, Manufacturer i purchases a component from his supplier and produce Product i. The market size, N, is a constant. And the utility of a consumer purchasing Product i can be set as:

$$V_i = a_i - a_q\Big[\lambda_{im}^0(1-e_{im}) + \lambda_{is}^0(1-e_{is})\Big] - a_v v_i = a_i^\theta + a_q\Big[\lambda_{im}^0 e_{im} + \lambda_{is}^0 e_{is}\Big] - a_v v_i$$

where

$$a_i^\theta = a_i - a_q\lambda_{im}^0 - a_q\lambda_{is}^0$$

Also, a_i is the attribute coefficient for the consumer preference for Product i. Since Product i competes with another products in the same market, the attribute coefficients for the consumer preferences for the product quality, a_q, and for sale price, a_v, are the same for those products in the same market. The utility level associated with non-purchase will be still denoted with V_0.

Since the utility of purchasing a product is not the same for all individuals, we assume an individual's utility from purchasing Product i is given by:

$$U_i = V_i + \mu\epsilon_i$$

ϵ_i is a random variable with zero mean and unit variance and μ is a positive parameter that represents the horizontal differentiation between products. By assuming ϵ_i is identically, independently Gumbel distributed, the market shares given by the multinomial logit model are

$$\text{Manufacturer } i\text{'s market share } (M_i) = \frac{W_i}{\Sigma_{j=1} W_j + W_0},$$

where

$$\begin{aligned} W_i &= e^{\frac{a_i^\theta + a_q(\lambda_{im}^0 e_{im} + \lambda_{is}^0 e_{is}) - a_v v_i}{\mu}}, \\ W_j &= e^{\frac{a_j^\theta + a_q(\lambda_{jm}^0 e_{jm} + \lambda_{js}^0 e_{js}) - a_v v_j}{\mu}}, \\ W_0 &= e^{\frac{V_0}{\mu}}. \end{aligned}$$

4.4 Analysis

In this section, we would like to study how to set up the optimal sale price and effort levels in order to maximize the potential profits in the competitive market. Let us assume that there are n manufacturers who compete against each other in the competitive market. The consumers make purchase decisions only based on the final product quality and the sale price.

Here we are more interested in how the manufacturers react in the one-stage decision process in the market. That is, they will make decisions of quality improvement, sale prices and other contract parameters at the same time, in order to maximize total profits. As for the two-stage process, the manufacturers and the suppliers will focus on minimizing their own costs without considering the market competition first. The unique optimal effort levels will be decided during the first stage, and the manufacturers will figure out the optimal sale prices at the second stage. Once the optimal quality improvement levels have been decided, there will be a unique optimal solution for the price competition. Bernstein and Federgruen (2004) showed the equilibrium model with price and service competition, and there will be a unique equilibrium for price competition if the service rates are fixed.

In this research, we would like to discuss the robustness of the optimal effort levels. By identifying the optimal effort level in order to maximize the expected profit, the optimal effort levels do not depend on the number of the competitors and their characteristics.

Theorem 1. *If* $\frac{\frac{\partial M_i}{\partial v_i}}{\frac{\partial M_i}{\partial e_i}}$ *is a constant where* M_i *is the market share for Manufacturer* i*, the optimal effort levels for the manufacturer will be independent of the number of the competitors and the competitors' decisions.*

Proof: Listed in the Appendix.

Theorem 1 states that the optimal effort is not influenced by the competitors if $\frac{\frac{\partial M_i}{\partial v_i}}{\frac{\partial M_i}{\partial e_i}}$ is a constant. That is, the manufacturer can decide the optimal effort levels without the market competition information. This is because on one hand, the increasing optimal effort level will reduce the unit profit, but the effort can result in the higher demand. The optimal effort level can also determine the expected costs due to the quality improvement and the warranty claims. On the other hand, the increasing price will cause the higher unit profit, but the lower demand. The price will influence neither the quality improvement nor the warranty claims. If the market share changes due to the price and the effort have the affine relation, that is, $\frac{\frac{\partial M_i}{\partial e_i}}{\frac{\partial M_i}{\partial v_i}}$ is a constant, the demand change due to the effort can be represented by the demand change due to the price. Then the optimal effort level will be used to minimize the cost and the optimal sale price will be used to maximize the demand and the profits.

