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Ecological Perspective on Error Orientations and Interactions in African-Centered Middle School Math Classrooms

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Abstract

It is widely understood that people's perceptions of themselves and tasks influence their engagement and effort. Further, these relationships are often viewed as the purview of the individual. In contrast, research on human development has documented the influences of participation in multiple (often overlapping) contexts on individual development. Processes of participation, prevailing meta-narratives and structures (political, economic, social) as well as resource allocation within and across these settings can pose challenges that individuals and groups must navigate to achieve their goals.

This dissertation focuses on mathematics learning, explicitly at the context in which student errors are investigated in the classroom using an ecological systems lens. This attention to mathematical errors is critical because errors can potentially position students as incapable, and thus how attention to mathematical errors unfolds in micro-level processes within classrooms can serve as one point in an ecological system to either support productive adaptive responses or confirm negative deficit self-assumptions. Overall, this study seeks to answer the following questions:

1. What is the relationship between students' error identities, racial identity, growth mindset, and how are these associated with macro-level messages about race?
2. How do teachers' beliefs and attitudes about errors index meta-structures and meta-narratives in their planning and instructional practices?
3. How do teachers' and students' orientations and dispositions influence their interactions within error moments?

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1. Introduction

This study focuses on mathematics learning, explicitly at the context in which student errors are investigated in the classroom using an ecological systems lens. This attention to mathematical errors is critical because errors can potentially position students as incapable, and thus how attention to mathematical errors unfolds in micro-level processes within classrooms can serve as one point in an ecological system to either support productive adaptive responses or confirm negative deficit self-assumptions. As Gholson & Wilkes (2017) noted:

> Certainly, mathematics has been described as a "gatekeeper" (Moses & Cobb, 2001; Stinson, 2004), but this term understates the pernicious discursive and structural machinations by which mathematics classrooms actively exclude particular groups of children (p. 229).

Other layers of the ecological system addressed in this study included micro-level processes within classrooms, belief systems brought into the classroom regarding perceptions of both teachers and students concerning mathematical errors, growth versus fixed mindsets around ability, and students' perceptions of racial identity. Belief systems and self-perceptions travel across and are developed across ecological settings (e.g., family, community sites, peer social networks, schools, and broader societal structures). In addition, the study includes data on the more general cultural practices within the school that serves as the site of the study.

Before describing the rest of the study, it is crucial to explain why studying errors using an ecological system lens was necessary. Existing research that explored errors in the classroom tended to focus on teachers' beliefs and orientations regarding errors with little consideration of how these orientations were enacted in practice, with the notable exception of Matteucci et al. (2015) and Tulis (2013) and none that explore how these orientations and beliefs were impacted by metanarratives and meta-structures surrounding race and intelligence. For example, consider the work of Tyler et al. (2006), who conducted think-a-loud interviews with 62 white female

teachers from two elementary schools in which the student population was 90% African American. The researchers gave each teacher learning scenarios that assessed their "perceptions and attitudes towards students who expressed specific cultural orientations in their classroom achievement behaviors" (p. 1000). Four different scenarios represented four cultural themes: two mainstream cultural themes, individualism & competition (Boykin et al., 2005) – a traditional schooling marker for success, and two African cultural themes, communalism and verve (Boykin et al., 2005). By design, all four scenarios showed academically successful students with the first line of each reading, "[Student Name] gets very high grades," and then described student behavior that reflected one of the cultural themes. To illustrate, the communalism student wants to share her ideas and help other students. The verve student likes the teacher to teach content in different ways and has multiple activities to explore her growing understanding. The individualism student enjoys working alone, and the competition student likes to see who received the best score and always wants to be the best. Their findings supported the argument presented by Boykin et al. (2005), in which they stated, "overall, teachers' perceptions of student motivation and achievement was significantly higher for mainstream cultural themes than for Afro-cultural themes" (p. 1002). The significance of these findings regarding teachers' perceptions was that even when students perform roughly the same if the expression of their knowledge did not fit into the macrosystem's beliefs about successful classroom practice, that student was seen as less motivated and less achieving. This orientation can then influence and mediate the teacher's microsystem-level interactions with these students. Overall, this study seeks to answer the following questions:

1. What are potential relationships among students' error identities, racial identity, mindset, and how are these associated with macro-level messages about race?

2. How do teachers' beliefs and attitudes about errors reflect meta-structures and meta-narratives in their planning and instructional practices?
3. What potential relationships exist between teachers' and students' orientations and dispositions and how are they reflected in classroom interactions within error moments?

Motivation for Study

While observing students at a school on the south side of Chicago, I often reflected on how my current role is inextricably tied to my own life experiences. Growing up in East St. Louis and labeled as a disadvantaged inner-city minority male youth, I frequently saw myself in the faces of these students and was reminded of my life through their stories. One such story I repeatedly heard in both my capacity as a teacher as well as a researcher was how students positioned themselves within the context of math: "I'm not a math person" or "Math is for white people." Papert (1980) emphasized this point in his book "Mindstorms" in which he wrote, "Difficulty with school math is often the first step of an invasive intellectual process that leads us all to define ourselves as bundles of aptitudes and ineptitudes, as being 'mathematical' or 'not mathematical,' 'artistic' or 'not artistic,' "musical" or 'not musical,' 'profound' or 'superficial,' 'intelligent' or 'dumb'" (p. 8). Echoed within this sentiment is the idea that math has become a gatekeeper for Black students that inhibits their ability to progress in the world (Moses & Cobb, 2002). However, on the other end of this spectrum, we see how students develop different stories if they can explore their racial identity more fully. Gholson and Martin (2014) provide examples of this regarding black girls in which "constructing Black girlhood meant the girls had more significant opportunity to affirmatively internalize their Blackness, i.e., not in negation or response to other ethnic or cultural norms (p. 20).

2. Literature Review

In this work I draw on three fields of study: ecological systems theory, identity development with an emphasis on race and ethnicity, and phenomenology. I first look to the work on ecological systems (Bronfenbrenner, 1994). Spencer (2007) provided a brief history of ecological systems explaining:

> The critical and long-term insights provided by early ecological psychologists such as Roger Barker, Herbert Wright, and Paul Gump (see Barker & Wright, 1949, 1954; Gump & Sutton-Smith, 1955; Wright, 1967), and later broadened and refined by Urie Bronfenbrenner (see Bronfenbrenner, 1985, 1992, 1993; Bronfenbrenner & Crouter, 1983), existed and continue to provide critical insights. Fieldwork by ecological psychologists in the United States and England obtained definitive findings concerning the reciprocal links between the individual and context: Conceptual contributions by Bronfenbrenner illustrated the indisputable interactions, and space psychologists such as Joachim Wohlwill (Wohlwill, 1985; Wohlwill & Heft, 1987) clearly explicated the reciprocal links further. Considered together, given the different though parallel methodological and conceptual strategies, they quite persuasively linked individual experiences with context character (p. 883).

Attention to ecological systems is essential because multiple interacting systems influence students' construals of themselves, tasks, and contexts. In particular, microsystems are the systems closest to the student and those in which the student has direct interactions, such as classroom interactions. Mesosystems are interactions between two or more microsystems, such as relationships between home and school. Exosystems are systems that the student does not directly interact with but indirectly affect the student, such as parental employment. Macrosystems are the cultural and societal norms within which the other systems are nested, such as heteronormativity or gender roles. Lastly, the chronosystem includes changes for the child and other systems over the child's life course, such as stability or shifts in economic standing over time. Within an ecological systems framework, students' engagement and knowledge construction are influenced by more than what happens in the classroom. For the population I plan to study (Black youth), meta-narratives, meta belief systems, and meta-level

institutional practices within these levels are deeply influenced by systems of racism and discrimination. Moreover, the mechanisms that "account for commonsense predictions, expectations, explanations, [and] judgments" that guide everyday interactions within these systems have to do with the meaning-making processes of human beings in which perceptions matter (diSessa, 1993, p. 105). These perceptions include the perception of goals, the self, the task's perceptions, etc. (Eccles, 2009).

Another example of an interaction between these systems and perceptions can be found in the work of Martin (2006), in which he discussed how parents' experiences with school mathematics could be interpreted as a "racialized form of experience" of what it means to be black and do mathematics. Students can take up these parent experiences and subsequently influence students' perceptions of mathematics. Martin's illustration is an example of how influences from what Bronfenbrenner calls the exosystem (experiences of socializers such as parents in which the children are not themselves directly involved) potentially influence participation in the microsystem (math classroom interactions) and highlight the role perception has on the meaning-making process of the parent that is then exposed to the student. This example also demonstrates how interacting levels can impact culture and vis versa. Medin and Bang (2014) illustrated this while arguing for a more operational definition of culture:

> If instead culture is seen as dynamic, contested, and variably distributed within and across groups, it is natural to see cultural learning as involving a reciprocal relationship between individuals' goals, perspectives, abilities, and values and their environment On this view, socialization partially depends on agents or others who are caregivers as well as an individual's interpretation of and reaction to their environment." (Medin p. 87).

With the population I plan to study, these perceptions can be tied to issues with the construction of race at all levels of the system (e.g., micro, meso, exo, macro, chrono). Steele (1997) described one example of this as stereotype threat or "stereotypes in the air." Steele noted

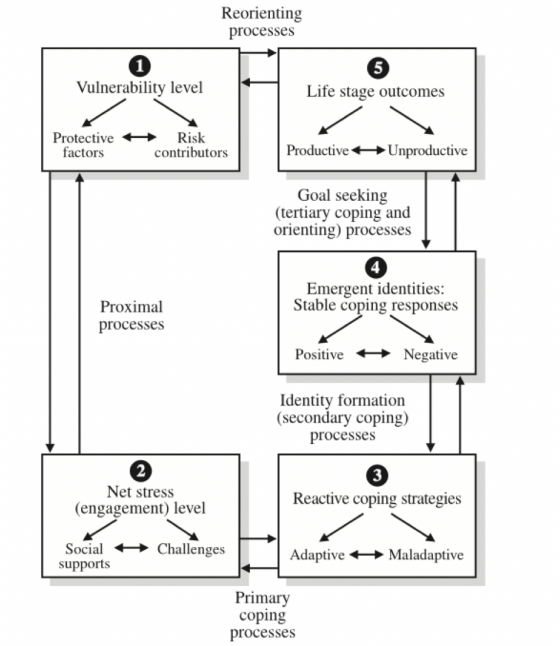
that adhering to and challenging these larger macrosystem level interpretations of perceived ability affects both the displays of the ability of individuals identified within a given stereotyped community and those who interact with members of these communities. The idea is that responding to perceived stereotypes takes up attention in short-term memory that can detract from the cognitive resources directly applicable to the problems to be solved. Steele & Aronson (1995) provided evidence for this hypothesis. They had black and white university students split into two groups and answered questions based on the verbal section of the GRE. The two groups had the same questions; however, what varied was how the researchers introduced the items to the students and for what purpose. One group was introduced to the task as a diagnostic of their ability. In contrast, the other group was presented to the same task as problem-solving activity unrelated to their ability or competence. Within the diagnostic group, white students outperformed black students. However, black students in the problem-solving activity (non-stereotype threat condition) performed equally with white students in the diagnostic group. Steele also noted that these macrosystem level interpretations of black students and their academic ability were so invasive that identifying their race on a demographic questionnaire before answering the questions (all groups in non-stereotype threat condition) affected students' cognition depressed their performance.

PVEST. One framework that integrates the previous theories mentioned and will be used as the cornerstone for this work is Spencer's (1995) Phenomenological Variant of Ecological Systems Theory or PVEST model. PVEST integrates ecological systems, the role of perceptions in humans' construal of the self and context, specific research on racial identity and stereotype threat, and how racial socialization can serve as a buffer or moderator to extant stereotypes and

experiences of discrimination, racism, etc. PVEST also introduces another important factor, wherein the life course is the learner.

PVEST considers explicitly how scholars must understand supports concerning the challenges faced regarding an individual's overall development within and across settings. Specifically, PVEST is a risk/resilience iterative model that includes five factors, (1) Net Vulnerability, (2) Net Stress, (3) Reactive Coping Processes, (4) Emergent Identities, and (5) Stage-Specific Coping Outcomes (shown in Figure 1) to understand "unique and cumulative individual-context interactions, such as the interaction between maturational influences and social experience-based cognitions" (Spencer, 2008, p. 698).

Figure 1: Process Emphasizing: PVEST (Spencer, 2008, p. 708, Figure 19.3)



Net vulnerability is described as "associated with the balance between risk factor burdens pitted against protective factor presence" (Spencer, 2008, p. 706) historically for the individual. Net vulnerability interacts with net stress by the quality of social supports or challenges present in interactions and encompasses the actual level of stress experienced by the individual. Reactive

coping processes "represent how the net stress is handled in the moment given unavoidable developmental task" (Spencer, 2008, pp. 709 & 710), in which these coping strategies could be both adaptive and maladaptive. This distinction of highlighting both ways these coping strategies can be present is critical for Spencer because she wants to argue against deficit assumptions within marginalized and underperforming communities. This decision allows for her analysis to include "the attainment of resiliency or good outcomes obtained in the face of significant and frequently overlooked challenges" (Spencer, 2008, p. 701) as well as note the barriers these students faced.

Emergent identities result from repeated expressions of coping processes that become "orienting behaviors" that stabilize over time. Stage-specific coping outcomes are the conscious and unconscious internalization of these identities at particular points in the life course. Overall, while this is a life-course model because of its focus on developmental stages, it can also be utilized within a specific developmental period and focus in a particular context.

In this study, the focus is on middle school-aged children (embodying two developmental thresholds, entering middle school and preparing to exit middle school) and charting the "sources and pathways of both productive and less productive coping processes, which, in turn, result in patterned life stage-specific outcomes" (p. 700) associated with committing errors in the classroom.

Historical Perspective of Error Analysis

Researchers have studied error analysis in mathematics for over 40 years (Radatz, 1980; Borasi, 1987; Santagata, 2004; Lannin et al., 2007; Moser et al., 2011; Steuer et al., 2013; Tulis et al., 2016;). Radatz (1980) defined errors as systematic, persistent, and derived from students' difficulties with a given mathematical concept. Radatz's study of errors was designed to "reveal

the faulty problem-solving process [of students] and provide information on the understanding of and the attitudes toward mathematical problems" (1980, p. 16). This type of research represented an early prevailing view in which errors were used as a diagnostic tool to identify student difficulties learning an academic topic and were then eliminated. That is, errors were seen as a barrier to understanding and learning. In contrast, a fundamentally different view emerged that viewed errors not as deficits but as a means for inquiry (Borasi, 1987). diSessa (1993) articulated this view when he stated that an error "does not need to be replaced so much as developed and refined" (p. 109). This approach referred to as the "error for inquiry model" (Borasi, 1987), aims to extend beyond simply correcting an error to helping students understand the source of the error and provide an opportunity to explore the mathematical ideas entailed within the error. The critical distinction here is that the error itself can serve as the foundation through which one attempts to understand and build upon students' mathematical knowledge.

These two schools of thought regarding addressing errors reflected existing differences in the behaviorist and constructivist views of learning, as Lannin, Barker, and Townsend (2007) noted. Behaviorist views were defined as valuing "positive reinforcement when students yield correct answers and [initiating] negative reinforcement or … [withholding] positive reinforcement to reduce student errors" (p. 44), while constructivism "builds on student sense-making, [and] utilizes errors as instructional opportunities to promote student learning" (p. 45). This dichotomy extended to views of errors themselves and expectations around how one should respond to students' errors during instruction. For example, the NCTM (1991) Standards for Mathematics Teaching, reflecting the "error for inquiry model," emphasized using student errors to promote robust debate about the mathematical content students learned.

Understanding why the identify and eliminate model remains prevalent is essential because it has been shown to devalue students' mathematical contributions by not interrogating the mathematical insights that lead to the error. In addition, some research suggests that adherence to the "identify and eliminate model" is due to a lack of depth and flexibility in teachers' mathematical content knowledge (Ma, 1999). Thus, it may be the case that shifting to an error for inquiry model can both support math identity development for students and strengthen and deepen teacher content knowledge.

Cultural Response to Errors

Despite the acknowledgment of the importance of the shift in orientation and practice regarding errors (Kramarski, B., & Zoldan, S. 2008; Lannin et al., 2007; Common Core, 2010; NCTM, 1991; 2000), identify and eliminate models have continued to guide practice in many spheres, with errors viewed as indicators of deficiencies in learning (Bray, 2013; Cohen, 1990; Santagata, 2004; Son and Sinclair, 2010) and to be avoided or eliminated upon being identified. Cross-cultural studies that compared U.S. teachers with teachers from Italy (Santagata, 2004), China (Ma, 1999; Schleppenbach et al., 2007), and New Zealand (Nuthall and Lawrence, 1965) also reflect the U.S. adherence to the identify and eliminate model. These researchers found that the dominant U.S. cultural response to errors focused on reducing the appearance of errors. This reduction or mitigation of errors was to limit the embarrassment or anxiety students faced because of the errors they produced (Bray, 2013). In contrast, other countries used the confrontation of errors to learn. Schleppenbach et al. (2007) provided further evidence of the persistence of the identify and eliminate model when they compared U.S. and Chinese teachers' responses to student errors. In particular, the researchers identified two types of teachers' responses to student errors: a student error is followed by a procedural statement from the

teacher statement, and a second in which a conceptual question from the teacher follows up a student error. U.S. teachers responded with procedural comments on average 34% of the time compared with 16% for Chinese teachers. The trend reversed with the U.S. averaged 66% compared to 84% of Chinese teachers for conceptual questions.

Another reason for U.S. teachers' reliance on the identify and eliminate model is their content knowledge. Ma (1999) demonstrated that the U.S. teachers in her sample seldom made connections across mathematical concepts and suggests that, as a result, teachers may struggle to engage with student errors. Thus, beyond shielding students from the embarrassment or anxiety associated with their errors, teachers may not build on student errors because they do not see the mathematical connections between the errors and the learning goals being examined at that moment.

As mentioned previously, while Matteucci et al. (2015) explored the teacher's beliefs about error analysis and error-handling strategies, these researchers did not investigate the impact that these views (when realized through practice) had on students. Tulis (2013), however, did focus on teacher attention to student errors in practice, and it is this work that I directly aim to extend.

Specifically, Tulis (2013) studied the potential impact of the "error climate" on students within mathematics classrooms. Tulis defined both a positive and negative error climate. This scholar described a positive error climate as allowing for "open communication about different solutions and as a result the sharing of error knowledge" (2013, p. 57) and an adverse error climate as closing off communication regarding errors and attributing errors to a lack of students' ability or skills. Tulis identified two general types of teacher responses that could lead to these different climates and labeled them adaptive or maladaptive. Adaptive responses promote a

positive attitude towards errors, while maladaptive responses punish or embarrass students for committing errors. Drawing on a review of studies conducted by Spychiger et al. (1998), Mindnich et al. (2008), Santagata (2005), Stigler et al. (1999), and Oser & Spychiger (2005), Tulis created a coding scheme with 11 categories for initial teacher responses to student errors, six maladaptive responses and five adaptive responses (Table 2.1)**.**

Table 2.1: Teacher Responses to Student Errors (Adapted from Tulis, 2013)

	Category/Type of Response	Examples
Maladaptive	Ignore the error	Switches without any comment to another topic
	Criticize the student	Negative evaluation of the student's error
	Redirect the question to another student	Teacher picks another student to correct the error made by the first
	Humiliate/Laugh at Student	Teacher laughs, makes jokes, humiliates the student
	Disappointment	Teacher is upset, shaking her head, grimacing with pain
	Correction by Teacher	Error directly solved by teacher
Adaptive	Discussion whole class	Teacher starts a discussion whole class
	Correction by student	Teacher repeats the question, gives hint to the student
	Wait	Teacher waits at least 5 seconds before giving hint or restating question
	Emphasize the learning potential	Emphasizes the learning protentional of the error
	Impede negative reactions from class	Teacher stops negative reactions from the class

Tulis aimed to "identify teachers' error management behavior in regular everyday classrooms" (p. 57) and assign them to either an adaptive or maladaptive response category and, in doing so, to characterize the nature of the error climate within a classroom. I believe, however, that to more fully understand the error climate in any school, we must look beyond teachers' initial responses to student errors and investigate the interaction surrounding an error as well. The inclusion of the exchange is necessary because we know that student learning does not end after

the teacher's initial response and extends within and beyond the interaction in which an error is a focus. Developing Tulis' methods, this study analyzed the first turns of talk and the *entire* exchange in which an error was the focus of the interaction. As I will demonstrate, this expansion provides a different picture of the teacher's practice than Tulis' analysis would have revealed and highlighted how individuals participate within these interactions.

Discourse In Classrooms

A focus on discourse is necessary to interrogate further the identify and eliminate and the error for inquiry models in math classrooms. Discourse is central to understanding many classroom interactions, and regarding math classrooms, this importance has been seen in shifts within the curriculum standards (Common Core, 2010; NCTM, 1991; 2000), researcher community (Knuth & Peressini, 2001; and for an extensive review see Walshaw & Anthony, 2008), as well as preservice instruction and professional development focused on implementing and improving discourse in the classroom (McDonald et al., 2013; Kazemi et al., 2016; Borko et al., 2008;).

Before I begin, I must define how I utilized discourse. The framework used to understand the discourse in this particular context came from Saxe et al. (2009) in which the researchers stated:

> The collective practices of the classroom support the emergence, reproduction, and alteration of mathematical ideas in students' problem solving and discussion. All participants contribute to the collective practices of the lesson – to the lesson's emerging structure, to the use of valued forms of representation and associated functions, and to the social positions of students and teachers" (p. 2).

These researchers discussed mathematical ideas that travel (discourse) using genetic analysis highlighting ontogenesis, microgenesis, and sociogenesis. They define these constructs as:

> Ontogenesis – marked both by continuity in the individuals' ways of understanding the experienced world and discontinuity as the individual structures new systems of understanding out of prior ones…Microgenesis is the process of moment-to-moment construction of representations as individuals work to turn representational forms into means to serve mathematical

> functions…Sociogenesis is the reproduction and alteration of ideas over time in the classroom community (pp. 7 & 9).

Because I did not conduct student interviews, I was not able to adequately represent the ontogenesis for student ways of understanding errors. Therefore, I used two of the three forms of genetic analysis as presented by Saxe et al. (2009). In addition, I operationalized them differently to be more in line with scholars of ecological systems theory discussed above. Bang (2015) provided valuable motivation for pursuing this work when she stated: "it is increasingly accepted that what people think and how people think are interdependent and sculpted by the daily activities, discursive practices, participation structures, and interactional processes in activity" (pp. 219 & 220).

3. **Study Design**

This research aims to apply an ecological systems lens to study teacher and student interactions within moments of errors in the classroom. This study also explores students' and teachers' values and orientations regarding errors. It then makes explicit how these values reflect different ecological system levels when enacted in the classroom setting. Lastly, by using two teachers as case studies, this dissertation seeks to understand how error climates are created and how different models for using errors in the classroom are maintained.

Overall, this dissertation highlights how micro and macro-level processes intersect and influence middle school students' perceptions about their abilities to navigate the challenge. Also, students' perceptions of themselves as mathematicians and their racial and ethnic identities (within the specific context of error moments) are essential because this population of students must navigate both micro and macro-level structures influencing opportunities to learn mathematics. To this end, this study interrogates how students' conceptions of themselves along multiple dimensions influence how they perceive errors in mathematics classrooms and how researchers can interpret teachers' classroom practices through meta-narratives and meta-structures. I look both within and beyond the classroom setting to do this work. This expanded focus is necessary because students' and teachers' perceptions of themselves, their ability, and their understanding of errors are contextualized by a combination of experiences in multiple settings. These experiences can include formal settings like school and informal settings like home.

As mentioned previously, the PVEST model (Spencer, 1995) is the theoretical framework I use to ground this work. This model extends beyond anchoring my literature review to inform my research questions and data collection and analysis. To reiterate, this study sought to examine

the following questions:

RQ1.) What is potential relationship among students' error identities, racial identity, mindset, and how are these associated with macro-level messages about race?

RQ2.) How do teachers' beliefs and attitudes about errors reflect meta-structures and meta-narratives in their planning and instructional practices?

(RQ3.) What potential relationships exist between teachers' and students' orientations and dispositions and how are they reflected in classroom interactions within error moments?

This dissertation looks at multiple data sources with the PVEST model as its core to answer these questions.

Study Site Description

I observed this study at Insight International Charter School (pseudonym). School Insight was located in an urban Midwestern city and is a K-8 charter school. Ninety-eight percent of the student body identify as African-American, and the school described itself as an African-centered school. One aspect of the school's mission was to integrate African cultural belief systems and practices into the curriculum to instill "a sense of history, responsibility, accountability, community, extended family, propriety, and of course, pride." The school seeks to socialize a positive sense of racial/ethnic identity among its students. School-wide cultural practices focus on building strong social and emotional relationships among students, staff, and families; develop a deep understanding of African, African-American, and African diaspora history, including cultural practices that have been sustained across the continent and the diaspora over time, as well as belief systems rooted in continental and diasporic history and traditions that socialize a positive sense of self and a commitment to communal interdependence. These belief systems include the Nguzo Saba or 7 Principles associated with the African-

American practice of Kwanzaa and the construct of Maat (the idea of human perfectibility) from ancient Egypt.

One such instantiation of this mission- the idea of the extended family - was those female teachers are called Mama, and male teachers were called Baba, which means father in Swahili. This naming convention is intended to reorient the relationship between teachers and students from an institutional connection to a family-centered one. African-centered schools have been discussed as a response to the "quick-fix mentality and single-solution approach" (Lee, 1992, p. 160) within school reform to address the needs of African-American students. This reorientation strives to dismantle the "pernicious discursive and structural machinations by which mathematics classrooms actively exclude particular groups of students (Gholson & Wilkes, 2017, p. 229) and provide a means to bypass the "gatekeeper" (Moses & Cobb, 2001; Stinson, 2004) to more beneficial life outcomes.

This study focused on one 8th grade and one 7th grade mathematics class at School Insight. School Insight used the EngageNY curriculum (Engage NY, 2012), a module-based learning program designed to help students develop extensive mathematical reasoning skills, reflect deeply on their learning, and connect standards for mathematical practice to standards for mathematical content. In essence, EngageNY provides a guide for the content being covered and how students should engage with the content in progressively more integrated and complex ways that correspond with their growing competencies and expertise. The 8th-grade class had 22 students and was taught by Mama Linne, an African American woman who had taught for seven years and the last five years was taught School Insight. Thirteen of the students consented to participate in this study. All thirteen consented students identified as African American, with seven students self-identified as female and six self-identified as male. The 7th-grade class had

26 students and was taught by Mama Elizabeth, also an African American woman. Mama Elizabeth was a teacher in her fifth year of teaching and first year teaching at School Insight. Within this class, 22 students consented to this study. All consented students identified as African American, with ten students self-identified as female and twelve self-identified as male.

Methods

Student Surveys Data Collection

Consented students completed three surveys throughout the study: the MEOS survey (Appendix B), the MIBI-t (Appendix C), and the Dweck Mindset Scale (Appendix D).

Modified Error Orientation Scale (MEOS)

The MEOS was given to gain a picture of their orientation towards errors. The original error orientation survey (Rybowiak et al.,1999) was designed with 37 questions that measured attitudes and coping with errors at work through seven scales: error competence, learning from errors, error risk-taking, error strain, error anticipation, covering up errors, and error communication. Because I planned to distribute the survey in a different context and with participants of another age group, I made modifications. In general, I replaced terms such as "work" with "class," and "colleagues" became "classmates." More specifically, I replaced all questions under the error competence scale with a single short answer question, modified two questions under the error risk-taking scale and three questions under the error anticipation scale, and eliminated two questions under the covering up errors scale. In addition, I removed five questions about error communication and added two new questions.

The new measure (MEOS) included 25 Likert-scale questions and one short answer question ("Think about the last time you made a mistake in math class and write about what

happened.") In light of these modifications, I conducted validation checks as part of a pilot study. Using SPSS and principal component analysis (Jolliffe, 2011), four scales were identified: error risk-taking, error strain, error embrace, and error ego. Error risk-taking measured students' attitudes toward the potential to make mistakes and their willingness to talk about those mistakes. The questions under this construct included the following: If I want to achieve in class, I have to risk making mistakes, I know I could make a mistake while doing classwork, I talk about my mistakes even if I'm the only one who notices them in class, It is ok to make my mistakes known to other people, I do not find it useful to talk about my mistakes in class, and When I make a mistake in class I tell others about it, so they do not make the same mistake. I calculated Cronbach's Alpha for all four scales as a reliability score. Cronbach's alpha score for the error risk-taking scale had good reliability at .805. Error Strain measured how much anxiety or fear was associated with making mistakes in the classroom. The questions under this construct included: I find it stressful when I make mistakes in class, I am often afraid of making mistakes in class, I feel embarrassed when I make a mistake in class, and If I make a mistake in class, I get upset. Cronbach's Alpha for this construct was .835. Error Embrace measures students' attitudes who view mistakes positively in their learning process and questions under this construct included the following: It is better to risk making mistakes than to do nothing at all in class, I expect to make mistakes from time to time with my work in class, Making mistakes is part of learning in class, and If I cannot fix my mistake by myself I ask my classmates for help. Cronbach's Alpha for this construct was .783. Lastly, Error Ego measures students' attitudes who view mistakes as blemishes, and questions under this construct included the following: Taking risks is part of learning in class, I don't expect to make mistakes as part of my classwork, If I cannot fix my mistake by myself, I ask my teacher for help, and After I have made a mistake in

class, I think about how I could have prevented it. Cronbach's Alpha for this construct was .662. All questions that needed to be reversed scored were done so before being combined with other questions.

Dweck Mindset Scale

The Dweck Mindset Scale measured students' beliefs about learning and talent as either malleable or fixed. There were a total of 16 statements (eight for each category), and students were assigned to a mindset by comparing the averages between the two beliefs about ability. Examples of statements reflective of a fixed mindset included the following: (1) You have a certain amount of intelligence, and you really can't do much to change it, and (2) Your talent in an area is something about you that you can't change very much. Examples of statements reflective of a growth mindset included the following: (1) No matter how much intelligence you have, you can always change it quite a bit, and (2) You can change even your basic intelligence level considerably. As with the MEOS and MIBI-t, all students responded on a Likert scale. Cronbach's Alpha for the growth and fixed constructs for this population were .906 and .805, respectfully.

Multidimensional Inventory of Black Identity-teen (MIBI-t)

I used the Multidimensional Inventory of Black Identity-teen (MIBI-t) to measure students' beliefs about black identity on three dimensions: Centrality, Regard, and Ideology. Regard had two sub-groups, Private and Public, while Ideology had four sub-groups, Nationalist, Oppressed minority, Assimilationist, and Humanist. Each of these constructs is presented below as defined by Scottham et al. (2008, p. 300):

> *Centrality* refers to the extent to which an individual normatively emphasizes racial group membership as part of their overall self-concept…*Public regard* is defined as the extent to which an individual feels that others view the African American community in a positive or negative manner. *Private regard* is defined as the extent to which an individual feels positively or

> negatively toward the African American community as well as how she/he feels about being a member of this community…*Nationalist ideology* emphasizes the uniqueness of being African American and is characterized by the support of African American organizations and preference for African American social environments. The *Oppressed Minority* ideology emphasizes the similarities between African American's experiences and those of other oppressed minority groups. *Assimilationis*t Ideology emphasizes the similarities between African American and mainstream American society. *Humanist* emphasizes the similarities among all people regardless of race (pg. 300; emphasis in original).

Example questions from each component are provided below in Table 3.1

Table 3.1: MIBI-t Constructs and Example Questions

Construct	Example Questions
Centrality	I feel close to other Black people.
	If I were to describe myself to someone, one of the first things that I would say is that I'm Black.
Private Regard	I am happy that I am Black.
	I am proud to be black.
Public Regard	Most people think that Blacks are as smart as people of other races.
	People think that Blacks are as good as people from other races.
Nationalism	Black parents should surround their children with Black art and Black books.
	Whenever possible, Blacks should buy from Black businesses.
Oppressed Minority	People of all minority groups should stick together and fight discrimination.
	There are other people who experience discrimination similar to Blacks.
Assimilation	It is important that Blacks go to White Schools so that they can learn how to act around Whites.
	I think it is important for Blacks not to act Black around White people.
Humanism	Being an individual is more important than identifying yourself as Black.
	Blacks should think of themselves as individuals, not as Blacks.

Students responded on a Likert scale in which each answer had a positive or negative connotation (neutral was not an option). The significance of this measure is its ability to note the different aspects of racial identity and how these might affect an individual's overall sense of ethnic, racial identity. Because of the population I chose to study, middle school children, the MIBI-teen was selected for its adherence to the larger constructs of the MIBI while also being

accessible to intermediate school-aged students. Cronbach's Alpha for the centrality construct was .583, regard .713, and ideology .730.

Teacher Interview Data Collection

For this study, I interviewed (Appendix A) both teachers and collected field notes and video data during classroom observations. Both interviews lasted approximately 30 minutes and focused on each teacher's understanding and use of student errors during instruction. Specifically, interview questions asked how the teachers defined errors and what purpose errors serve (if at all) within and outside of the classroom setting.

Classroom Observations Data Collection

I visited Mama Linne's 8th-grade classroom once a week for classroom observations. I recorded seven full class periods between January 2017 and March 2018, and for Mama Elizabeth, I recorded eight full class periods between November 2019 to February 2020. In line with approval from Northwestern University's Institutional Review Board, I recorded the entire class and removed the faces and comments of students who did not consent before analysis. For the first month of both sets of observations, I observed without videotaping to establish a rapport with the teacher and students. I created field notes on these introductory days and throughout the remainder of the observation timeframe. Across the observations, I captured approximately nine hours of classroom instruction on video for Mama Linne and eight hours of instruction for Mama Elizabeth. For this study, the analysis focuses on four hours and fifteen minutes of video, reflecting three class periods for Mama Linne and four hours of video, reflecting three class periods for Mama Elizabeth. I selected these class sessions for analysis because my field notes indicated extended error interactions in multiple parts of the lessons.