The linear, general attraction and multi-nominal Logit demand functions can all satisfy the condition of $\frac{\frac{\partial M_i}{\partial v_i}}{\frac{\partial M_i}{\partial e_i}}$ as a constant. When we use these functions as the demand functions, we will get the optimal effort level without the market competition, according to Theorem 1. Then optimal effort levels can be obtained, and it is unique and completely invariant to market competition, due to the number of the competitors and the changes in the characteristics of any competitors in the market. That is, we will get the same effort level for a manufacturer in the monopoly or competitive market. The manufacturer and the supplier will optimize their own effort, based on their components failure information, effort cost function, and the consumer preference. Then the sale price will be obtained by maximizing the final profit with the given optimal quality.

4.4.1 Centrally coordinated supply chains in the market

If each supply chain is centrally coordinated, manufacturer i's profit, Π_i^C will be:

$$\Pi_i^C = \max_{v_i, e_{im}, e_{is}} N\Big\{v_i - \omega_i[\lambda_{im}^0(1-e_{im}) + \lambda_{is}^0(1-e_{is})] - C_{im}(e_{im}) - C_{is}(e_{is})\Big\}M_i. \quad (4.1)$$

Theorem 2. *For a centrally coordinated supply chain in the competitive market, there is a unique equilibrium existing in this game. The optimal effort levels in the one-stage decision process will be obtained by solving:*

$$\omega_i = \frac{1}{\lambda_{im}^0}\frac{\partial C_{im}(e_{im})}{\partial e_{im}} - \frac{a_q}{a_v}, \quad \omega_i = \frac{1}{\lambda_{is}^0}\frac{\partial C_{is}(e_{is})}{\partial e_{is}} - \frac{a_q}{a_v}, \quad \textit{where } i \in \{1,2\}. \quad (4.2)$$

Proof: Listed in the Appendix.

The optimal effort levels will be very similar to the ones in a monopoly market (in Chapter 3). They will be the same if the attribute coefficients for the consumer preference are the same in both the monopoly and competitive markets. Those optimal effort levels depend on the effort cost function, repair cost, and the consumers' preference for the product quality and sale prices. As the repair cost, the effort cost, or the ratio of consumers' preference for the quality increase, the optimal effort levels will increase. However, the sale prices will be determined by the market competition and it will not influence the optimal effort levels in this one-stage decision process, or even in the two-stage decision process, where the optimal effort level is equal to the one in the monopoly market in Chapter 3. The sale prices will be justified in order to reach the equilibrium with maximum profits for both manufacturers. If we assume that the quality improvement effort cost is the total expense on the quality improvement,t and it does not depend on the number of improved units, the optimal effort level will be influenced by the market share and the sales price. Basuroy and Nguyen (1998) assumed the market expenditure is a lump-sum and the product marginal cost is constant. The market share is determined by the logit model with the sale price and the marketing expense. They found that when the market share is shrinking, it is better to reduce the sale price and optimal market expenditure. That is, the optimal market expenditure (like the improvement effort in our paper) will be influenced by the market share and the sale price. However, in our paper, since the effort cost is the unit cost, and the final quality will also change the unit marginal profit, the optimal effort policy is to balance the warranty cost reduction, effort cost expenditure, and the market preference term between the price and effort. Therefore, the optimal effort level will be a constant either in a monopoly or in multi-player market. Meanwhile, the maximum profit can be achieved by optimizing the sale price, which changes as the market competition changes.

We can get the sale price functions like the ones that we got in the monopoly market (in Chapter

3):

$$v_i^C = c_{im} + c_{is} + \omega_i[\lambda_{im}^0(1 - e_{im}^C) + \lambda_{is}^0(1 - e_{is}^C)] + C_{im}(e_{im}^C) + C_{is}(e_{is}^C) + \frac{\mu}{a_v}\frac{1}{1 - M_i^C}. \qquad (4.3)$$

If the number of the providers on the market increases, the competition becomes more intensive. Then market share will drop down and the price will be lower in the market. Therefore, the profits in the competitive market will go down.

Symmetric manufacturers' information

If the manufacturers' information is symmetric, the suppliers' types and decisions will determine the final product quality, and the total profits where manufacturers will exert the same effort levels. The better the final supplier's component quality is, the higher the market share is. The lower sale price is, the higher final profit is. If suppliers' production costs are the same, a supply chain with the lower supplier's effort cost constants will be more profitable. For any set of parameters, we can find the unique solution for the effort levels for the manufacturers and the suppliers, and the sale prices. Then, we can compare the profitability between manufacturers. Here we can also apply the result from Theorem 6 in Chapter 3 to compare the profitability between these two supply chains. If we can get $a_q(\lambda_{is}^0 e_{is}^{C*} - \lambda_{js}^0 e_{js}^{C*}) \geq a_v(v_i^{C*} - v_j^{C*})$, we can show that choosing Supplier i will will be more profitable than choosing Supplier j in the market with symmetric manufacturer information.

If both manufactures' and suppliers' information are symmetric, then the profits and market shares will be the same for all manufacturers. Their effort levels are also the same and are equal to those in the monopoly market (shown in Chapter 3). However, the sale prices will be lower than one in the monopoly market because of competition where more consumers purchase their products. Thus, the sum of all manufacturers' market shares will be greater than one in the monopoly market. Thus, the total market shares will be expanded in the market because of competition.

4.4.2 Decentralized supply chains in the duopoly market

Can quality cost-sharing contracts generate extra profits in the decentralized duopoly market?

In this section, we would like to identify whether the quality cost-sharing contract can generate more profits than lowering the purchase price. That is, should the manufacturer ask his suppliers to lower the purchase price, or to work together on the quality improvement, in order to maximize the profits? The traditional purchase decisions depended on the lowest bidding. By setting up the minimum accepted quality requirement, the manufacturers choose the suppliers with the lowest bidding prices in order to minimize the purchase expenses and to increase the profits. Does the lower purchase price generate higher profits?

Here we assume that two supply chains, with same failure information and cost functions, compete against each other in the same market. Manufacturer 1 will not only pay the supplier the component production cost, but also the extra incentive to push Supplier 1 to improve the component quality and to share the external quality cost based on a quality cost-sharing contract. Although Manufacturer 1 will increase the purchase cost, it will result in the reduction of potential warranty cost. On the other hand, Manufacturer 2 will buy the component from Supplier 2 with the lowest price, which is equal to the supplier's component production cost. Then Manufacturer 2 can save money on purchasing, but the final product quality may be worse than the other supply chain.

When Manufacturer 1 asks Supplier 1 to participate the cost-sharing contract, there are two important points of information which the manufacturer needs: one is the failure information of the supplier's component, and the other is the supplier's effort cost function. Chao, Iravani and Savaskan (2007) have discussed the influence of failure information on the performance of the quality cost-sharing contract. The supplier will accept the contract when he can provide the better quality than what the manufacturer expected within the cost-sharing contract. By providing the better quality components, both the manufacturer and the supplier can be better off; especially for the contract with root cause analysis. Even though the supplier failure rate is not clear to the manufacturer, by implementing the cost-sharing contract, the manufacturer can increase his profits and assure his profits as he expected.

Now we would like to discuss the influence of the supplier's effort cost function on the manufacturer's profit change. Figure 4.1[1] shows how the profit change from no sharing for both supply chains[2] to the cost-sharing contract for Manufacture 1 and no sharing for Manufacturer 2. Figure 4.1 shows that the manufacturer can generate extra profits by adopting a quality cost-sharing contract; especially, when the supplier's effort cost constant is small, that is, it is less costly to improve the component quality. When the cost-sharing contract is enforced, the supplier will exert the effort to improve the component quality in order to avoid the extra expense for external failure. Since the manufacturer has given the incentive to the supplier, the supplier is willing to exert the optimal effort level in order to maximize profits, and the supplier's extra effort can be compensated by the incentive. The better product quality will decrease the potential warranty cost, increase the competitiveness of the product in the market, and result in the higher demand. That is why the profit will increase. However, when the supplier's effort cost constant is larger, the quality improvement is limited. Thus, the extra demand, due to the better quality, is also insignificant. Therefore, the profit increase will not be so attractive when the supplier's cost constant is large. On the other hand, the profits for the manufacturer choosing not to share the quality with the supplier will be eroded. Since the competitor can provide better product, which can expand the market share, the profit for the supply chain with no-sharing will be reduced as shown in Figure 4.1. The profit difference increases as the effort cost constant decreases. Moreover, in our example, the consumer preferences for both manufacturers (a_1 and a_2) are the same, and the cost-sharing contract is more beneficial. As time goes by, consumers will tend to buy the product from the supply chain with cost-sharing contract and the benefit will become more significant eventually.

This result can validate the empirical findings by Dyer (1996) and Dyer and Hatch (2006) that the

[1] The y-axis is the ratio of manufacturer's profit under some cost-sharing contract to the manufacturer's profit under no-sharing case.