Student Survey Data Analysis

There was a total of 93 students who completed each survey given. Quantitative methods were used to analyze the surveys. Data used for analysis were the average scores among the specific constructs between the surveys. The first analysis completed was a univariate analysis that highlighted the number of surveys collected (Obs), the average of the scores across individual constructs (Mean), the standard deviation from the mean within each Construct (Std. Dev.), and the average of the lowest (Min) and highest (Max) scores observed for each construct. The subsequent analysis was a correlation matrix between all of the constructs presented in each survey. The correlation table allowed for bivariate analysis of the significance of associations between the variables across the surveys.

Teacher Interview Data Analysis

I analyzed both teachers' understanding and utilization of errors using qualitative analysis of the interview transcript. In my initial coding of the interview, I divided the interview data into chunks according to when the topic of conversation shifted. With this chunking, one or multiple sentences can contain the idea units. For example, when the topic of conversation shifted, I identified a new unit. After chunking the transcript, I sought to identify comments that revealed Mama Linne's and Mama Elizabeth's beliefs about the role of student errors in the mathematics class using bottom-up (descriptive) coding (Miles & Huberman, 1994). Within bottom-up coding, I identified 24 idea units for Mama Linne and 26 idea units for Mama Elizabeth. I then created provisional codes that embodied the broad themes for both instructors' answers regarding errors. These codes included the relationship between errors and learning, the impact of errors on student self-esteem, and identifying and responding to errors that arise during instruction. Multiple coding within idea units occurred during this process, and some codes were absent from

whole idea units. I captured the resulting percentages in the tables below. When I collapsed these provisional codes, I identified two major themes reflected in the codes, overall attitudes and orientation towards errors and approach to errors during instruction. After I identified these themes, I then looked at the idea units coded as related to these categories and whether they were consistent or inconsistent with the error for inquiry or identified and eliminated model. Table 3.2 & 3.3 illustrates these themes and codes from the interview data..

Table 3.2: Mama Linne Interview Responses

Themes	***Error Codes***	***Example***	***Idea Units Aligned With Error/Inquiry***	***Idea Units Aligned With Identify/Eliminate***
Attitude and Orientation toward Errors	Relationship between Errors and Learning	"It's just another opportunity to learn."	33%	13%
	Impact of Errors on Student Self-Esteem	"A lot of times you have those students who are academically amazing and they're used to getting everything right and so when they're face with something more challenging they tend to you know, panic, get stressed out."	29%	4%
Approach to Errors During Instruction	Identifying Errors that Arise during Instruction	"You can see an error when the answer is wrong."	42%	17%
	Response to Errors that Arise during Instruction	"So if it's going impede what they need to know in the long run we are going to stop and focus on that."	21%	8%

Table 3.3: Mama Elizabeth Interview Responses

Themes	***Error Codes***	***Example***	***Idea Units Aligned With Error/Inquiry***	***Idea Units Aligned With Identify/Eliminate***
Attitude and Orientation toward Errors	Relationship between Errors and Learning	"[Students] may not know that they're making mistakes and if you're not calling them on it, they may continue to do the same thing"	19%	46%
	Impact of Errors on Student Self-Esteem	"We all make mistakes and I know that students do feel embarrassed and it's just a matter of letting them know it's nothing to feel embarrassed about."	19%	15%
Approach to Errors During Instruction	Identifying Errors that Arise during Instruction	"I like to grade papers so I can see the patterns I like to see what the common mistakes are."	19%	46%
	Response to Errors that Arise during Instruction	"With individuals addressing mistakes would be as its happening while they're making the mistake."	12%	27%

Classroom Videotaped Lessons Data Analysis

The second data source I analyzed was recorded classroom instruction. The first phase of analysis mirrored the methodology presented by Tulis (2013). I noted those moments in which the teacher identified a student error and then coded the teacher's responses in each case as either adaptive or maladaptive. In defining what counted as an error, I used Lischka et al.'s (2018) definition in which errors "can include misconceptions, erroneous solution processes, ineffective problem-solving strategies, or incomplete mathematical arguments" (p. 434). I identified and

coded 199 teacher responses to student errors for Mama Linne and 43 for Mama Elizabeth in this initial phase. (See Table 2.1 for a list of the coding categories).

The second phase of analysis built upon the first by coding extended interactions that I refer to as error episodes in which an error was the focus of the interaction. Specifically, for this analysis, error episodes were defined as introducing an error, the interaction resulting from the error, and the completion of the interaction in which the error was the focus. Error episodes became the bounded events (Bloome et al., 2004) in which I sought to understand the discourse within. Bloome et al. (2004) further defined interactions as "events," as "a way to place emphasis on the dynamic and creative aspect of what people do and accomplish in interactions with each other…a bounded series of actions and reactions that people make in response to each other at the level of face-to-face interaction" (pp. 5-6). For discourse, I used van Leeuwen's (2008) definition in which discourse is "the sense of social cognition, of a socially constructed knowledge of some social practice, developed in specific social contexts, and in ways appropriate to these contexts (p. 6). I also borrowed van Leeuwen's definition of social practice as "socially regulated ways of doing things," and in this case, regulation was provided by the teacher or "through the influence of experts and charismatic role models" (pp. 6 & 7).

I argue that within this particular type of event (error episodes), students understand there is an appropriate way to enter and interact within this discourse. van Leewuen (2008) emphasized this point and highlighted the need to study discourse in varying contexts (in this case within error episodes) in which he said "different social contexts offer writers and speakers different amounts of freedom. And the rules, strategies, or best practice models they follow are not autonomous linguistic structure potentials, but modalities of institutionalized social control that should themselves be studied as different kinds of practices" (p. 10). This quote was not to

say that students uniformly adhered to this discourse, and later in this paper, we see how a student used her knowledge of the discourse to enter within it and repurposed it for her own goals.

Specifically, for this analysis, I traced the sociogenesis of the specific discourse (represented as the dominant form of communication that members of the classroom community participated in) created within error episodes by the interrogating microgenetic moments (individual interactions within whole-class discourse). For this analysis, microgenesis represented the interaction between the teacher and students or student to student interactions in which these thoughts were brought into contention or agreement with one another, and sociogenesis represented the dominant form of communication that was created and supported by the members of the classroom community within these specific interactions. As noted above, I bounded these events by introducing an error, the exchange around the error that unfolded, and the completion of the interaction in which the error was the focus. From Mama Linne's 199 initial teacher responses, I identified and coded 55 events, while Mama Elizabeth only had 43 instances in which I coded six events. Each event extended beyond the initial response and created an error episode. I coded each of these error episodes along several dimensions: length of interaction, evidence of nonverbal acknowledgment of error (teacher mark on student work), evidence of verbal acknowledgment of error (the actual message unit), results from acknowledging error (start a whole class discussion, provide a direct answer, etc.), mathematics topic being discussed (systems of equations, operations with integers, etc.), type of interaction (one-on-one, whole class, small group, etc.), and kind of error produced (procedural or conceptual). I then conducted line-by-line coding (within each extended error interaction) to identify moments in which Mama Linne's and Mama Elizabeth's responses aligned with either

the error for inquiry or the identify and eliminate model. For this dissertation, I focus on analyzing one event for each teacher in which each event was one of the longest in the entire dataset. I determined if an error interaction was more aligned with an error for inquiry or identify and eliminate model by first coding how the exchange was created (initial type of interaction, evidence of error acknowledge, type of interaction produced) and then within the line-by-line analysis, I matched each teachers' statements as reflective of either model. I coded the length of time within each error interaction to highlight the teacher's overall commitment to exploring errors in their class. It was also used as an indicator of alignment with either an error for inquiry model or identify and eliminate model. By their nature, more time within error moments would indicate an error for inquiry model because the model promotes using errors as a means for exploration and understanding. In comparison, less time within error moments would align with an identify and eliminate model because the goal of this model is to remove errors as quickly as possible. With this coding system, it is possible that teachers embody both models within one interaction (present in our analysis below) and is more evidence of the need to look beyond initial reactions to student errors to more fully understand how these interactions can influence learning.

4. Interactions of Identity, Mindset, and Orientations

In this chapter, I explore the following research question: "What are potential relationships among students' error identities, racial identity, growth mindset, and how are these impacted by macro-level messages about race?" This question is necessary to understand better how meta-narratives impact students' perceptions of themselves, their ability, and their relationship to errors.

I draw on multiple surveys to answer this question: The Modified Error Orientation Scale (MEOS), the Multidimensional Inventory of Black Identity-Teen (MIBI-t), and the Dweck Mindset Scale. These surveys were designed to draw on students' unique perspectives and explore their own lived experiences while responding to each set of questions. Using these three surveys also allowed students to express their views across varying levels and is necessary for this work because an ecological perspective requires interrogation of the interaction between multiple levels.

Findings

To preface the findings, overall, this data supported the hypothesis that factors influenced students' perceptions and attitudes about errors outside of the immediate classroom setting and that these external and internal factors (societal expectations of African American students and mindsets around intelligence) had a measurable relationship to the error constructs within the MEOS survey.

Analysis of Descriptive Statistics

Before discussing how I reached these findings, it is important to note how students responded to each construct individually. To this end, drawing from the univariate analysis, summary statistics of the measures across all three years are presented in Table 4.1. These

summary statistics suggest several vital results. First, across student responses to the MEOS questions, students tended to feel more positive than negative about errors. This positive orientation was seen in the highest average across the error constructs, being error embrace (4.7, *SD* 0.84 and the lowest being error strain (3.7, *SD* 1.27). The standard deviation for error embrace shows the student averages were more closely grouped than the error strain averages. Specifically, 66% of students fell within two points (one above and one below) of the mean for error embrace. In contrast, for Error Strain, students in the data set reported more variation in the amount of anxiety they experienced as opposed to the other error measures while students clustered closer together and, on average, had higher scores for the Error Embrace measures.

Table 4.1: Constructs Variable Summary

Survey	Variable	Obs.	Mean	Std. Dev.	Min	Max
MEOS	Error Risk-Taking	93	4.211828	0.8594831	1.83333	6
	Error Strain	93	3.747312	1.273952	1	6
	Error Embrace	93	4.706989	0.8353791	2.5	6
	Error Ego	93	4.392473	0.787468	2.25	6
Dweck Mindset	Fixed Mindset	93	3.05914	1.283199	1	5.75
	Growth Mindset	93	4.86828	0.8484652	2.5	6
MIBI-t	Centrality	93	4.677419	1.035437	2	6
	Private Regard	93	5.430108	0.7766064	1.666667	6
	Public Regard	93	3.889247	1.427196	1	6
	Nationalism	93	4.778495	1.047053	1	6
	Humanism	93	4.275986	1.156512	1.333333	6

	Assimilation	93	2.462007	1.45386	1	6
	Oppressed Minority	93	4.824373	0.97256	1	6

Note: Obs is the number of observations collected, Mean is the average of the scores across individual constructs, Std. Dev is the standard deviation from the mean within each construct, Min is the average lowest scores observed for each construct present in the three surveys and Max is the highest scores observed for each construct present in the three surveys.

Second, students completed the Dweck Mindset survey to measure students' beliefs about whether their own and others' intelligence is fixed or malleable. As seen in Table 4.1, the mean growth mindset score was 4.9 with a standard deviation of .848, indicating that students on average believe that intelligence is within their power to change.

Third, the MIBI-t survey results indicate that overall, students held a positive self-image of blackness. Specifically, the four highest average scores on the MIBI-t, as seen in Table 4.1, are Centrality (4.7, *SD* 1.035), private regard (5.4, *SD* 0.777), nationalism (4.8, *SD* 1.047), and oppressed minority (4.8, *SD* 0.973). The high averages in Centrality, Private Regard, and Nationalism reflect a positive self-image of blackness. Recall that Centrality relates how central blackness is to one's identity, Private Regard refers to the personal belief that Blacks are intelligent, and Nationalism reflects the idea that Blacks should support other Blacks. In addition, the MIBI-t results suggest that students share a desire to support one's community and other communities that are viewed as similarly oppressed. The difference between the private and public regard scores is a 1.5-point difference. It could indicate students' awareness of the societal expectations discussed previously regarding stereotype threat and other harmful tropes around blackness. The influence of the school may also be seen as impacting the results of these constructs. As mentioned previously, Insight Interactional Charter School is an African-centered

school that believes in academic excellence and cultivating and celebrating Black culture. Perhaps the biggest testament to the influence of the school on these scores was shown when the students roundly rejected the notion of assimilation as a means of bettering one's situation and station with the lowest score (2.5, *SD* 1.45) among all the constructs.

Relationships Among Constructs

As stated above, because I am using three surveys and have a relatively small sample size, I chose to examine bivariate rather than multivariate associations among the variables (i.e., Pearson correlations). The correlation table is included below. Within the following table ** next to a value represented a statistically significant relationship while * represented marginal relationship or trends among constructs. For the male variable, one described a student as male, and 0 represented a female student. The teacher variable one represented students who had Mama Elizabeth as a teacher, and 0 represented Mama Linne as their teacher. Within the following table, *r* coefficients are located above and noted with the symbol β, while *p* values are noted in parentheses within in the table.

Table 4.2: Construct Correlation Matrix (1 of 2)

	Male	Teacher	Fixed	Growth	Error Risk-taking	Error Strain	Error Embrace
Male	1						
Teacher	β 0.1578	1					
	(0.1307)						
Fixed	β -0.0017	β -0.2568	1				
	(0.9869)	(0.013**)					
Growth	β 0.0809	β 0.1148	β -0.4615	1			
	(0.4407)	(0.2733)	(0**)				

Error Risk-taking	β -0.0837	β -0.0436	β -0.1439	β 0.2329	1		
	(0.4253)	(0.6783)	(0.1689)	(0.0246**)			
Error Strain	β -0.168	β -0.0242	β 0.1973	β -0.1964	β -0.1121	1	
	(0.1076)	(0.8177)	(0.058*)	(0.0592*)	(0.2849)		
Error Embrace	β -0.1324	β -0.0417	β -0.2584	β 0.4017	β 0.5668	β -0.0627	1
	(0.2059)	(0.6916)	(0.0124**)	(0.0001**)	(0**)	(0.5507)	
Error Ego	β -0.1036	β -0.1064	β -0.3214	β 0.1946	β 0.4775	β 0.0532	β 0.5867
	(0.3229)	(0.3098)	(0.0017**)	(0.0615*)	(0**)	(0.6125)	(0**)
Centrality	β 0.1985	β 0.1434	β 0.0436	β 0.2267	β 0.1896	β 0.0195	β 0.2516
	(0.0564*)	(0.1704)	(0.6785)	(0.0289**)	(0.0687*)	(0.8532)	(0.015**)
Private	β 0.0383	β 0.2209	β 0.0068	β 0.1822	β 0.2718	β -0.1523	β 0.2091
	(0.7152*)	(0.0334**)	(0.9482)	(0.0804**)	(0.0084**)	(0.1451)	(0.0443**)
Public	β 0.1801	β -0.0364	β 0.2199	β -0.0646	β 0.0578	β 0.0011	β -0.0455
	(0.084)	(0.7293)	(0.0342**)	(0.5386*)	(0.5822)	(0.9914)	(0.6648)
Nationalism	β -0.1355	β 0.1972	β -0.0611	β 0.1468	β 0.1728	β 0.0089	β 0.2621
	(0.1952)	(0.0582*)	(0.5609)	(0.1602)	(0.0977)	(0.9324)	(0.0112**)
Humanism	β -0.1656	β -0.0334	β 0.0781	β -0.0094	β 0.1512	β 0.1778	β 0.1284
	(0.1127)	(0.7503)	(0.4571)	(0.9291)	(0.1481)	(0.0883)	(0.22)
Assimilation	β 0.0641	β -0.1014	β 0.3926	β -0.281	β -0.0875	β 0.1551	β -0.1507
	(0.5415)	(0.3334)	(0.0001**)	(0.0064**)	(0.4044)	(0.1376)	(0.1492)
Oppressed Minority	β 0.0385	β 0.0626	β -0.0707	β -0.0827	β 0.2542	β 0.1539	β 0.1065

	(0.7143)	(0.5511)	(0.5007)	(0.4308)	(0.0114)	(0.1409)	(0.3094)

r coefficients are noted with β, while *p* values are noted in parentheses

(Table 2 of 2)

	ErrorEgo	Centra~y	Privat~d	Public~d	Nation~m	Humanism	Assimi~n
Error Ego	1						
Centrality	β -0.1286	1					
	(0.0587*)						
PrivateRegard	β 0.1164	β 0.443	1				
	(0.0664*)	(0**)					
PublicRegard	β -0.0604	β 0.1266	β 0.1453	1			
	(0.565)	(0.2266)	(0.1645)				
Nationalism	β 0.1198	β 0.4043	β 0.5303	β 0.0373	1		
	(0.2528)	(0.0001**)	(0**)	(0.7225)			
Humanism	β -0.0281	β -0.1412	β 0.0391	β 0.0312	β 0.0448	1	
	(0.789)	(0.1771)	(0.71)	(0.7663)	(0.6701)		
Assimilation	β -0.348	β -0.1855	β -0.1977	β 0.2473	β -0.0871	β 0.1635	1
	(0.0006**)	(0.0751)	(0.0575*)	(0.0168**)	(0.4066)	(0.1173)	
Oppressed Minority	β 0.2802	β 0.1228	β 0.3008	β 0.1894	β 0.2912	β 0.242	β 0.0912
	(0.0065**)	(0.241)	(0.0034**)	(0.069*)	(0.0046**)	(0.0194)	(0.3844)

r coefficients are noted with β, while *p* values are noted in parentheses

This correlation table allowed me to see patterns amongst the data. First, males trended towards higher centrality and private regard, but overall, gender had no statistically significant relationships among the constructs. This is especially important because gender has been associated with mathematical ability and performance (Gholson & Martin, 2014). As such, one

might have expected gender to have a most pronounced impact on student error identities and mindset scores. Specifically, female students could have internalized macro messages about their gender and math ability (Gholson, 2014) and hold a negative association with errors in the classroom but that is not the case. While gender did not have any statistically significant relationship with the constructs, the teacher did. Students who had Mama Elizabeth were more likely to have a fixed mindset when it came to thoughts of intelligence as opposed to students who had Mama Linne.