[2] $N = 1,\ \omega_1 = \omega_2 = 100, \lambda^0_{1m} = \lambda^0_{2m} = 1, \lambda^0_{1s} = \lambda^0_{2s} = 0.6, c_{1m} = c_{2m} = 50, c_{1s} = c_{2s} = 30, \gamma_{1m} = \gamma_{2m} = 100, a_q = 10, a_p = 1, a_1 = a_2 = 400, V_0 = 50, \mu = 50.$

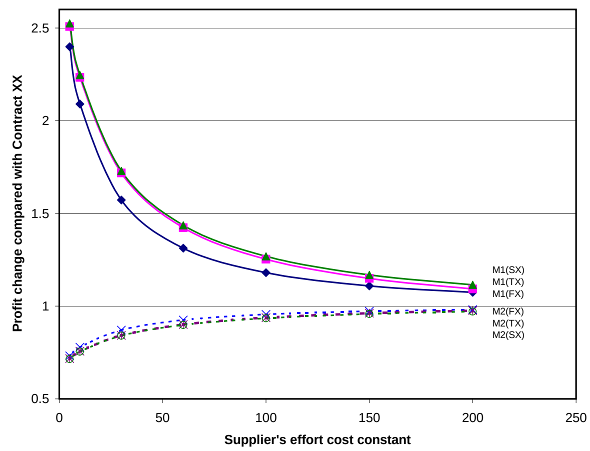


Figure 4.1: Profit Change under the Influence of Supplier's Effort Cost Constant in the Duopoly Market. M1(SX) represents the manufacturer 1's profit change where Manufacturer 1 and 2 adopt Contract S and X, respectively.

supplier network resources have a significant influence on firm performance. The traditional inter-organizational routines and policies will act as barriers to knowledge transfers within the supply chains. The company should work together with his suppliers, share the knowledge, and share the responsibilities, like warranty cost, in order to improve the performance and profits. That is why U.S. automakers, like GM, Ford, and Chrysler, cannot compete with Japanese automakers, like Toyota and Honda, in terms of the product quality, the rate of learning, market shares, and the profits. If the supplier receives more direct assistance in terms of days of visits, knowledge transfer, and asset specificity from the manufacturer, this supply chain will behave more like a centrally coordinated supply chain and each member can still retain his own autonomy. Even though, if there are some issues prohibiting the extensive information-sharing within the supply chain, the supply chain can still benefit from the adoption of the cost-sharing contract to drive the supply chain members to exert some effort to maximize the profits.

Cost-sharing contract comparisons in the duopoly market

From the previous section, we know that the manufacturer should implement the quality cost-sharing contract in the supply chain in order to increase his competitiveness and profits on the market. In this section, we will compare two supply chains in the duopoly market where they adopt one-stage cost-sharing contracts in the decentralized environment. That is, they will choose either Contract F, T, or S. The profit functions for the different contracts in the duopoly market will be similar to the ones in the monopoly market, except for the definition of the market share. From Chapter 3, we find the optimal effort levels for one stage decision in the monopoly market depend on the initial failure rate, repair cost, effort cost function and consumers' preferences for the sale price and market share. With the same derivation in Chapter 3, we can get Corollary 1 for the optimal quality improvement effort in the duopoly market.

Corollary 1. *With the same consumer preferences in the monopoly and duopoly markets, the optimal effort levels for a certain quality cost-sharing contract in the duopoly market are equal to those in the monopoly market.*

Like in the monopoly market, in addition to the warranty cost and the initial failure rate, the optimal effort levels will also depend on the market information (consumer preferences). The optimal effort levels will reach the balance on the decrease of expected warranty cost, increase of effort cost, and the increase of profit due to the enhanced product quality. Thus, if the consumer preferences are the same in the monopoly and duopoly markets, their optimal effort levels will be the same.

Then we can get the similar effort levels and sale price analysis results like in Chapter 3 for an individual manufacturer. Since there is one competitor in this duopoly market, the market share in the duopoly market will be lower than one in the monopoly market. Accordingly, the profit in the duopoly market will be lower than one in the monopoly market.

Table 4.1 and 4.2 show the profits for two manufacturers[3] under the different contracts in the

[3] $N = 1,\ \omega_1 = 100, \lambda_{1m}^0 = 1, \lambda_{1s}^0 = 0.6, c_{1m} = 50, c_{1s} = 30, \gamma_{1m} = 100, \gamma_{1s} = 80, \omega_2 = 80, \lambda_{2m}^0 = 1.2, \lambda_{2s}^0 = 0.8, c_{2m} = 40, c_{2s} = 25, \gamma_{2m} = 80, \gamma_{2s} = 60, a_q = 10, a_p = 1, a_1 = 400, a_2 = 320, V_0 = 50, \mu = 50.$

Table 4.1: Total Profits for Manufacturers under the Different Contracts in the Duopoly Market (low analyst cost, $c_r = 1$). *Left*: Manufacturer 1's profit; *Right*:Manufacturer 2's profit.