While there are many statistically significant relationships among the varying constructs (such as fixed mindset and public regard), the goal of this dissertation was to focus on if/how the constructs in the MIBI-t and Dweck Mindset scale are associated with error orientations. To this end, I will not explicitly note relationships between the two surveys mentioned above but still include the relationships between them in the correlation matrix for reference.

Constructs within MEOS

Within the error constructs, we saw statistically significant relationships between error risk-taking and embrace and error ego and error ego and error embrace. Positive scores with error embrace predicted a positive association with error risk-taking. Negative scores with error embrace indicated a negative association with error risk-taking, and positive scores with error embrace predicted a negative association with error ego. For example, if students viewed errors positively (error embrace) in their learning experience, they were more likely to engage in discussions in which their misunderstanding could be made public (error risk-taking). Conversely, if students negatively viewed errors, they were less likely to engage in the same discussion. The relationship between error ego and error risk-indicated if students did not want their errors known to the class or were ashamed of making errors, they might avoid instances in

which these blemishes could be made visible. The opposite was also true -- if students were not as ashamed of their errors, they were more likely to engage in moments in which these errors could present.

Mindset and Error Orientations

Noted in the correlation table, there were statistically significant relationships between constructs on the Dweck Mindset scale and the MEOS. I noted one such relationship between growth mindset and error risk-taking and error embrace. This relationship also had a positive coefficient which indicated that as the value of the growth mindset score increased, the mean of the error embrace and error risk-taking scores also improved. These relationships could signal that students' who believed that errors were beneficial for learning also viewed intelligence as malleable and something that could change over time. These same students could also think that errors could directly support change in intelligence and were more likely to engage in complex content in which errors were more likely to occur. Growth mindset also trended negatively with error strain, as evident from the negative coefficient. Negative coefficients suggested that as growth mindset scores increased, the value of the mean for error strain decreased. In this case, students became less anxious about mistakes the more they believed intelligence was something that could change. I observed the opposite in the statistically significant negative relationship between fixed mindset and error embrace and ego. For these students who consider intelligence to be fixed, they are more likely to avoid situations in which their mistakes can be known or believe that by committing errors, they showed the limits of their mathematical knowledge (thinking they could not exceed those limits) and were more anxious and nervous about committing errors in general.

MIBI-t and MEOS

As mentioned previously, I included racial identity and mindset measures because African American students are potentially impacted by larger societal influences such as stereotypes regarding African Americans' performance within mathematics (Martin, 2006). To test the hypothesis that students' experience with societal expectations (macro) around doing math as well as their interpersonal interactions in different settings (meso) can have an impact on the culture around errors and students' attitudes and beliefs about errors in the classroom.

The first statistically significant relationship of note was a positive relationship between error embrace and centrality. The higher a student scored on error embrace, the more likely there were to score high on centrality. This relationship could be viewed as a rejection of macro-level messages about blackness and intelligence. I noted this relationship as a rejection of macrolevel messages such as stereotypes because the more a student associated blackness as a core to his/her identity, the more likely he/she was to embrace making mistakes and does not see these mistakes as blemishes to his/her black identity or intelligence. Also, as a result, having a high centrality value may buffer against macro-level messages, and the micro-level interactions students experience in the classroom. Error risk-taking also trended positively with centrality, supporting the claim made above. These same students, who hold blackness close to the core of their identity, were more likely to engage in tasks in which errors were more likely to occur. The negative trend between error ego and centrality could be an internalization of macrolevel messages about blackness and intelligence. I noted this relationship as an internalization because, as opposed to the previous relationship, the more students associated blackness as a core to their identity, the more acutely they recognized macro-level messages and could see their errors as diminishing that Black excellence. This diminishing could occur because errors can be seen as

blemishes to their intelligence when centrality is high, and thus errors are viewed negatively and to be avoided.

Within the Regard category, there was the constructs public and private regard. Within these constructs, we saw a statistically significant positive relationship between private regard and error risk-taking and a positive relationship between error embrace and private regard. There was also a negative marginal relationship between error ego and private regard. The relationship between private regard and error embrace and risk-taking can also be seen as a rejection of macrolevel messages around blackness and ability. As seen with centrality, students who feel positively toward the black community and see themselves positively as members of this community have a more favorable view of errors in general and are more likely to engage in a task where errors are more likely to occur. Again, as with centrality, having high private regard might act as a buffer for error orientations regarding ideas about mathematical ability and competence. The marginal relationship between error ego and private regard was similar to that between error ego and centrality. If students held a positive internal view of blackness, they might not want to make their mistakes known because not only does it have the possibility of blemishing their intelligence. Beyond the individual threat of appearing not intelligent, this relationship also had the danger of students believing their errors confirmed negative stereotypes of black people within mathematics beyond the student who committed the error.

The final category within the MIBI-t was Ideology and had the constructs of nationalism, humanism, assimilation, and oppressed minority. Nationalism had a statistically significant positive relationship with error embrace, while error ego had a negative statistically significant relationship with assimilation and a positive statistically significant association with oppressed minority. The relationship between nationalism (emphasizes the uniqueness of being Black) and

error embrace is further evidence that if blackness was central to a students' identity, he/she valued their blackness. If a student enjoys positive black spaces (such as the school in this case), he/she is more likely to see errors as positive regarding his/her learning and development. The relationship between error ego, assimilation, and oppressed minority is different in the sense that for students who do not view their blackness positively and want to adhere to an identity more closely associated with dominant (white) culture then they are less likely to value errors and seek to avoid them in the classroom. This can serve as an internalization of negative macro-level messages of black students and math. They believe and seek to distance themselves from these stereotypes to better fit in or be seen as fitting in.

For this school, while teaching students about black excellence and embracing their culture was internalized, this could also have harmed how they viewed errors in the classroom if a teacher did not have a positive error climate. This claim was supported through direct observations of classroom instruction. I argue this data was evidence that the support systems put into place within the school that reaffirmed students' racial and ethnic identities had an overall positive effect on their error orientations. This positive effect could also buffer negative associations with their race and ability, such as stereotype threat as seen in the relationships noted between the MEOS constructs and the MIBI-t constructs, specifically centrality, private, and public regard. The teacher's role (as seen through observations) seemed to either bolster or limit these buffers by their interactions and the error climate they create and maintain.

5. Exploration of Teacher Beliefs

To answer the research question "How do teachers' beliefs and attitudes about errors reflect meta-structures and meta-narratives in their planning as well as their instructional practices?" I examined teachers' conceptions of errors using interviews. Within this semi-structured interview protocol, teachers explored their reasoning for engaging (or not) with errors to interrogate how these answers could be indexed by macro, meso, and micro experiences surrounding student ability. Lastly, these interviews were designed to draw on teachers' perspectives and explore their lived experiences while responding to questions surrounding their utilization and orientation towards errors.

Findings

Teacher Interviews

The central finding for this chapter is that in the interview, Mama Linne more closely aligned with the error for inquiry model. At the same time, Mama Elizabeth was more closely aligned with the identify and eliminate model. This alignment is seen in the frequency in which errors are made focal in the classroom, 199 instances in Mama Linne's observed math lessons versus 43 cases in Mama Elizabeth's math lessons, and how both teachers conceptualize errors in general. For example, within the interview code *relationship between errors and learning*, Mama Linne spoke more often about errors as exploration to enhance learning. In contrast, Mama Elizabeth generally said that errors as impediments to understanding and knowledge could only occur once the error was corrected.

Analysis of the answers given by Mama Linne in the teacher interview indicated overall alignment with the error for inquiry model. In contrast, answers given by Mama Elizabeth aligned more closely with the identify and eliminate model. I noted four categories that

demonstrate alignment to either model as mentioned above. These four categories were captured by two themes: (1) teachers' attitudes and orientation towards errors and (2) teachers' approach to errors during instruction. Each of the four categories and two themes is expanded below, with each teacher discussed in turn.

Attitude and orientation toward errors: Mama Linne

Tables 5.1 and 5.2 noted that two codes were identified under the theme of attitude and orientation toward errors: highlighting the relationship between errors and learning and identifying the role errors can play in student self-esteem. This teacher expressed the belief that errors can support student learning when she stated**,** "[Errors] are just another opportunity to learn something new." She continued, "Everyone makes errors, if you haven't made errors you are a very lucky person ((Laughs)), you're one in a million but going through life you're more than likely going to make a lot of error and it's nice to learn how to handle those and grow and learn through those." In this statement, the teacher highlighted the learning potential of errors on two separate occasions: to learn something new from the error made and to learn simply by making an error. Other responses also embodied the error for inquiry model. One such response came when Mama Linne said: "I made a error, where did I go wrong and how can I fix it and get better from there?" In this response, Mama Linne again discussed the errors' learning potential and allowed the student to have agency in locating, identifying, and learning from the error made. Moreover, in all the responses mentioned above (and the majority of the entire interview), Mama Linne noted errors as central in helping her and her students learn and grow. However, I must note that while most of Mama Linne's comments aligned with the error for inquiry model, two moments aligned closer to the identify and eliminate model in which she stated, "When an answer might've been wrong, I…let them know they need to go back, reevaluate, and find their

error" and "Did they not subtract properly, is it a calculation error, do we know what exactly is the misconception and so we work through a lot of problems like that."

Alignment with the error for inquiry model was also seen within comments coded relating to how errors affect student self-esteem. This teacher both acknowledged and challenged the notion of errors affecting student self-esteem with the response: "I don't think I have an environment where you're crucified if you're wrong. I think because in math they tend to make a lot of errors and we discuss those errors on a daily basis [so] it's not a big deal if you mess up. And they're often really quick to try and help each other try and figure it out. So yeah I don't think we have that issue." This response addressed how an individual student might perceive an error and perceptions other students committing the same error. This comment can also be understood as a response to concerns voiced within the discourse of error exploration about the impact of highlighting an error on the student who committed the error (Bray, 2013; Santagata, 2004). Mama Linne likely knows that an error can carry a stigma historically and maybe even currently for her students. Also, the teacher may be trying to be explicit that she believes such a stigma does not exist within her classroom and is not imposed on her students. By saying that she does not have that type of environment, she acknowledges that such environments exist but then explains how she minimized this type of environment without minimizing error exploration, as evidenced by the distribution of codes above.

Attitude and orientation toward errors: Mama Elizabeth

Unlike the previous instructor, Mama Elizabeth explicitly expressed how errors impede student learning and understanding. This is evident with responses such as "Hey, I know you've been doing this, but that's not the way you should be doing it" and "I know you guys are going to want to do this but don't, that would make the question wrong, the strategy wrong, so do it this

way as opposed to doing it that way." The importance of highlighting statements such as these lies in the fact that the instructor notes that even though some mathematical understanding backs the error, "I know you've been doing this" and "I know you guys are going to want to do this," what's most important is the correct procedure given by the instructor as opposed to exploring why students have been or want to do a problem a certain way. Also, another feature of these answers that align with an identify and eliminate model is how the teacher explicitly closes off alternative avenues for solving these problems using the words "don't" and "that's not the way." With statements like these, it primes students to believe that there is only one correct solution path, and they should only consider that solution path. Other messages that align with the identify and eliminate model are seen when Mama Elizabeth states, "Yeah, so if you always used to not simplifying and you keep getting points off of it, while you have the answer right, I need you to simplify. That's a common mistake. So, at this point after you've lost so many points now you're going to remember oh I need to simplify". While the mistake here is relatively simple (not simplifying), how Mama Elizabeth pairs the error with a direct negative outcome, loss of points, can affect the student's relationship with errors and emphasizes that errors are to be avoided. While Mama Elizabeth's responses overwhelmingly align with an identify and eliminate model, she has two answers that could show alignment with an error for inquiry model within this code when she stated, "It's about learning from your mistakes" and "[Errors] are just apart of learning."

Alignment with the identify and eliminate model was also seen within comments coded relating to how errors affect student self-esteem. For Mama Elizabeth, she viewed errors as a source of embarrassment for students several times in her interview. She avoided making students' errors known to the class or exploring errors publicly because of this. She explicitly

states this when she says, "We all have questions, we all make mistakes, and we can just move past it" and "You are going to feel embarrassed at some point or another." Within the first statement, she not only diminishes the mistake made, but she also states her preferred response of moving past it instead of interrogating the why of the mistake. Moreover, the second statement again pairs a negative outcome to mistakes in which Mama Elizabeth says that eventually everyone will make a mistake, and as a result, everyone will feel embarrassed. This pairing reinforces the idea that errors are to be avoided and, if they do come up to move past them as quickly as possible.

Approach to errors during instruction Mama Linne

In discussing her approach to errors during instruction, Mama Linne's comments also showed alignment with the error for inquiry model, specifically as she described how she identifies and responds to errors that arise during instruction. For example, Mama Linne explained, "…typically like with the math homework, if someone struggles with it and they put it on the board, you know they can step back and look at it. And if they work it all the way out like they did in their homework and they got it wrong, they can step back, they can look at it, their classmates can look at it, and everyone tries to identify 'Ok, where did they go wrong?'" Here the teacher discussed who was is expected to identify the error. This statement mirrored what Cobb et al. (1992) described in one classroom in their study of characteristics of math classroom norms as a "procedural description," that is to retrace your steps, correct the procedure, and by doing so correct the error. However, the difference here is who is allowed to do the intellectual work and moving power and authority to identify and address errors from the teacher to the students. In this response, the responsibility for determining the error belonged to the whole class —the student at the board, the other students in the class, and the teacher herself. This self and

collective responsibility for error identification demonstrated a belief that it was not the teacher's sole responsibility to identify and correct errors and spoke to the environment the teacher had set up to safeguard students' self-esteem.

Lastly, how Mama Linne discussed responding to student errors during instruction also embodied error for inquiry constructs. This can be seen in the response given by Mama Linne in which she stated: "I think if it's an error that's going to affect the long-term goal then it's an impending error and that's something we need to address. If it's a calculation error than I can talk to you on the side about that" as well as the following example she provided: "[in] the math challenge we had today, there were two problems that were really really challenging so instead of continuing to make the one group struggle we split the class up into two groups, put the problems on the board and let them worked through them together so it became more of a group effort versus more of a "this one group can't get this problem," they kind of shared the struggle" Within the first response, the teacher evaluated the type of error produced and used that evaluation as the basis for the kind of response to give. Here, Mama Linne distinguished between an "impeding error" as an error that needs immediate and focused attention as opposed to a "calculation error" that she could address at a later time without fear of it impacting the learning happening at the moment. Also, of note in this response, is that an "impeding error" is something discussed with more than just the student who committed the error with the use of "we" as opposed to the individual correction denoted with the help of the words "you" and "on the side" for the "calculation error." Also, within the first response is the notion that the error itself should be evaluated and used in the meaning-making process of the students learning. In the second quote from Mama Linne, she described doing an on-the-spot evaluation of student progress and refocused the struggle being made by one group to be shared with the collective class. Overall,

Mama Linne explicitly noted how she does not dismiss the error in these moments but uses it as the starting point for a much larger discussion. Thus, instead of isolating the error, she believes that learning for both the student and the class can result from the error being explored.

Approach to errors during instruction: Mama Elizabeth

As with the previous categories, Mama Elizabeth discussed how she approached errors generally aligned with the identify and eliminate model regarding errors. As opposed to Mama Linne who discussed using errors to further group discussion, Mama Elizabeth prefers to address errors individually and only publicly only after grading an assignment or when over half the students have the same error. This is seen when she stated "What I usually do is I pull them to the side, unless it's a common mistake like more than like half the class is doing it then I address it as a class. But if it's a specific student that I know knows what they're doing but they just made a mistake I'll call them to the side like hey you forgot your negative or hey you forgot to simplify or hey you missed this extra step here did you realize you were supposed to divide, or just pulling them to the side and talking about it as opposed to just calling them out in front of the whole class". This statement is also essential because of the who being discussed. In her view, even though she believes the hypothetical student is competent, sharing the error whole class would be perceived as "calling them out," which is explicitly negative. Mama Elizabeth also embodies an identify and eliminate model in how she frontloads errors in her instruction. Mama Elizabeth noted how she captured errors when she stated, "When it's time to grade then I take notes" and how she addressed errors when she said, "I address common misconceptions before I start a lesson or umm doing a lesson so if it's something I know I've seen in previous years then I would be like hey, while I'm teaching the lesson I would address it." For Mama Elizabeth, the only errors that need to be addressed publicly affect the majority of the class and only at specific

times during instruction. She tries to note the misconceptions, addresses them directly with a correct procedure, and then moves on. While both Mama Linne and Mama Elizabeth draw upon previous years of teaching experience to anticipate common misconceptions, they operationalize these misconceptions at opposite ends of the spectrum.

In summary, across Mama Linne's comments in the interview, she seems to hold a consistent "error for inquiry" stance and discussed that errors were not only helpful in learning mathematics but can also act as the catalyst for said learning. On the other hand, Mama Elizabeth embodied the identify and eliminate model in which errors were barriers to learning that were to be avoided or replaced as quickly and quietly as possible.

6. **Exploration of Teacher Practices**

To answer the research question "What potential relationships exist between teachers' and students' orientations and dispositions and how are they reflected in classroom interactions within error moments?" I conducted classroom observations to capture these interactions in situ. This attention to teacher moves within moments of mathematical errors is crucial because teachers can potentially position students as incapable or credible sources of mathematical knowledge. Thusly, how teachers pay attention to mathematical errors unfolding in micro-level processes within classrooms can serve as one point in an ecological system to either support productive adaptive responses or confirm negative deficit self-assumptions students might hold.

Findings

Teacher Classroom Practices

Applying Tulis' (2013) categories for initial responses to student errors, Mama Linne met the criteria for creating a positive error climate while Mama Elizabeth did not. For Mama Linne her responses to student errors in the classroom were overwhelmingly adaptive and she spent a significant amount of classroom time within error interactions while Mama Elizabeth spent less time within error interactions and her responses were mostly maladaptive. As stated previously, using Tulis' (2013) coding convention across three classrooms for both teachers, I identified 199 responses to student errors for Mama Linne and 43 for Mama Elizabeth.

Data analysis revealed that Mama Linne applied adaptive responses to student errors 89% of the time across all coded moments (Table 6.1). In general, when an error happened, the teacher would often give a hint or provide a question to allow students to correct the error themselves or use that error as an opportunity for a whole class discussion. Maladaptive responses accounted for only 11% of the total responses to student errors, with the bulk of that

evenly split (4.5%) between "teacher correcting the error herself" and "having another student correct the error." Moreover, because the teacher only scarcely had to impede negative reactions from class (1.5%), the fact that she emphasized the learning potential of errors (8.5%) almost as much as she engaged in all maladaptive practices combined (11%), and her responses in total were overwhelmingly adaptive (89%). A few examples of these types of reactions would be when the teacher allowed the student who gave the incorrect answer time to respond while providing a hint "watch your signs," or when she emphasized the learning potential of errors stating, "we have to learn from these minor errors." Given the adaptive nature of these responses, I can view this class as having a positive error climate and embodying the ideologies presented by the teacher in her interview in which she wants students to self-identify, self-correct, and be comfortable sharing their errors whole class. Mama Linne also effectively guarded students' self-esteem regarding error expression in her class by making a point to highlight the learning potential of such errors and making this type of error expression and communication commonplace, if not an outright expectation.

Table 6.1: Mama Linne's Initial Responses to Student Errors

	Category/Type of Response	Frequency	Percentage
Maladaptive	Ignore the error	1	0.5
	Criticizing the student	0	0.0
	Redirecting the question to another student	9	4.5
	Humiliating/Laughing	1	0.5
	Disappointment/Hopelessness	2	1.0
	Correction by Teacher	9	4.5
Adaptive	Discussion with whole class	46	23.0
	Correction by the student	76	38.0
	Waiting	35	18.0
	Emphasizing the learning potential	17	8.5
	Impeding negative reactions from class	3	1.5

In contrast, the data analysis for Mama Elizabeth revealed she applied adaptive responses to student errors only 14% of the time (Table 6.2). In general, when an error occurred, the most likely response was she directly corrected the error (46%) quickly before moving on. I must note that the number of instances of analyzed errors was dramatically smaller for this teacher and is more evidence of her approach's impact on students' willingness to commit errors in this classroom. Also of note is how the teacher redirected the question to another student. When a student made an error 23% of the time, Mama Elizabeth would acknowledge the mistake with a blunt "no" or "incorrect" and then ask the same question to another student or call another student's name. These two categories (*correction by a teacher* and *redirecting to another student*) accounted for almost 70% of all the noted responses to errors and are indicative of an adverse error climate. Lastly, as three of the adaptive categories show no responses is further evidence of the type of error climate present.

Table 6.2: Mama Elizabeth's Initial Responses to Student Errors

	Category/Type of Response	Frequency	Percentage
Maladaptive	Ignore the mistake	6	13.6
	Criticizing the student	1	2.3
	Redirecting the question to another student	10	22.7
	Humiliating/Laughing	1	2.3
	Disappointment/Hopelessness	0	0
	Correction by Teacher	20	45.5
Adaptive	Discussion with whole class	3	6.8
	Correction by the student	3	6.8
	Waiting	0	0
	Emphasizing the learning potential	0	0
	Impeding negative reactions from class	0	0

Also, beyond their initial responses to student errors, I also tracked how much time each teacher engaged within error interactions in their classrooms, as shown in Tables 6.3 and 6.4.

Table 6.3: Distribution of Talk Around Errors: Mama Linne

	Lesson 1	Lesson 2	Lesson 3
Number of Error Interactions	11	22	22
Number of Minutes in Error Interactions	9	19	22
Percent to Total Classroom Time	12%	25%	29.3%
Range of Time within Error Interaction	5 seconds to 2 min	7 seconds to 6 min	7 seconds to 8 min

Table 6.4: Distribution of Talk Around Errors: Mama Elizabeth

	Lesson 1	Lesson 2	Lesson 3
Number of Error Interactions	28	13	3
Number of Minutes in Error Interactions	22	4	0.5
Percent to Total Classroom Time	18%	15%	0.7%
Range of Time within Error Interaction	2 seconds to 3 min	4 seconds to 4 min	3 seconds to 25 seconds

For Mama Linne, looking across the three days observed, the amount of classroom time dedicated to error exploration ranged from 12% to 29%. By allowing a significant amount of classroom instruction time to the discussion of errors, Mama Linne demonstrated the value she places on error exploration. Within these moments, students were often going to the board to highlight their errors. This process of going to the board to work through student errors reinforced the idea of errors as having learning potential, and the public display shows errors as having learning potential for all and not just the student who committed the error. Taken together, these indicators suggest Mama Linne's alignment with the error for inquiry model.

Mama Elizabeth was on the opposite end of this spectrum with less overall interactions as well as less time spent within said interactions. As noted earlier, Mama Elizabeth planned her classroom instruction in order to limit these moments and as a result, students' mathematical

contributions were more routinely ignored or tabled for a later and private discussion. In doing so, Mama Linne aligned more closely with the error for inquiry model Mama Elizabeth aligned more closely with the identify and eliminate model. Taken at face value Mama Linne's orientations, dispositions, and classroom instruction would be what previous scholars looked for and highlight as markers of an error for inquiry model and the goal would be to move Mama Elizabeth closer to the orientations and practices held by Mama Linne. However, even though both teachers appeared to embody different models for utilizing errors in their interviews, in practice, within the extended interaction presented below, their interactions within these moments were largely similar. This is important because even though both teachers appear on opposite end of the spectrum, they move closer together and more strongly aligned with an identify and eliminate model the longer an error interaction continues. This comparison between these two teachers during instruction is important because it highlights the weakness of only looking at dispositions and initial reaction to errors and provides evidence that teachers can move between models as an interaction unfolds.

Understanding Fluctuations Within Microgentic Moments

To better understand these fluctuations, I analyzed error interactions in each of the classrooms. For Mama Linne I analyzed one extended error interaction that occurred as the class reviewed the previous night's homework from a unit on linear equations. The homework included a series of twelve problems in which students were asked to write and solve equations related to angles of unknown measures. The routine for this type of class discussion centered on highlighting student errors and making these errors visible to the class as a learning opportunity. This structure was most clearly evident when a student attempted to breach this norm.

Michael: *Can I put something on the board?*

Mama Linne: *Do you have a problem you didn't understand?*
Michael: *No, I understood all of them.*
Mama Linne: *Ok, then no, thank you so much.*

Mama Linne would typically announce the beginning of this portion of the lesson by going over the answers for the homework and then asking the students to "raise your hand if you struggled with any of the problems." Students would then be asked to go to the board and write down the problems they found difficult. It was at this point Mama Linne reminded Michael of this norm when he wanted to participate. This section of instruction was also significant because of how it positioned both the student and the error itself. By dedicating specific classroom time to error analysis, the teacher privileged the mathematical contributions of students regardless of accuracy. This time also served to normalize the notion that students were expected to both make errors as well as to share those errors with everyone in the class. This approach to setting up discussions aligned with the error for inquiry model and was also reflected in Mama Linne's interview answers in which one of her goals was to destigmatize errors in her classroom and safeguard student self-esteem by making errors both visible and welcomed. For example, she explained "I try and create an environment where the kids are comfortable being themselves and not afraid to make errors" and "We all make errors, but [do] not …let that diminish your self-worth."

This type of structure is not the norm for every classroom and should be recognized for the collective effort it took to create and maintain such an environment. However, while this structure allowed for error expression, it did not always allow for error exploration. Following the exchange with Michael, the class focused on a problem that a student Kimi raised. The images below show Kimi's answer before she went to the board (left image) and the changes she made after she exited the interaction (right image) for the following question: "The measure of

one angle is described as twelve more than four times a number. Its supplement is twice as large. Find the measure of each angle in degrees." Elenchothy et al. (2010) noted algebra concept such as simplifying and expanding algebra expressions (required to solve the present problem) is required in almost every study of mathematics and "students need more algebraic scaffolding to build their mathematics skill to solve problems and perform better in mathematics" (p 363). More importantly as noted by scholars "word problems are notoriously difficult to solve…children perform 10 to 30% worse on arithmetic word problems than on comparable problems presented in numeric format" (Cummins et al., 1988, p. 405). One reason for this difficulty has been noted as the misalignment between the comprehensive phase and the solution phase. Koedinger & Nathan (2004) defined these phases as followed:

> In the comprehension phase, problem solvers process the text of the story and create corresponding internal representations of the quantitative and situation-based relationships expressed in the text. In the solution phase problem solvers use or transform the quantitative relationship that are represented both internally and externally to arrive at a solution…the comprehension and solution phases typically are interleaved rather than performed sequentially (p. 131).

Breakdown between these two phases tend to occur from a conceptual basis as opposed to an arithmetic misunderstanding (Cummins et al., 1988) in which flawed "external representations may influence further comprehension in later cycles" (Koedinger & Nathan, 2004, p. 131). As seen with this student the mechanics of solving the equation was not a concern but what was most important was figuring how does one decide what gets clustered together within parenthesis and why. In other words, the central feature that was at issue for the student was the function of parenthesis and knowing conceptually when and how to apply them in a multi-step equation. For this student, if her mathematical conceptions are not adequately addressed here then her future mathematical progress could also be impeded.

Figure 2: Kimi answer before going to the board:

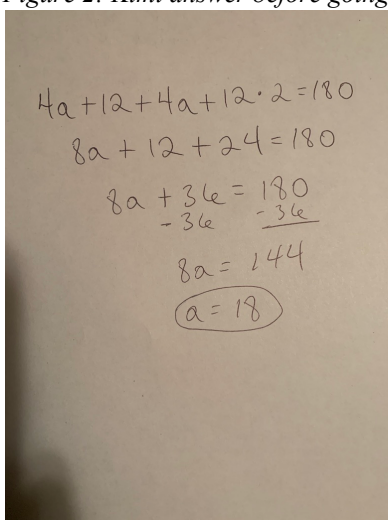
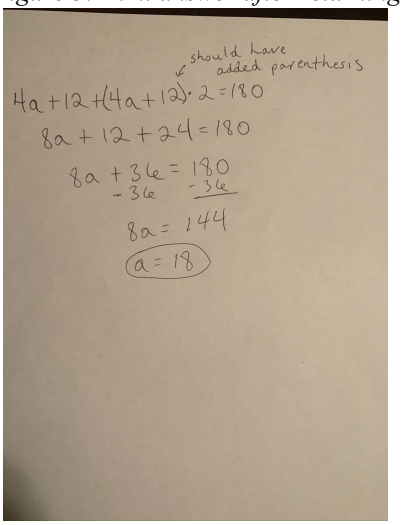


Figure 3: Kimi answer after returning from the board:



After Mama Linne read the answers from the assignment Kimi realized her solution was incorrect. What was her error? Kimi understood that she was to add the measure of two angles and that the two angles were supplementary and therefore their measures equaled 180 degrees. She also knew something about how to represent the angles in the equation, one being "12 more than 4 times a number" or 4a + 12, and the second being "twice as large." While Kimi wrote the expression 4a + 12 · 2 for the second angle, her error was that she did not multiply the entire quantity of the second angle by two as in (4a + 12)2.

Mama Linne Extended Interaction

For this excerpt, Kimi was the student, Mama Linne was the teacher of the class, and Student stood for students whose voices I captured but were not explicitly named.

Excerpt 1

Line 1 Kimi: ((raises hand))
Line 2 Mama Linne: Yes you have a question?
Line 3 Kimi: Umm no I just well
Line 7 Kimi: I just need help with solving the equations that we set up
Line 8 Mama Linne: Okay
Line 9 Kimi: It's like
Line 10 Mama Linne: So did you struggle with solving the equation for number two?
Line 12 Kimi: Yes
Line 13 Mama Linne: Yes put your equation on the board
Line 14 Kimi: ((walks to the board)) I just think I did the steps in the wrong order
Line 15 Mama Linne: Okay lets see
Line 16 Kimi: and I think I should've wrote a parenthesis

Analyzing, the beginning of this excerpt Tulis (2013) and I would agree that the teacher responded adaptively to the student. This is evidence in how the teacher used the misunderstanding of the student as an opportunity to discuss whole class. This categorization is where Tulis's analysis of the interaction would end. However, what is missed is the meaning-making process of the participants within the interaction. Tulis' approach also glosses over the nature of the error itself. Because all errors were treated equally rather than considering, for example, whether they were computational or conceptional in nature, a deep analysis of the teacher response was not necessary. Within Tulis' framework the focus was highlighting the *what* of the teacher's response, however my desire is to also explain *how* the teacher responded, requiring deeper analysis.

Within this excerpt the teacher immediately attempted to define the boundaries of Kimi's question as shown in line 10 in which she specified the problem at hand "So did you struggle with solving the equation for number two?" Kimi has a pause of three seconds here and her elongation of her answer "yes" in line 12 could reflect her apprehension of agreeing with the

teacher that her misunderstanding was with solving the problem. In line 13 we see the teacher respond not by asking what Kimi's misunderstanding was but as a directive to go to the board and work through her misunderstanding by working through the problem. Kimi describes two pieces to her misunderstanding in lines 14 and 16 — "I think I did the steps in the wrong order" and noting that parenthesis was missing from her equation. I noted these two lines as the crux of Kimi's misunderstanding but as we see later in the transcript one is continually reemphasized while the other (which is more conceptually-oriented) is abandoned entirely.

Excerpt 2

Line 23 Mama Linne: She is going to solve number two and we are going to see if we can help her out
Line 24 Kimi: ((K writes 4a + 12 + 4a + 12 • 2 =180))
Line 25 Mama Linne: Ok there is one slight problem with your equation the way it reads is you have twelve times two not twelve times 4a plus twelve
Line 26 Student: Not 2 times
Line 27 Mama Linne: Show me what your next step was
Line 28 Kimi: That's my problem I didn't have parenthesis I
Line 29 Mama Linne: Correct
Line 30 Kimi: So when do you know when you should have parenthesis because I've been adding them and getting the wrong answers

In excerpt two the teacher foregrounds solving the problem to address the misconception in line 23 "She is going to solve number two." The focus was not on the mistake that Kimi initially had and voiced, but on the correct procedure Kimi was tasked to do. In line 25 the teacher attempted to identify Kimi's mistake but defaulted to a procedural way of processing the problem in line 27 after taking several seconds to consider the range of possibilities for Kimi's utterance. Of note here, the teacher did not ignore the student speaking in line 26 but did not hear her because of the low volume of the utterance as well as the distance this student was from the teacher. The pause and direct response to Kimi in line 27 suggest that the teacher was considering what Kimi had just written on the board and not a moment of dismissal for the other student. Kimi broke from the procedural processing method of understanding in line 28 in two

ways. First, she did not immediately respond to the teacher's directive to write her next step down from line 27 and second, she re-identified her error which was her incorrect use of parenthesis. The teacher seemed to acknowledge Kimi's error in line 29 in which she said "Correct." Line 29 was the first time the teacher addressed or recognized Kimi's mistake verbally, and Kimi took this recognition as approval to further explain her misconception in line 30. In line 30 Kimi's question went beyond the problem at hand and addressed a larger issue of "When do you know when you should have parenthesis?" Kimi also attempted to keep her turn and not allow time for the teacher to respond to her question before further explaining her broad concern about the function of parenthesis. She did this by stating "I've been adding them and getting the wrong answers."