		Manufacturer 2							
		Contract C		Contract F		Contract T		Contract S	
M 1	C	\$51.41	\$20.63	\$53.27	\$18.68	\$51.63	\$20.39	\$51.63	\$20.39
	F	\$47.94	\$21.47	\$49.71	\$19.43	\$48.23	\$21.20	\$48.15	\$21.22
	T	\$51.00	\$20.73	\$52.80	\$18.81	\$51.22	\$20.49	\$51.60	\$20.40
	S	\$51.00	\$20.73	\$52.85	\$18.76	\$51.03	\$20.70	\$51.22	\$20.49

duopoly market. The profit does not depend only on the contract type, but also on the competitor's movement. The best scenario for a manufacturer is to choose Contract S and his competitor chooses any contract but not Contract S, where Contract S will result in the highest profits. However, since both players are risk-neutral, all of them will seek for Contract S for the maximum profits.

When the analysis cost is low, as shown in Table 4.1, the root cause analysis will economically identify the failure responsibility and generate higher profits effectively. Thus, Contract T is the better choices while Contract S behaves as Contract T. Both manufacturers will choose Contract S when the analysis cost is low. When the analysis cost is high, as shown in Table 4.2, the root cause analysis will erode their profits, especially for Contract T. If one manufacturer can "persuade" his competitor to adopt Contract T with high root cause analysis cost, he will benefit more from that, especially when he adopts Contract F or S. (Here Contract S behaves as Contract F) Under this situation, conducting any root cause analysis is not a wise decision, and whoever adopts the high-cost root cause analysis will be punished, and lose the profits and market shares. Thus, by implementing Contract S, the manufacturer can increase extra profits from the low-cost root cause analysis, and he can also prevent extra expenses for the expensive analysis cost.

Table 4.2: Total Profits for Manufacturers under the Different Contracts in the Duopoly Market (high analyst cost, $c_r = 50$). *Left*: Manufacturer 1's profit; *Right*:Manufacturer 2's profit.

		Manufacturer 2							
		Contract C		Contract F		Contract T		Contract S	
M 1	C	\$51.41	\$20.63	\$53.27	\$18.68	\$59.62	\$12.66	\$53.27	\$18.68
	F	\$47.94	\$21.47	\$49.71	\$19.43	\$55.74	\$13.16	\$49.71	\$19.43
	T	\$35.75	\$24.98	\$37.13	\$22.59	\$41.96	\$15.27	\$37.13	\$22.59
	S	\$47.94	\$21.47	\$49.71	\$19.43	\$55.74	\$13.16	\$49.71	\$19.43

Benefits of cost-sharing contracts under consumer preferences

From the previous section, we know both manufacturers will choose Contract S to maximize their profits. Which manufacturer can benefit more from Contract S? There are many factors influencing the final profits, e.g. the failure rates, production cost, effort cost, repair cost, and market situations. Here we assume that two manufacturers compete each other with the same products but different product quality in the duopoly market[4]. The manufacturer with a higher quality product (with lower failure rate) will spend more on the costs of purchase, production, and improvement. Here we observe how the market situations influence the profitability for these two manufacturers.

When the consumers do not care about the price, (that is, the consumer preference for the price, a_v, is low) both manufacturers can generate higher profits (as shown in Figure 4.2 *left*), and there is no difference between adopting Contract X and Contract S (as shown in Figure 4.2 *right*). This is because both manufacturers will enhance their product quality as much as they can, and the effort cost can be compensated by the higher sale prices because of low a_v . Thus, both manufacturers can increase their profits by increasing the sale prices. The profit increase due to the Contract S will be limited. Thus, there is no significant benefits to conduct Contract S under low consumer preference for price.

[4] $N = 1$, $\omega_1 = 100, c_r = 1, \lambda^0_{1m} = 1, \lambda^0_{1s} = 0.6, c_{1m} = 50, c_{1s} = 30, \gamma_{1m} = 100, \gamma_{1s} = 80, \omega_2 = 80, \lambda^0_{2m} = 1.2, \lambda^0_{2s} = 0.8, c_{2m} = 40, c_{2s} = 25, \gamma_{2m} = 80, \gamma_{2s} = 60, a_q = 10, a_p = 1, a_1 = 400, a_2 = 320, V_0 = 50, \mu = 50.$