Excerpt 3

Line 31 Mama Linne: So for this problem the way it read::s chu chu chu chu chu chu chu it says It's supplement is twice as large right so the first equati- or the first expression is going to give you your angle measurement right and its supplement it's supplementary angle is twice as large as that so you have to use the entire equation so you have to multiply that entire equation by two
Line 34 Kimi: °Ok
Line 35 Mama Linne: So is that where you went wrong?
Line 36 Kimi: Yeah but it's just because I've done this exact same thing before except when I added the parenthesis I kept getting the wrong answer like it didn't check out and I didn't understand
Line 37 Mama Linne: So put the parenthesis in and let's see
Line 38 Kimi: ((Kimi writes parenthesis around 4a + 12)) °So then ok
Line 39 Mama Linne: Keep going
Line 40 Kimi: ((Kimi solves the problem))
Line 41 Mama Linne: Ok
Line 42 Kimi: Ok
Line 43 Mama Linne: So where did you go wrong on your equation=because that's the correct answer

In excerpt three the teacher did not address the larger issue presented by Kimi in the previous turn but attempted to redirect the error back to the problem, "so for this problem" and back to the procedure, "so you have to multiply the entire equation by two." Initially, Kimi reluctantly accepted this explanation in line 34 with an "ok" spoken in a low volume. Subsequently, however, she rejected this explanation when it Mama Linne attempted to use it to

address her misunderstanding in line 35, "is that where you went wrong?" In line 36 Kimi stretched the word "because" in an attempt to both re-affirm the point of her misconception as well as show that her error was not limited to the problem on the page and had to do with more than her procedural understanding of the question. The teacher again reverted to her procedural method for understanding in line 37 while disregarding Kimi's attempt to identify her misconception. Still, one could argue that the teacher was hoping to address Kimi's larger misconceptions of the functionality of parenthesis by providing a localized understanding of her use within this problem. However, this argument loses validity in the next excerpt.

Excerpt 4

Line 44 Kimi: I mean
Line 45 Mama Linne: Look at your equation look what you wrote
Line 46 Kimi: Yeah
Line 47 Mama Linne: No look at what you wrote on your paper
Line 48 Kimi: It's because I didn't have the parenthesis so everything kinda just
Line 49 Mama Linne: Oh ok it's because you ***did not*** have parenthesis
Line 50 Kimi: Yeah but I don't I get kinda confused on when parenthesis should be there and when they shouldn't be there

At this point in excerpt four, Kimi has solved the problem correctly, and the teacher saw this solution as a resolution to her misunderstanding. In line 44, she did not allow Kimi to highlight her misconception because in the space where she would, (line 43 from excerpt three) "where did you go wrong on your equation" the teacher circumvented a response from Kimi by highlighting the correctness of the procedure produced, "because that's the correct answer." Kimi again was hesitant to accept this by pausing before she responded and as if recognizing this apprehension, the teacher re-focused Kimi not on her conceptual misunderstanding but on the procedural differences between what she wrote on the board and what she wrote on her paper in lines 45-47. In line 48 Kimi could be responding to the directive provided by the teacher in lines 45-47 in which she noted the difference between what she had on her paper and what she had

produced on the board, or she could be responding to her previous uses of parenthesis as she noted in line 30. Regardless of how Kimi meant her response on line 48, the teacher interpreted this utterance on line 49 to be about the problem at hand and stressed what she identified as Kimi's error as well as the resolution of that error, "Oh ok it's because you ***did not*** have parenthesis." Kimi feigned acceptance of the teacher statement in line 50 with a "yeah but" before outright rejecting (after a brief pause) the resolution the teacher offered in the previous line. Kimi's utterance in line 50 reinforced two ideas. First, that her misconception was not resolved and second that it extended beyond the procedural processes used to solve the problem at hand.

Excerpt 5

Line 51 Mama Linne: So you have to look at what exactly you're multiplying if you're multiplying I mean you can use parenthesis anytime it just helps you ((makes parenthesis shape with hands)) identify what exactly you are multiplying by a certain number does that make sense?
Line 52 Kimi: Ok
Line 53 Mama Linne: Does that make sense?
Line 54 Kimi: I mean it makes sense but it's just like (inaudible)
Line 55 Mama Linne: Even if it was just two times twelve you could put twelve ((makes parenthesis shape with hands)) in parenthesis if you wanted to
Line 56 Kimi: It's just for certain equations I don't really understand I believe it was like on some of the equations we were doing yesterday like I didn't really understand where to add the parenthesis and where not to add the parenthesis
Line 57 Mama Linne: So you add the parenthesis anywhere you multiply ((makes parenthesis shape with hands)) to isolate what exactly you're multiplying
Line 58 Kimi: °Ok, ok
Line 59 Mama Linne: °Ok
Line 60 Mama Linne: Yes Ma'am ((points to another student))
Line 61 Tracy: Do you always have to add parenthesis?
Line 62 Mama Linne: Umm in this case I would so you don't make the mistake of multiplying one number times two so you know you have to multiply that entire equation. Does that make sense?
Line 63 Tracy: Yeah

Excerpt five was the first time the teacher attempted to respond to Kimi's misunderstanding beyond the specific problem but she did so in a procedural way. Mama Linne highlighted what you could do with parenthesis but not Kimi's misunderstanding of when and more importantly when not to use parenthesis. In line 51 the focus on the what of parenthesis (to

multiply) does not address the why of parenthesis (why do parenthesis in the function the way they do) demonstrates this teacher's orientation toward error interaction. In line 53 the teacher did not accept Kimi's "Ok" from line 52 as a clear marker that her explanation was understood and asked explicitly here if that "makes sense." Kimi pushed back on the explanation given in line 53 but upon taking a pause the teacher used this break in line 55 to restate her explanation in concrete terms "twelve times two" and also as a bid to have Kimi come to an understanding of how to use parenthesis "you could put twelve in parenthesis." Kimi again rejected this procedural explanation and restated her misunderstanding beyond the terms and examples provided by the teacher. After one more explanation provided on line 57, Kimi relented, and in lines 58 and 59 both the teacher and student ended their interaction with a reserved "Ok." Because this interaction happened publicly, the error interaction did not conclude on line 59 but continued when another student revoiced Kimi's question on line 61"Do we always have to use parenthesis?" Like Kimi's utterances, this question was not necessarily tied to the problem. However, the teacher referred this student back to the problem at hand "in this case I would" and also to the procedural process of understanding "so you know you have to multiply that entire equation." The error interaction ended on line 63 when the error introduced by Kimi was no longer the focus of the interaction.

Summative Analysis: Mama Linne

While there were a multitude of aspects to note in this interaction, for present purposes, it is important to understand how Kimi positioned herself upon entering the interaction and how this maneuvering spoke to her understanding of why this positioning was necessary. Kimi responded to the teacher asking her "Do you have a question" with "I just need help with solving the equation that we set up." The argument I am making here is that this response was in

recognition of how the classroom previously dealt with mistakes. This interpretation was also noted because Kimi did not enter the interaction vocalizing her mistake but entered with her understanding of procedure equals correctness and correctness addressing the mistake formula in mind. I argue this identification is the case because, as noted previously, this interaction was one of 55 coded for Mama Linne in which 41 of these interactions follow this type of pattern. However, there was a breach in Kimi's response in that her utterance could have been interpreted to be for multiple problems and not just problem two. The teacher addressed this ambiguity with her statement in line 9 in which she stated: "So did you struggle with solving the equation for number two?" This statement solidified that the moment they were in was an error episode and Kimi responded with the acceptable discourse around error episodes in line 13 when she said: "I just think I did the steps in the wrong order." This utterance again highlighted the acceptable way of discussing mistakes in which the procedure to solving the problem should be the focus and not the conceptual mistake itself. In other words, what was acceptable is to highlight the steps rather than explore the conceptual mistake she had. This process-oriented response was repeated again in line 15 in which Kimi noted one particular procedural step she did not take in writing the parenthesis as well as within the teacher comments on lines 21 and 25. Neither time (lines 21 and 25) was Kimi's mistake emphasized or recognized but what was noted was the process for solving the problem and focus on "next steps." It was not until line 28 that Kimi uttered her confusion she wanted resolved of not knowing *when* to add parenthesis. Again, Kimi likely did not lead with this because she recognized that the norm for discussing errors was to focus on the correct solution path as opposed to interrogating the error itself. Before this moment twice Kimi used her understanding of the discourse to try and frame her mistake within it. In line 15 as well as line 26 Kimi attempted this reframing before subverting this discourse outright in line 28. By

Kimi demonstrating her ability to solve the problem in the moment tells us that Kimi did not need help solving the problem or getting the correct steps. However, because her mistake was not found within the procedural steps of solving the problem, it could not be addressed procedurally.

Kimi recognized this discourse and first gained access to the interaction by framing her language exclusively with this discourse (Lines 6, 13, and 15). After Kimi gained access to the discourse, she then subverted the discourse in an attempt to accomplish her goal of addressing her mistake of not wholly understanding the functionality of parenthesis. We see this in how she first presented her question and then how she continued to attempt to reframe the conversation to her misunderstanding that was not solution based. While the full interaction did not fulfill this goal for Kimi, it was of note that she identified the discourse surrounding error episodes, responded appropriately to gain access to this discourse and then tried to reframe before outright rejecting this discourse. Kimi recognized and challenged this discourse in her interaction with the teacher.

Extended Interaction Mama Elizabeth

For Mama Elizabeth I analyzed one extended error interaction from when students were reviewing a worksheet they previously completed on finding surface area. Unlike the previous teacher ,Mama Elizabeth does not have time set aside during the lesson for explicitly exploring errors. Instead, error interactions typically occur at the beginning of a lesson after she noted serval mistakes from the previous days lesson or homework. This portion of the class is framed as a review that focuses on solution steps as opposed to highlighting the types of errors she noticed while grading student papers. The following moment is evidence of this type of interaction.

This problem tackles issues of measurement and the relationship between the properties of geometric shapes. Measurement is one of the major topics in mathematics curriculum and students have been shown to have difficulties in measurement units such as area (Chappell & Thompson, 1999). Also, understanding how to evaluate shapes from their visible representations is important because students with struggle to visually recognize and identify the properties of geometric shapes and solids are less likely to be able to perform calculations regarding 2-D shapes and 3-D objects (Chiphambo & Mtsi. 2021). "Moreover, measuring of area not only expands students' understanding of spatial measurement but also provides foundation for the development of students' understanding of multiplication, fractions, algebraic multiplication and enlargement" (Sisman & Aksu, 2015, p. 1296)

For solving prism problems specifically, it has also been showed that students fell into four board error categories: (1) errors in understanding the problem, (2) errors in thinking of a plan, (3) errors in implementation of the plan, (4), errors in review. Hasanah and Yulianti (2020) further defined each of these groups with the examples that follow:

> [Errors in understanding the problem] The student can write what is known or asked. However the student wrote the formula…it can be concluded this student did not understand the problem and committed the error in using formula. [Error in thinking of a plan] Student wrote the correct formula…but they could not identify which one was the height…In other words this student cannot interpret a pyramid when it is presented in other form. [Error in implementation] Student had plotted the answer well by writing down what is known and asked…However student made mistakes in calculations. [Error in review] Students experience errors in checking answers…students only thought that the most important thing was that they got answers from the given mathematical problem (pp. 4 & 5)

We see in the following interaction that the teacher tries to address student errors that fall into categories one, two and three. However, while acknowledging that the student's purposed central conflict is her misunderstanding of identifying the different components of surface area for a

rectangular prism (length, width, and height) the larger issue that is not address in the above four categories as well as within the teacher response to this student is why the formula for solving surface area works to represent the figure described. As noted "it is clear that both knowing *how* to do measure and knowing *what* and *why* measure are so crucial for meaningful understanding of measurement" (Sisman & Aksu, 2015, p. 1311). It is unknown if the student also struggles to conceptualize the idea of surface area itself but because of the nature of the classroom a focus on the mechanics of solving the equation is the primary focus.

Within this interaction there were four named students here whose voice was captured: LaShawn, Tonya, Marcus, and Keith. Student stood for students whose voices I captured but were not explicitly named.

Excerpt 1

Line 1: Mama Elizabeth: Do ya'll know the formula for finding the surface area
Line 2: Lashawn: Yeah
Line 3: Multiple Students: No
Line 4: Mama Elizabeth: I hear a lot of no's and one yes, so, Marcus I'm up here. That being said for those of you who are not, Quinton, LaShawn, Keith thank you, for those of you who are not aware, Roger, for those of you who are not aware for finding the formula for surface area, it is two times your width times your length, plus two times your height times your length, plus two times your height times your width. Ok
Line 5: Tonya: What?
Line 6: Mama Elizabeth: That's why I wrote it on the board. This is
Line 7: Tonya: I'm just going to get that one wrong.
Line 8: Mama Elizabeth: This is, why would you get it wrong when I'm over here teaching you how to do it?

As with the previous interaction Tulis and I would have classified this as an adaptive response because the teacher addressed the students' misunderstanding, as indicated by their response of "no," by making the conversation whole class. However, once the interaction began we see that when confronted with the possibility of more student confusion from Tonya in lines 5 and 7 the teacher did not interrogate why this confusion existed, but redirected Tonya to the formula on the board as a means to address her misunderstanding. Furthermore, in Line 8 Mama

Elizabeth presented the dominant structure for many of her interactions in which procedure equals correctness and correctness can address any mistake.

Excerpt 2

Line 10: Mama Elizabeth: So, on 2-1, I'm sorry 2-2, everybody should be on page 47. Ok where it says find the surface area of the rectangular prism. So for the first one, I'm going to go over the first one with you and you can do the second one by yourself. Ok on the first one, ((attempts to draw figure on the board)). I can't draw that ((erases shape from board)). For the first one can somebody tell me what the width is?
Line 11: Student: Three
Line 12: Mama Elizabeth: ((Writes three next to the words width on board)) What about the length?
Line 13: Student: Six
Line 14: Mama Elizabeth: ((Writes six next to the words length on board)) Six. The length is going up down. And what about the
Line 15: Female Student: Height
Line 16: Mama Elizabeth: I'm sorry ((erases the incorrect numbers and writes the correct numbers for length and height)).

In this excerpt we see what was prioritized in these types of interactions by what was captured on the board and what gets erased. In line 10 the teacher erased a poorly drawn figure of a rectangular prism and instead refocused the student's attention on the formula previously written. This could demonstrate to students that only correctness, even pictorial representations, can be captured and thusly are valued. By erasing the image she devalued the need to have it because she could not draw it correctly. We see this again later in lines 14 and 16 in which the teacher mistook the length for the height and instead of explaining how this error occurred she erased the incorrect numbers and replaced them with the correct ones. This last act also directly tied into how students came to understand these moments by what was expected of them within these interactions. The teacher only ever asked the students for factual information to be captured on the board. However, as soon as an error was introduced (an error the teacher herself reinforced), the teacher stopped asking students even for factual information and filled in the rest of the information herself before readdressing the class.

Excerpt 3

Line 18: Mama Elizabeth: First off y'all shouldn't be playing. Everybody should be looking at the board, Keith you're looking the wrong way you should be looking at the board. Turn your body around. Ok so for the first one,

for the first one, your width is three miles, your length is five miles, and your height is six. Now what you are going to do is plug in these numbers into the formula. So for number one, your surface area is two times your width times your length, which is three times five, plus two times your height times your length, which is six times five, plus two times your height times your width, which is five times three.
Line 19: Student: What?
Line 20: LaShawn: Oh my god.
Line 21: Student: I got a question
Line 22: Mama Elizabeth: Ok so we're multiplying and then adding. So, two times three times five is what, what's two times three?
Line 23: Class: Six
Line 24: Tonya: 15
Line 25: Mama Elizabeth: What's six times five
Line 26: Class: 30
Line 27: Marcus: She said what's two times three and she said 15
Line 28: Mama Elizabeth: And we're moving pass it
Line 29: Tonya: I thought she said what's five times three.
Line 30: Mama Elizabeth: What's two times six
Line 31: Class: Twelve.
Line 32: Mama Elizabeth: Twelve, and what's twelve times five?
Line 33: Keith: 60
Line 34: Student: Who said 15?
Line 35: Mama Elizabeth: 60 plus what's two times five, we're pass that already.
Line 36: Class: Ten
Line 37: Mama Elizabeth: Ten and what's ten times three
Line 38: Class: 30
Line 39: Mama Elizabeth: ((Adds the numbers up on the board)) So your surface area is ((points to the answer on the board))
Line 40: Keith: 120
Line 41: Mama Elizabeth: 120
Line 42: Marcus: So the answer is 120 ((writes that answer on his assignment))

The first aspect to note in this excerpt was the readdress to the class after the error was quietly corrected on the board. In line 18 the teacher provided the correct measurements to the class and immediately began to insert these numbers into the formula. Nowhere in this line of talk were students invited to contribute. This could be a result of the previous error being captured on the board and the teacher not allowing room for that type of scene to play out again. The lack of student contribution was more evident in lines 19, 20, and 21 in which three separate students responded audibly to the teacher both with frustration (lines 19 and 20) and with a question (line 21). The teacher did not explicitly acknowledge any of the three students, but her next comments conveyed the notion that the procedure will answer their question. This was evident because it was this procedure that became the teacher's focus. Within this same line (line

22) we see the teacher again limited the type of contribution students gave to factual responses, in this case computational responses. In line 24 we see another error introduced in which Tonya responded to the question of what is two times three with an answer of 15. Here the teacher only noted the correct answer of six on the board and ignored Tonya's answer completely. It was not until Marcus brought the error up again to the whole class in line 27 that the teacher addressed the error and the way she did was to further dismiss the mistake and move on "And we're moving pass it." At this point (line 29) Tonya explained the reasoning behind her mistake to the class as a means to defend herself unprompted by the teacher and not for any mathematical exploration. The teacher again ignored the error actively being discussed of Tonya saying 15 was the answer to two times three in line 35 and moved on to the next computation in the sequence. What was particularly interesting in this excerpt was who was responsible for what type of knowledge. Throughout this excerpt students were only allowed to give computation answers out loud when those computations were relatively straight forward, however the teacher did not allow students to contribute once the problem advanced to the final step of adding the different components together. In line 39 the teacher added up the expression on the board and only when the answer was provided did she reengage students for their contribution, going as far as pointing to the answer on the board before asking the class. Again, it must be noted here that this was supposed to be a review of a completed assignment, but Marcus wrote the answer on his paper for the first time in line 42 only after the answer was given to the whole class by the teacher.