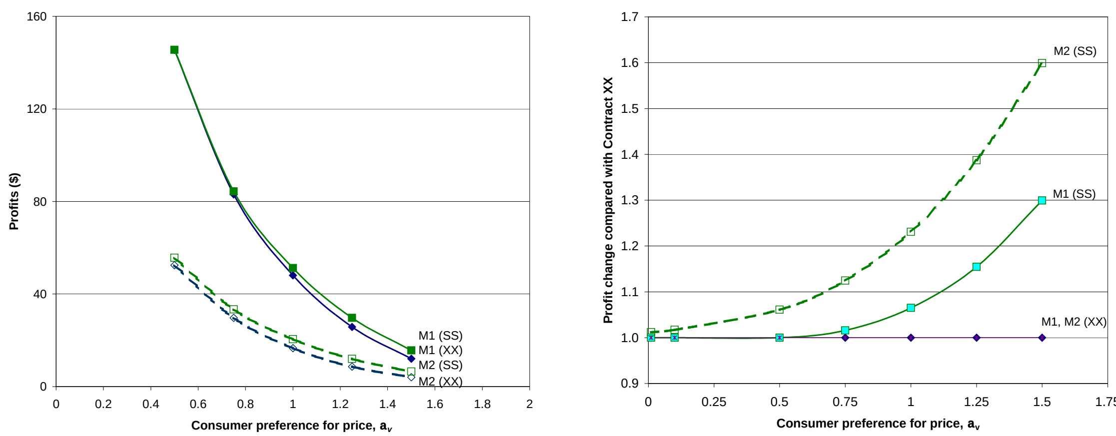


Figure 4.2: Profits (*left*) and Profit Change (*right*) under the Influence of Consumer Preference for Price in the Duopoly Market. M1(SS) represents the manufacturer 1's value where both Manufacturer 1 and 2 adopt Contract S.

On the contrary, when the consumer preference for the price increases, both profits will decrease dramatically. Consumers are sensitive to the price change, and the manufacturers cannot raise the price easily without changing the market share. Thus, the extra quality improvement cost will be covered by the high sale price. In that way, both manufacturers would like not to improve product quality so much, like what they did under low a_v. With high a_v, Contract S can help both manufacturers work with their suppliers to improve their final product quality, in order to squeeze more profits from the market. Especially for the manufacturer with lower effort cost (that is Manufacturer 2 in our example), he can benefit more by adopting Contract S than his counterpart.

On the other hand, the consumer preference for the quality, a_q, will also influence the profitability. When a_q increases, both manufacturers' profits drop almost linearly as shown in Figure 4.3 *left*. Moreover, when a_q is high, the manufacturer with the worst product quality will benefit more by adopting Contract S than his counterpart, as shown in Figure 4.3 *right*.

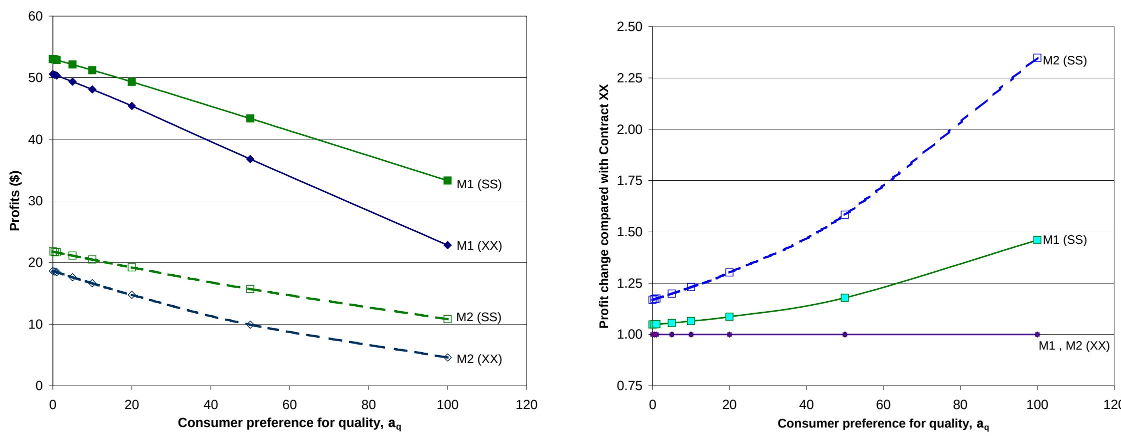


Figure 4.3: Profits (*left*) and Profit Change (*right*) under the Influence of Consumer Preference for Quality in the Duopoly Market. M1(SS) represents the manufacturer 1's value where both Manufacturer 1 and 2 adopt Contract S.

4.5 Summary & Managerial Insights

In this section, we will highlight the insights about the cost-sharing contract and the pricing strategy in the market competition. Most results in the monopoly market (see Chapter 3) can also be validated in the market competition. The manufacturers had better ask their suppliers to share the external costs. The cost-sharing contract can enhance the product quality, strengthen the market competency and assure the market share.

Optimal effort levels are independent from the also price and the market competition. A company can maximize the profits by reducing the costs or increasing the revenues. The optimal effort level can help a company minimize his quality cost under the influence of the quality improvement cost, and consumer preferences. The price and the market competition can maximize the revenue with the optimal improved product quality.

Cost-sharing contract with the selective root cause analysis will the best choice in the decentralized environment. Even though the total cost allocation can result in the better product quality and keep the player in the market, the high improvement cost erodes the profits. Not adopting any cost-sharing contract will endanger the manufacturer's future because of the decreasing profits. By adopting the selective root cause analysis, on one hand, the supply chain can use the root cause analysis to identify the liability; on the other hand, the supply chain can lessen the high root cause expense due to the selective examination.

Consumer preferences will dominate the attractiveness of the quality cost-sharing contracts. If the consumers are quality and price- sensitive, the quality cost-sharing contract will be the powerful tool to maximize the profits and fortify the market competency for manufacturers. Manufacturers cannot change consumers' preferences. However, by adopting the cost-sharing contract with the optimal price and quality improvement, manufacturers can increase their profits.

The lower quality provider will benefit more from the implementation of the cost-sharing contract. It is not necessary to improve the product quality for the manufacturer with the worst product quality in order to be as good as the best quality product on the market. However, the cost-sharing contract can result in the better quality, the lower cost, and the strong competency on the market; especially for the manufacturer with the lowest quality.

APPENDIX

Proof of Theorem 1 Let's assume that the demand function for Manufacturer i is $M_i(\mathbf{v}, \mathbf{e})$ where $\mathbf{v} = \{v_1, v_2, \ldots, v_n\}$ and $\mathbf{e} = \{e_1, e_2, \ldots, e_n\}$. $G_i(e_i)$ is the corresponding effort cost functions for e_i. Then we can assume the profit function as

$$\Pi_i = (v_i - G_i(e_i))M_i(\mathbf{v}, \mathbf{e})$$

We seek for maximum profits; therefore, the first order condition will be

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial v_i} &= M_i(\mathbf{v}, \mathbf{e}) + \Big(v_i - G_i(e_i)\Big)\frac{\partial M_i}{\partial v_i} = 0 \\
\frac{\partial \Pi_i}{\partial e_i} &= -G_i'(e_i)M_i(\mathbf{v}, \mathbf{e}) + \Big(v_i - G_i(e_i)\Big)\frac{\partial M_i}{\partial e_i} = 0
\end{aligned}$$

Then

$$\frac{1}{G_i'(e_i)} = -\frac{\frac{\partial M_i}{\partial v_i}}{\frac{\partial M_i}{\partial e_i}}$$

Since $\frac{\frac{\partial M_i}{\partial v_i}}{\frac{\partial M_i}{\partial e_i}}$ is a constant in our model. Then $G_i'(e_i)$ is a constant. The the optimal effort level is independent of the other competitor's characteristics and decisions. **QED**

Proof of Theorem 2 Centrally coordinated supply chain in a duopoly market

We will prove this theorem in two steps. The first step is to prove the existence of the unique optimal solution for the effort levels. The second step is to prove the price competition is a supermodular game and there is an unique equilibrium.

First, we will prove the unique optimal effort levels. This proof is very similar to the proof for Scenario (*ii*) in Theorem 1 in Chapter 3. With the same method, ,we can get Manufacturer i's optimal effort level:

$$w = \frac{1}{\lambda_{im}^0}\frac{\partial C_{im}(e_{im}^*)}{\partial e_{im}} - \frac{a_q}{a_v} \quad \text{and} \quad w = \frac{1}{\lambda_{is}^0}\frac{\partial C_{is}(e_{is}^*)}{\partial e_{is}} - \frac{a_q}{a_v} \tag{4.4}$$

Moreover, they are unique.