Excerpt 4

Line 43: Keith: Wait what we on?
Line 44: LaShawn: I thought the answer was 126?
Line 45: Mama Elizabeth: It is, I'm looking at what I did wrong.
Line 46: Keith: Ya'll slow wait what are we doing LaShawn?
Line 47: LaShawn: Because it's supposed to be 30 plus 60 plus 36 because is two times six times three.
Line 48: Mama Elizabeth: Why 36
Line 49: LaShawn: Because it's supposed to be two times six times three.
Line 50: Keith: It's 90 guys

Line 51: Mama Elizabeth: Thank you
Line 52: Keith: How is it not 90?
Line 53: Mama Elizabeth: ((Erasing the incorrect numbers and replacing them on the board)) Because that's 36. Thank you LaShawn.
Line 54: Keith: What area are we doing?
Line 55: Student: Surface Area.
Line 56: Mama Elizabeth: You have four people in your group
Line 57: Keith: They don't know.
Line 58: Mama Elizabeth: She ((pointing to LaShawn)) literally just said the surface area.
Line 59: Keith: How did she get that?
Line 60: Tonya: Wait I had 90, how is that the answer I had 90. Mama Elizabeth, Mama Elizabeth, I had 90.
Line 61: Keith: See when I ask for help you never help me.
Line 62: Mama Elizabeth: Keith what's the problem?
Line 63: Keith: °I just wanted to ask you a question.
Line 64: Mama Elizabeth: What's the question?
Line 65: Keith: How is it not 90?
Line 66: Mama Elizabeth: How is it not 90 ((points to the board))? You have to use the formula, the formula for finding the surface area of a rectangular prism, that's the formula you have to use.

In this excerpt we see immediately that students recognized something was wrong with the problem. In lines 43 and 44 both Keith and LaShawn responded that the answer given differed from the answer they had, LaShawn more directly than Keith. The teacher was also aware that the solution differed from her answer sheet but the use of the word I in line 45 was striking because of how knowledge was hoarded by the teacher throughout the interaction. Keith was so sure about his answer that the only rationale he came up with to why their answers differ is that they were doing a different problem in line 46. LaShawn ignored Keith in a similar way the teacher ignored Tonya and proceeded to have a one-on-one conversation with the teacher while Mama Elizabeth was still at the board. Keith tried twice to insert himself in this interaction on lines 50 and 52 and the progression of his utterances were of note. In line 50 Keith was proclaiming with confidence his answer, but once his answer was not taken up by the teacher or even acknowledged, he knew his answer was perceived as incorrect and then changed his utterance to challenge this perception (line 52). For Mama Elizabeth, her main priority was getting the correct answer on the board and erasing any errors so her entire attention was devoted to hearing LaShawn's explanation and making the appropriate changes. Only once these changes

were made did Mama Elizabeth address Keith. However, instead of trying to understand how Keith got his answer she first directed him to LaShawn (who in the previous moment demonstrated her mathematical worth by provided the correct answer), line 58, and after he rejected this offer and repeated questioning, she then directed him to the equation on the board in line 66. While she did engage with Keith, Mama Elizabeth ignored Tonya completely who had the same answer as Keith and this could be explained by Mama Elizabeth being reluctant to give Tonya a public voice when she committed the error in the interaction previously. Moreover, because at least two students came up with the same solution there might have been a misconception shared between them, and possibly more students, that the teacher chose to completely dismiss because it did not align with the solution given.

Summative Analysis Mama Elizabeth

Taken collectively these excerpts show how errors were generally treated, as well as how students were expected to participate once within these interactions. The knowledge that is captured only reflects correctness and never was there an opportunity to explore misunderstandings or misconceptions. Another consequence of this type of interaction was that students came to see only the final answer as valuable and thusly the steps to get to that answer, both conceptually and computationally, lost merit. We saw this manifest in two different ways with Marcus and Keith. Marcus accepted the final answer given by the teacher as correct without doing the work himself and proceeded to stop engaging with the interaction completely. This was most striking when the answer was shown to be false and a correct answer was given but Marcus did not change it because the interaction was completed on line 41 when the teacher gave the answer of 120. Keith had an answer before this interaction, but never did he stop to compare his process to the one given on the board until the final product was produced. Keith pushed back

on the answer given several times, but his focus was only on the difference in product and the teacher's response was to redirect him to the formula on the board.

Summative Analysis Both Teachers

For both teachers it is noted that regardless of the environment the error occurred in and no matter how open (or closed) the teacher was to explore errors, each one focused on procedure knowledge as opposed to conceptual knowledge.

> If a learner commits an error that is a misapplication of the rule, the learner's understanding of the rule is procedural. The rule is understood conceptually if the learner if the learner is in a position to see why the rule does not apply. This is so because: procedural knowledge refers to mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms, and definitions. Conceptual knowledge refers to knowledge of the underlying structure of mathematics (Morapeli & Luenta, 2017, p. 246).

Neither student was allowed to know the "underlying structure of the mathematics" or know when to or not apply a rule (parenthesis and formula) because each teacher focus almost exclusively on the rule being violated or the computational skills needed to answer the specific question at hand.

Language of Procedures

This chapter aimed to identify the dominant discourse in two middle school math classrooms within error episodes using two extended interactions as case studies. My finding suggested that such a discourse not only existed but was recognizable to members of the classroom community. This discourse, which I named Language of Procedures, represented the dominant form of communication created and supported by members of both classroom communities within these specific interactions. As evidenced by 41 out of 55 interactions following this pattern for Mama Linne and all of the interactions following this pattern for Mama

Elizabeth. I traced the Language of Procedures within these classrooms by looking at the microgenetic moments (individual interactions within whole-class discourse) involving errors for Kimi and Tonya.

This discourse emphasized that for a misconception (error, mistake, etc.), the best way to address it was by not confronting the misconception itself but by giving the correct procedures (steps) to solve the problem at hand. For students who understood that their misconceptions could not be addressed with the correct procedure (Mama Linne), they had to navigate this discourse to try and force it to resolve their mistake by reframing or rejecting the discourse, but for students who have no flexibility in manipulating this discourse (Mama Elizabeth) they conform to the expectations within.

There are severe consequences to teaching and learning having the Language of Procedures as the discourse within error episodes. The first is what was foregrounded and what was backgrounded. Within this discourse, procedure and correctness were privileged to address and correct mistakes. As a result, the student and the error made became virtually irrelevant. If the procedure that provided correctness could address any error, then the student who made a mistake and the type of mistake made do not matter. For students, this could have the effect that they lose sight of the value of mistakes as a meaning-making process (Borasi, 1987) and reduce these interactions to the identify and eliminate model (Radatz, 1980). Secondly, students lose intellectual agency with their mathematical understanding because they are no longer seen as intellectual authors of their knowledge if their thoughts (mistakes) are disregarded and not seen as relevant for interrogation or contribute to the overall understanding within their classroom community (Engle, 2012). Within this discourse, the student's initial conceptions remained

unaddressed. For learning to occur, a student's initial understanding that may have led to the mistake must be addressed before a new or alternative conception can be realized.

Beyond the practical implications for teachers, systemic issues need to be addressed. I use the word systemic because even though Mama Linne and Mama Elizabeth were on opposite ends of the spectrum for their approach to errors set by contemporary error scholars, I found a much closer orientation and practice once I extended the analysis to the entire interaction. This similarity was most evident in the fact that both classrooms developed the same type of discourse around errors despite overarching differences in their error orientations in theory (teacher interview) and practice (classroom instruction). The methods are taken up here (tracking the sociogenesis of discourse using microgenetic moments) proved that earlier studies that focus on initial responses are insufficient and can paint an overly generous portrait of classroom learning and engagement.

7. Conclusion

It is widely understood that people's perceptions of themselves and tasks influence their engagement and effort. Further, these relationships are often viewed as the purview of the individual. In contrast, research on human development has documented the influences of participation in multiple, often overlapping, contexts on individual development. Processes of participation, prevailing meta-narratives and structures (political, economic, social) as well as resource allocation within and across these settings can pose challenges that individuals and groups must navigate to achieve their goals. This dissertation sought to unpack influences among micro-level processes of participation among multiple interlocutors because "without aerial, macro views of social life through historical time, or without ground-floor, micro views of social interaction in specific settings, an array of analytic voices is rendered unavailable" (Espinoza & Vossoughi, 2014, p. 292).

In particular, I addressed perceptions of the self among middle school children who faced developmental challenges because of where they are in the life course, as well as because of how they self-identify (African-American), such that their normative developmental challenges are complicated by deficit meta-narratives around race. These racialized developmental challenges are further enhanced in the context of schooling because of persistent meta-narratives around what is entailed in learning mathematics. These include the prevalent belief that excellence in mathematics is due to a fixed intelligence and that particular populations, both in terms of race/ethnicity and gender, are less able to become proficient in the study of mathematics. Thus, this dissertation sought to interrogate the broader problem of the influences of participation and practices within and across multiple ecological settings on perceptions of the self with regard to achieving ego-related tasks for a specific population, in this case, African-American students. To

be clear, I defined cultural learning (in regard to African-American students within math class) based on the definition provided by Medin and Bang (2014):

> If instead culture is seen as dynamic, contested, and variably distributed within and across groups, it is natural to see cultural learning as involving a reciprocal relationship between individuals' goals, perspectives, abilities, and values and their environment On this view, socialization, partially depends on agents or others who are caregivers as well as an individual's interpretation of and reaction to their environment. (p. 87)

This definition forced me to resist the urge to treat all the African-American students in this study as a monolith and guided my choices for data collection and analysis. The data collection and analysis choices explored above allowed for multiple lines of inquiry. First was a focus on the students and teachers as both individuals in the classroom as well as agents within a community that collectively work toward a shared objective. Secondly this inquiry allowed me to take into consideration larger societal expectations that might impact the learning environment in which these interactions take place.

Overall, in this chapter, I reflect on the findings presented throughout this dissertation. Drawing on results shown previously in this study, I note how this research contributes to our understanding of the benefits of taking an ecological systems approach to interrogating and understanding teacher knowledge, student identity, and error unitization. I will also discuss the implication of the findings for classroom instruction and practice. Lastly, I discuss the methodological contributions of this work, followed by limitations and future directions for this research.

Student Orientations

This research makes several contributions to our understanding of classroom instruction and practice in moments of error. First, this work empirically illustrates relationships among racial identity, beliefs about ability, and attitudes toward errors. There has been research documenting

that positive racial identity is associated with broad academic outcomes (e.g., grades, graduation) (Rowlet et al., 1998; Chavous et al., 2003; Byrd & Chavous, 2011), but little work has connected micro-level learning processes in specific academic content areas. As noted in a previous chapter, this research supports the hypothesis that factors outside of the classroom can influence students' perceptions and attitudes about errors, and these factors correlated to students' black identity and beliefs about intelligence. On average, students tended to feel more positively about wrestling with errors, believed that ability is something within their power to change, and held a positive self-image of blackness. Within this work, I saw positive statistically significant relationships between students' error orientations and their beliefs about intelligence being malleable and having a positive orientation toward their racial identity. Students who hold a negative orientation to their racial identity or who tend to think of intelligence as fixed have more of a negative association within their error orientations. I expected these two findings; however, this work also discovered a complex relationship between these two points. For example, having a high centrality value may act as a buffer against macro-level messages regarding race and ability. Still, that same high centrality value might also exacerbate these messages. As noted previously, the more students associate blackness as a core to their identity, the more acutely they may recognize macro-level messages and see their errors as diminishing Black excellence. This diminishing could occur because students view errors as blemishes to their intelligence, and thus errors are to be avoided.

Overall, while these complex relationships exist, this research shows the benefit of positive racial identity and the belief that ability and intelligence is malleable on error orientations. These orientations can impact students' willingness to engage with a complex topic and have lasting implications on how students view themselves and how they come to view mathematics in

general. This is especially salient because this study was conducted in an African centered school in which centrality of racial identity was reaffirmed at both the classroom level as well as within the broader school climate. I suspect results might differ significantly in a more traditional classroom in which identity is not held in such importance. Lastly, as we see in the following section, the teacher's error culture also influences how these constructs correlate.

Teacher Disposition and Practice

Based on interview questions and direct observations of each teacher's instructional practices, I can argue that Mama Elizabeth aligned with the identify and eliminate model. In contrast, Mama Linne aligned with the error for inquiry model in her planning and instruction. I documented these alignments in how they discussed their orientations to errors, with Mama Linne describing errors as opportunities for students to learn and grow. In contrast, Mama Elizabeth described errors as impediments to student achievement. I also document these associations to either model in the breadth and depth of error interactions within each classroom. Mama Linne had 199 instances in which she makes errors the focal point of class in the lessons analyzed, while Mama Elizabeth only had 43 of these moments. Mama Linne also spent significantly more time within these error moments, averaging 22% of her entire class time exploring errors. In comparison, Mama Elizabeth only spent an average of 8% of her classroom instruction doing the same. Also, the way each teacher responds to errors varies greatly. Within these moments, Mama Linne reacts positively to these errors, as defined by Tulis, 89% of the time, while Mama Elizabeth responds positively only 14% of the time. Lastly, students who had Mama Elizabeth as their teacher were more likely to have a fixed mindset when it came to ability and intelligence as opposed to students who had Mama Linne. As demonstrated, having a fixed

mindset on intelligence correlated to having low racial identity scores and negative error orientations.

Taking the previous findings collectively, I argue that Mama Linne created and maintained a positive error climate in her classroom while Mama Elizabeth did not. However, by looking beyond the initial reaction to errors, this research complicates our understanding of what teachers bring as knowledge and dispositions to the uptake of errors in math classes, moving beyond simple categories of either/or. This distinction is crucial because even though both teachers appear on opposite ends of the spectrum, they move closer together during an extended error interaction. Both teachers more strongly aligned with an identify and eliminate model the longer an error interaction continues. These results demonstrate both a methodological and conceptual contribution to the field. Methodologically this work contributes to our understanding for a need to move beyond immediate error response in qualitative analyses of talk in math classrooms around errors. Conceptually this can be achieved by moving beyond dichotomies in classifying teachers' attitudes and behaviors towards errors. Overall, in real time instruction attitudes and behaviors towards errors by teachers are connected to their content and pedagogical content knowledge.

Implications on Classroom Interactions

Both teachers engage in what I call the Language of Procedures. Regardless of the error climate they create, the way the teacher explores the errors focuses on procedural knowledge instead of conceptual understanding. This research also demonstrates the complex ways in which students respond to errors in classrooms in which this discourse is present. For Kimi (in Mama Linne's classroom), she recognizes the type of discourse present. First, she enters the interaction using a procedural language before subverting this discourse to address her conceptual

misunderstanding. While unsuccessful, Kimi challenges this discourse directly. For Mama Elizabeth, this discourse is never challenged in the same way Kimi does, resulting in students prioritizing the final product above all else. Even when students issue challenges, they almost exclusively revolved around differences in answers or solution path. This way, students are not allowed to examine the underlying structure of the mathematics that would lead to an understanding beyond computational.

Interaction of Findings within PVEST Model

The PVEST model offers insights into how macro-level narratives (in this case around race/ethnicity, ability, and mathematics) can be internalized or rejected by students. It also offers insights into how perceptions of the self around race/ethnicity and ability are associated with errors in math classes. Lastly, I have illustrated how perceptions of errors in mathematics by teachers and students interact in dialogic processes that are reflective in their coping strategies and emergent identities. In contrast, I do not have data to support any claims about life stage outcomes. I will address this gap in the future research section.