Secondly, since the effort levels are unique and the original profit functions can be simplified into the functions of the sale prices only. If we want to make sure this game is supermodular. we have to prove that $\frac{\partial^2 \Pi_i^C}{\partial v_i \partial v_j} \geq 0$, and $\frac{\partial^2 \Pi_j^C}{\partial v_i \partial v_j} \geq 0$.

$$\text{Assume} \quad K_i^C = \omega[\lambda_{im}^0(1 - e_{im}) + \lambda_{is}^0(1 - e_{is})] - C_{im}(e_{im}) - C_{is}(e_{is})$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_i^C}{\partial v_i \partial v_j} &= N\frac{a_v}{\mu}M_iM_j - N(v_i - K_i^C)(\frac{a_v}{\mu})(1-2M_i)(\frac{a_v}{\mu})M_iM_j \\
&= N\frac{a_v}{\mu}M_iM_j - N(v_i - K_i^C)(\frac{a_v}{\mu})(1-M_i)(\frac{a_v}{\mu})M_iM_j \\
&\quad +N(v_i - K_i^C)(\frac{a_v}{\mu})(\frac{a_v}{\mu})M_i^2M_j \\
&= N(v_i - K_i^C)(\frac{a_v}{\mu})^2M_i^2M_j = N\frac{1}{1-M_i}(\frac{a_v}{\mu})M_i^2M_j \geq 0
\end{aligned}
$$

where $(v_i - K_i^C)(\frac{a_v}{\mu})(1-M_i) = 1$ from $\frac{\partial \Pi_{i)}^C}{\partial v_i} = 0$

Thus, this effort game is supermodular and there is at least one Nash equilibrium existing in this game. If we want the equilibrium is unique, we need to prove that $-\frac{\partial^2 \Pi_i^C}{\partial v_i^2} \geq \frac{\partial^2 \Pi_i^C}{\partial v_i \partial v_j}$.

$$
-\frac{\partial^2 \Pi_i^C}{\partial v_i^2} = NM_i\frac{a_v}{\mu} \geq N\frac{a_v}{\mu}M_i^2 \geq N\frac{a_v}{\mu}M_i^2\frac{M_j}{1-M_i} = \frac{\partial^2 \Pi_i^C}{\partial v_i \partial v_j}
$$

Thus, the equilibrium is unique. This concludes the proof for Theorem 2. **QED**

Chapter 5

Summary and Future Research

We introduced the quality cost-sharing contract in Chapter 2 where we minimize the total external quality cost on the supply chain. We find the cost-sharing contract will result in the better quality and the lower quality cost. By setting the pooled contract or the menu of contract, the manufacturer can diminish the inefficiency due to asymmetric information from the suppliers. With the selective root cause, the manufacturer can avoid over or under-investments due to the fixed share rate contract and the total cost allocation contract, respectively. Especially, when the supplier is new or a product is during the introduction period, the manufacturer had better implement this quality cost-sharing contract to ask the supplier to share the quality responsibility.

The improved product quality and the lower quality cost will strengthen the market competency for the manufacturer. In Chapter 3, we discuss how to set up the pricing strategy with the cost-sharing contract in the monopoly market. We find that the manufacturer should make the decision for the optimal quality improvement effort levels and the sale price at the same time. Meanwhile, the optimal effort levels are determined by the effort cost functions and the consumer preferences but independent from the sale price. The optimal effort levels can minimize the potential external quality cost and increase the market demand which is assumed as the Logit function of product quality and the sale price. The optimal sale price will help the manufacturer maximize the profits.

With the market competition, the optimal effort levels will not be influenced by the competitors' characteristics or decisions. The optimal effort levels in the market competition are the same with those in the monopoly market. The selective root cause analysis contract is still the best choice for the manufacturer. However, the consumer preferences will dominate the attractiveness of the cost-sharing contract. If consumers are price and quality-sensitive, the cost-sharing contract will be the powerful tool for the manufacturer to increase the profits. Otherwise, the benefits due to the cost-sharing contract will be not so striking.

Who should adopt the cost sharing contract? We find the same results in Chapter 2, 3, and 4 that the supply chain with the worse product quality should adopt the cost-sharing contract. We also find the cost-sharing contract with selective root cause analysis will be used to identify the liability and push the liable party to exert more effort on the component quality improvement. Here we assume all failures could be identified and studied thoroughly and perfectly. To enrich this study, we would like to explore the undetermined liability case. We might use the fixed share rate contract and the root cause analysis for those undermined and identified liabilities, respectively.

In addition to the supplier, the manufacturer can also let the consumers get involved in the reduction of external quality cost. With the regular periodic examination and replacement, the number of potential warranty claims will be shrinking. With the introduction of the preventative maintenance into this cost-sharing contract, the optimal effort levels for the manufacturer and the supplier, the optimal sale price, and the optimal product examination period for consumers will be decided.

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