Methodological Contributions

In addition to having implications for instruction and practice, this research makes several methodological contributions. First, this study offers new ways of examining the relationships between racial identity, belief in intelligence, and error orientations. More broadly, this research demonstrates the need to expand how scholars study how errors are presented and taken up in math classrooms. Prior studies have focused on the initial reaction to denote alignment to either an identify/eliminate model or an error for inquiry model or indicators of a positive error climate (Bray, 2013; Cohen, 1990; Santagata, 2004; Santagata, 2005; Son and Sinclair, 2010; Tulis, 2013; Matteucci et al., 2015). However, this research demonstrates how

limited this approach can be and highlights the consequences of not more fully exploring error interactions in the classroom for both teacher practice and student engagement and learning.

Limitations

This study has several limitations that are important to note. First, the students discussed between the two classrooms were not captured in the same year. This gap could impact not recognizing time-specific issues that might have only affected one teacher or one set of students. Second, inter-rater reliability was not established for coding the video data. I will prioritize this when I conduct this work in the future on the remaining video data. Next, both teachers were female, and the lack of male teachers raised questions about gender as a factor regarding classroom practices and instruction that I cannot answer. Due to the pandemic, I could not get interview data from my focal students in Mama Elizabeth's class to compare to students from Mama Linne's class, so I removed that data from this study. Lastly, due to consent issues, I could not observe students in out-of-school spaces, which severely limited my ability to speak to meso level systems students participated in from an ecological system standpoint.

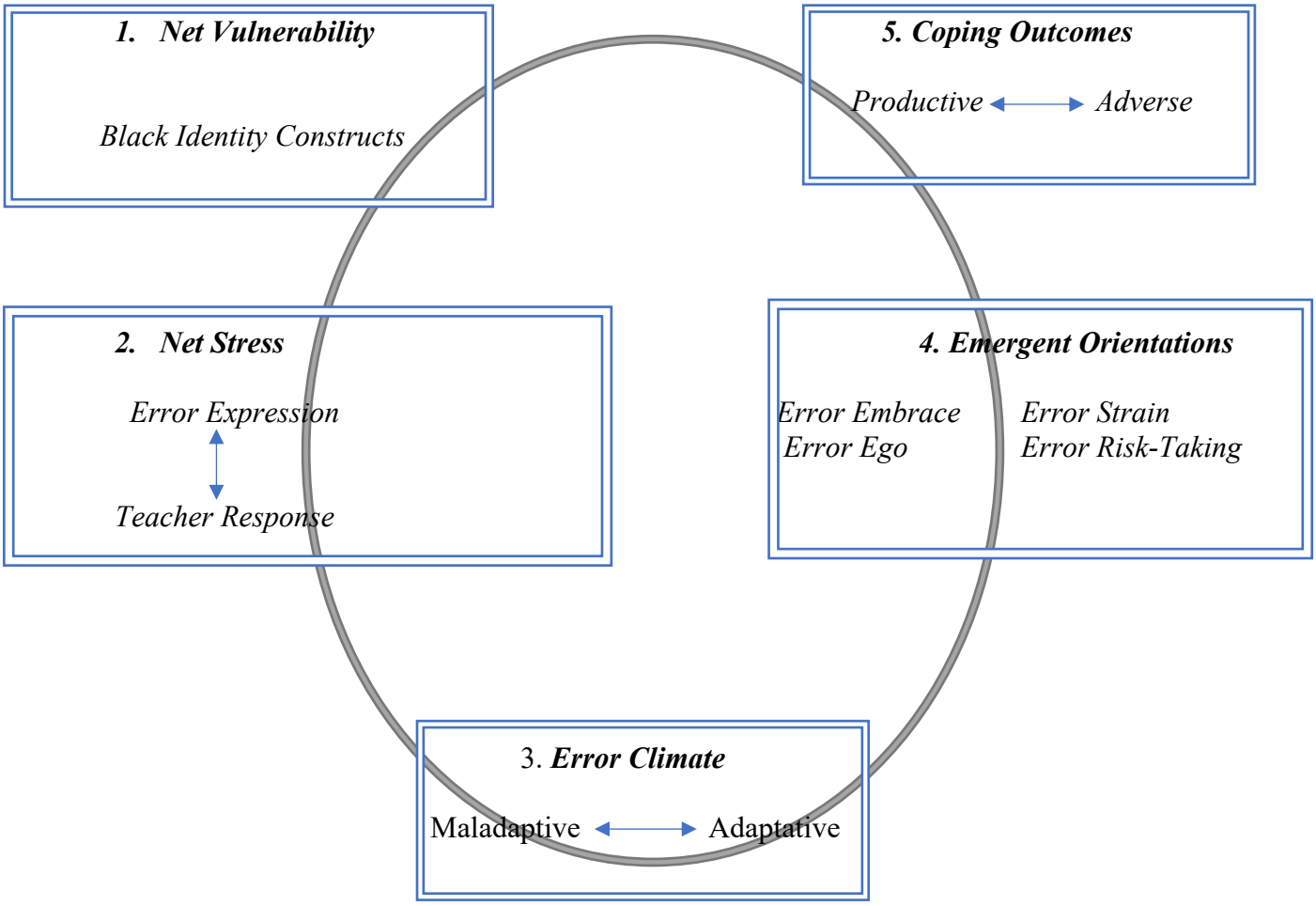
Future Research

Within this study, I attempted to answer three questions: (RQ1.) What are potential relationships among students' error identities, racial identity, mindset, and how are these impacted by macro-level messages about race? RQ2.) How do teachers' beliefs and attitudes about errors reflect meta-structures and meta-narratives in their planning and instructional practices? (RQ3.) What potential relationships exist between teachers’ and students’ orientations and dispositions and how are they reflected in classroom interactions within error moments? To understand classroom errors and make sense of the varying data sources collectively, I used the Phenomenological Variant of Ecological Systems or PVEST framework (Spencer, 1995).

PVEST provided a framework through which I could begin to understand and interpret my data findings. As mentioned previously, PVEST (Spencer, 1995) takes a broad human development perspective over the life course. For this population, PVEST is applicable because these students are beginning an important developmental transition point that introduces new challenges and vulnerabilities. As mentioned before, PVEST included five factors, (1) Net Vulnerability, (2) Net Stress, (3) Reactive Coping Processes, (4) Emergent Identities, and (5) Stage Specific Coping Outcomes (shown in Figure 1) to understand "unique and cumulative individual-context interactions, such as the interaction between maturational influences and social experience-based cognitions" (Spencer, 2008, p. 698).

For future research, I plan to more thoroughly study the use of my modified Error PVEST Model in classroom interactions among all core subject areas (Figure 4 Below).

Figure 4: Error PVEST Model



For the Error PVEST model net vulnerability considers the interaction of risk contributors and protective factors for an individual's development. For this work, I identify one feature as either risk or a protective factor, internalization of their black identity. To ascertain how students view their own black identity, I will continue to draw on the Multidimensional Inventory of Black Identity-teen survey (MIBI-t). MIBI-t is suited for this purpose because it "explicitly differentiates the importance of race to individuals from the meanings individuals ascribe to their group, allowing examination of the individual effect of each component" (Chavous et al., 2003, p 1078) as well as being specifically designed (in both question framing and length) for children in

this age group. These constructs can be either protective factors or risk contributors depending on the salience of each construct within the MIBI-t.

Net stress measures the interactions between social supports and challenges as people experience participation in real-world settings. This participation will occur in the classroom at the micro level for future studies within the moment-to-moment interactions. I will note the interaction between social supports and challenges in how the teacher responds to student errors. Teacher responses to student errors measure the specific nature of that response. These two features work collectively and could become a pathway or a barrier for students' current and future engagement. While this study looked at these interactions in one classroom future studies will attempt to track focal students over the course of one year to analyze how their net stress develops. These responses also index macro-level messages a teacher might internalize about ability and belief toward the benefit (or lack thereof) and use of errors in their instruction. Overall, students' perceptions of the task, their abilities, perceived relevance of the academic work, or their perceptions of critical socializing agents matter for the effort they put forward and their engagement for the content being covered (Eccles, 2005).

Reactive coping strategies represent the adaptive or maladaptive strategies resulting from how net stress is handled within a given context. As these interactions continue to happen, they indexed a specific climate around errors as either adaptive or maladaptive, and I noted this as error climate. While this identification was important more work is needed to measure student's resiliency against this error climate by further interrogating their classroom experiences.

Emergent identities will result from repeated experiences within the coping process that become orienting behaviors. Future research will use longitudinal data for students across multiple years to interrogate how these error orientations stabilize or shift over time. The bi-

directional relationship between constructs on the Error PVEST model is essential and more work is needed to make these relationships more clear.

Lastly, stage-specific outcomes are the internalization (conscious or otherwise) of these identities in the life course. These stage-specific outcomes will be essential to study underrepresented students precisely because of the potential impact on their cognitive, self-identity, and motivational development (Wigfield et al., 2005). Further study is needed to understand and adequately address stage-specific outcomes that impact students within and beyond the classroom. To do this work, I will need to observe students in multiple settings and across multiple years to identify how these coping outcomes are negotiated and expressed in these varying settings, especially within developmental thresholds. This consequence of this component is particularly salient for African-American students, in which deficit meta-narratives around race can complicate student identity development and learning experience. These racialized developmental challenges are further enhanced in schooling because of persistent meta-narratives around what is entailed in learning mathematics that teachers can hold and express within their teaching practice. As evidenced in this study, these notions included the prevalent belief that excellence in mathematics was largely procedural, and little time was spent exploring computational learning for students. I can argue that this procedural focus resulted from the teacher's belief in a fixed intelligence and their belief this type of knowledge construction was the best way for students to become proficient in mathematics. More broadly, findings from this study indicate the need for an expanded methodological approach to study error climates and teachers' and students' orientations to errors in the classroom, and I believe the Error PVEST model will allow for such exploration.

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Appendix A

A. <u>Definitions</u> "The first couple questions are general ones…"

A1. Tell me about your overall teaching style?

A2. How would you define a student error in your classroom?

A3. How do you talk about errors in general, if at all?

B. Purpose of Errors. "Now we'll shift to some questions about your conceptions of errors".

B1. Imagine I'm a new student coming to your class and I've never made a mistake. I've never even heard the term mistake, can you pretend I'm that student and explain to me what is a mistake?

B2. What do you think students gain from committing errors in the class, if anything?

B3. How useful are mistakes for improving students' work, if at all?

B4. When planning your lessons, do you take students' mistakes into account?
How often does the thought of students making mistakes impact your planning of lessons?

B5. How often do students' mistakes redirect or change the flow of a lesson?
How often do you worry about students making mistakes while teaching a lesson?

B6. How often do you worry about students feeling embarrassed or frustrated by the mistakes they make?

C. **<u>Mistakes</u>** "Now we'd like to shift to some questions about how you utilize mistakes in your classroom.

C1. How do you go about addressing errors once they happen in class?

C2. Can you talk about the last time you addressed an error and walk me through that interaction?

C3. How often do you address an error in the moment?

C4. How often do you wait to address an error until a later time?

C5. What goes into your decision to either address errors in the moment or at a later time?

C6. Do you more often address mistakes whole group or with the individual student?

D. Errors Outside the Classroom

D1. I just have a couple of final questions about your daily life. Think about your own day-to-day life in what ways if any do you have to deal with errors?

D2. Can you think of anybody in your life whose errors you correct and whose you choose to ignore?

E. Final Questions

E1. Can you describe your classroom culture?

E2. How would describe how you established this culture for a new teacher?

E3. Lastly, is there anything you would like to tell me that I haven't covered in this interview?

Appendix B

Modified Error Orientation Scale

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
I am often afraid of making mistakes in class	○	○	○	○	○	○
I feel embarrassed when I make a mistake in class	○	○	○	○	○	○
If I make a mistake in class, I get upset	○	○	○	○	○	○
While working in class, I expect to make mistakes	○	○	○	○	○	○
I expect to make mistakes from time to time with my work in class	○	○	○	○	○	○
I know I could make a mistake while doing class work	○	○	○	○	○	○
Making mistakes is part of learning in class	○	○	○	○	○	○
I don't expect to make mistakes as part of my classwork	○	○	○	○	○	○
I talk about my mistakes even if I'm the only one who notices them in class	○	○	○	○	○	○
It is ok to make my mistakes known to other people	○	○	○	○	○	○
I do not find it useful to talk about my mistakes in class	○	○	○	○	○	○
I would rather keep my mistakes to myself in class	○	○	○	○	○	○
When I make a mistake in class, I tell others about it so they do not make the same mistake	○	○	○	○	○	○
If I cannot fix my mistake by myself, I ask my teacher for help	○	○	○	○	○	○
If I cannot fix my mistake by myself, I ask my classmates for help	○	○	○	○	○	○

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
After I have made a mistake in class, I think about how I could have prevented it	○	○	○	○	○	○

"Short Answer
Think about the last time you made a mistake in math class and write about what happened in the space provided."

Submit

Appendix C

MIBI-t

Name

First Last

Date

MM / DD / YYYY

Please answer the following questions:

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
I feel close to other Black people	○	○	○	○	○	○
I have a strong sense of belonging to other Black people	○	○	○	○	○	○
If I were to describe myself to someone, one of the first things that I would say is that I'm Black	○	○	○	○	○	○
I am happy that I am Black	○	○	○	○	○	○
I am proud to be Black	○	○	○	○	○	○
I feel good about Black people	○	○	○	○	○	○
Most people think that Blacks are as smart as people of other races	○	○	○	○	○	○
People think that Blacks are as good as people from other races	○	○	○	○	○	○
People from other races think that Blacks have made important contributions	○	○	○	○	○	○

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
Black parents should surround their children with Black art and Black books	○	○	○	○	○	○
Whenever possible, Blacks should buy from Black businesses	○	○	○	○	○	○
Blacks should support Black entertainment by going to Black movies and watching Black TV shows.	○	○	○	○	○	○
Being an individual is more important than identifying yourself as Black	○	○	○	○	○	○
Blacks should think of themselves as individuals, not as Blacks	○	○	○	○	○	○
Black people should not consider race when deciding what movies to go see	○	○	○	○	○	○
It is important that Blacks go to White Schools so that they can learn how to act around Whites	○	○	○	○	○	○
I think it is important for Blacks not to act Black around White people	○	○	○	○	○	○
Blacks should act more like Whites to be successful in this society	○	○	○	○	○	○
People of all minority groups should stick together and fight discrimination.	○	○	○	○	○	○
There are other people who experience discrimination similar to Blacks.	○	○	○	○	○	○

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
Blacks should spend less time focusing on how we differ from other minority groups and more time focusing on how we are similar to people from other minority groups	○	○	○	○	○	○

Submit

Appendix D

Dweck Mindset Scale

Name

First Last

Date

MM / DD / YYYY

Please answer the following questions:

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
You have a certain amount of intelligence, and you really can't do much to change it	○	○	○	○	○	○
Your intelligence is something about you that you can't change very much.	○	○	○	○	○	○
No matter who you are, you can significantly change your intelligence level.	○	○	○	○	○	○
To be honest, you can't really change how intelligent you are	○	○	○	○	○	○
You can always substantially change how intelligent you are	○	○	○	○	○	○
You can learn new things, but you can't really change your basic intelligence	○	○	○	○	○	○
No matter how much intelligence you have, you can always change it quite a bit	○	○	○	○	○	○
You can change even your basic intelligence level considerably	○	○	○	○	○	○

Dweck Mindset Scale

Name

First Last

Date

MM / DD / YYYY

Please answer the following questions:

	Strongly Agree	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Strongly Disagree
You have a certain amount of intelligence, and you really can't do much to change it	○	○	○	○	○	○
Your intelligence is something about you that you can't change very much.	○	○	○	○	○	○
No matter who you are, you can significantly change your intelligence level.	○	○	○	○	○	○
To be honest, you can't really change how intelligent you are	○	○	○	○	○	○
You can always substantially change how intelligent you are	○	○	○	○	○	○
You can learn new things, but you can't really change your basic intelligence	○	○	○	○	○	○
No matter how much intelligence you have, you can always change it quite a bit	○	○	○	○	○	○
You can change even your basic intelligence level considerably	○	○	○	○	○	○

Appendix E

Math Self Score

This is your form description. Click here to edit.

Name

First Last

Date

MM / DD / YYYY

Please answer the following questions:

	1	2	3	4	5	6	7
How good at math are you? (1=not at all; 7=very good)	○	○	○	○	○	○	○
If you were to rank all the students in your math class from the worst to the best in math, where would you put yourself? (1=the worst; 7=the best)	○	○	○	○	○	○	○
How well do you think you will do in math this year? (1=not at all; 7=very well)	○	○	○	○	○	○	○
How well do you think you will do in math this year? (1=not at all; 7=very well)	○	○	○	○	○	○	○
How successful do you think you'd be in a career that required mathematical ability? (1=not very; 7=very successful)	○	○	○	○	○	○	○

	1	2	3	4	5	6	7
How social are you in math class? (1=not at all; 7=very)	○	○	○	○	○	○	○
If you were to rank all the students in your math class from the most popular to the least popular, where would you put yourself? (1=the least; 7=the most)	○	○	○	○	○	○	○

Submit