TRANSPORTATION CENTER

ROUTING AND DISPATCHING MULTIPLE VEHICLES TO AN EMERGENCY SCENE

by

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RESEARCH REPORT

NORTHWESTERN UNIVERSITY EVANSTON • ILLINOIS

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ABSTRACT

A model of the distribution of arrival time at the scene of an emergency for the first of n vehicles is developed for the case in which travel times on the links of the network are Normally distributed and the path travel times of different vehicles are correlated. The model suggests that the probability that the first vehicle arrives at the scene within a given time may be increased by reducing the path time correlations, even if doing so necessitates increasing the mean path travel time for some vehicles. The model with Normally distributed travel times is compared with a more complex model that assumes Erlang distributed travel times and the results are shown to be in close agreement.

1. Introduction

The provision of emergency services to the public is one of the most important functions of local governments. Such services include fire protection, police protection and emergency medical services. The private sector is often also involved in the provision of backup emergency medical care and emergency road services for automobiles. Whether operated in the public or private sectors, the perceived quality of these services is often directly related to the time between when a need for service is first reported and the arrival time at the scene of the first unit responding to the emergency. In this research, we are concerned with estimating the distribution of the arrival time of the first emergency vehicle at the scene when multiple units are simultaneously dispatched to the scene and when travel times on the network are stochastic. The results have implications on the routes emergency vehicles should take in travelling to the scene of an emergency, on the locations from which they should be dispatched and, ultimately, on the sites at which facilities should be located. In this paper, we focus on the routing and dispatching implications, leaving the location aspects to future research.

The literature on emergency services is vast. However, most previous studies have focused on locating facilities on a network using a variety of objectives, including minimization of the number of facilities needed to attain a given level of service [3, 14, 24, 25], maximization of the number of demands served by a given number of facilities [2, 10, 13, 15] and minimization of mean travel times [4, 17, 26]. In almost all cases, the studies assumed that travel times were deterministic, that the nearest available vehicle is dispatched to the emergency, and that the demands for service can be satisfied by a single vehicle.

Several authors have examined travel times in detail. Mirchandani and Odoni [23] treated travel times as random variables. In their formulation, the source of randomness is such that the travel time between any vehicle site and a given demand location at a particular time is known; however, for the same vehicle site and the same demand location, the travel time at a future point in time may differ from the present travel time for the vehicle and demand combination. They found that travel time variability of this form can significantly influence vehicle siting decisions. Chelst and Jarvis [9] proposed using Larson's hypercube queuing model [21, 22] to estimate the distribution of travel times for emergency services. They assumed that the travel times between any two atoms or nodes in the network is deterministic and known. The sources of randomness in their approach include the location of the demand for service, the availability of the different emergency vehicles, and the location of the unit selected to respond to a particular demand. Finally, several researchers [18, 19, 20] have investigated the relationship between travel distance and travel time and have found that travel time is a non-linear function of travel distance for short trips.

Carter, Chaiken and Ignall [5] studied vehicle dispatching policies and showed that dispatching the nearest available vehicle may increase the system-wide expected response time for future demands so much that it would be better to dispatch a more remote vehicle to respond to the current demand and to preserve the capability to respond adequately to future demands. We show below that dispatching the two nearest available vehicles when multiple vehicles are sent to a single emergency may be suboptimal even in terms of serving the present demand, regardless of the impact on future demands.

Several papers have also examined the problem of multiple vehicle dispatches. Chaiken [6] developed a Markov model that may be used to estimate the number of vehicles that are busy at the scene of an emergency. Chelst and Barlach [8] extended the hypercube model to account for demands that require two vehicles to be dispatched. They assumed that service times for the two vehicles follow independent identically distributed exponential distributions. With this assumption, they modified the hypercube model to obtain a model whose solution provides the steady-state state probabilities of finding a unique combination of busy and free vehicles. From the state probabilities they computed a variety of system performance measures including the mean response times for the primary and secondary vehicles and for the first and second arriving units. In all these computations, travel times between atoms or nodes of the network are assumed to be deterministic. Chelst [7] has developed a model to predict travel times of the first and second arriving units that accounts for vehicle availability. The model is based on geometric probability results and is used to compare travel times of one and two officer police patrol units, as well as the manpower requirement impacts of a change from one-officer to two-officer units.

In this study we adopt a different approach; we begin by focusing on vehicle routing and dispatching decisions rather than facility location decisions. We assume travel times are random variables and that multiple vehicles are to be dispatched to the emergency scene. In the next section we outline our basic assumptions and develop a model to estimate performance under these assumptions. Section 3 presents a simple example of the use of the model showing the implications of the results on routing and dispatching decisions. In section 4 we relax the assumption of Normally distributed link travel times made in Section 2 and compare the results of Section 2 with those obtained using a more realistic Erlang distribution for link travel times. Section 5 contains our conclusions and recommendations for future study.

2. A Multivehicle Response Time Model

The problem we address is that of finding the distribution of the arrival time of the first emergency vehicle at the scene of an emergency. We assume that demands for service are generated at the nodes of a connected network and that the travel times on the links are random variables. The sources of this randomness include traffic delays, delays due to the inability or unwillingness of drivers to yield to the emergency units and delays due to such rare events as train crossings or traffic accidents.

Let X_k be a random variable denoting the travel time on *link* k. We assume (until Section 4 below) that X_k is Normally distributed with mean μ_{X_k} and variance $\sigma^2_{X_k}$ that the variance is proportional to the mean. That is, we assume

$$\sigma^2_{X_k} = \gamma\mu_{X_k} \tag{1}$$

where γ is the variance of travel time per unit mean travel time or, more simply, the *unit variance*.

This final assumption relating the mean and variance is not strictly needed, though it simplifies our presentation in Sections 3 and 4. In principle, the unit variances may differ from link to link. In particular, there is no *a prior* reason to assume that the unit variances on expressways, arterials and local streets are the same.

Let T_i be a new second random variable denoting the travel time on a path or sequence of links. Under the above assumptions for X_i, T_i is also a Normally distributed random variable with mean, μ_{T_i}, and variance, $\sigma^2_{T_i}$, given by

$$\mu_{T_i} \quad = \quad E(T_i) \quad = \quad \sum_k a_{ik}\, \mu_{X_k} \qquad (2)$$

$$\sigma^2_{T_i} \quad = \quad Var(T_i) \quad = \quad \sum_k a_{ik}\, \sigma^2_{X_k} \qquad (3)$$

where

$$a_{ik} = \begin{cases} 1 & \text{if link } k \text{ is on path } i \\ 0 & \text{otherwise} \end{cases}$$

The covariance in path travel $\sigma^2_{T_i,T_j}$, between paths i and j is given by

$$\sigma^2_{T_i,T_j} \quad = \quad \sum_k \delta_{ijk}\, \sigma^2_{X_k} \qquad (4)$$

where

$$\delta_{ijk} = \begin{cases} 1 & \text{if link } k \text{ is on both path } i \text{ and path } j \\ 0 & \text{otherwise} \end{cases}$$

That is, we assume that the covariance in path travel times is equal to the sum of the variances in the link travel times for the links that the two paths share in common.

To summarize, let $\underline{T}$ be a vector of random path travel times from the sites of all available emergency vehicles that will be dispatched to a particular emergency scene. We note that the paths need not be the minimum paths from the vehicle sites to the emergency location and, as discussed below, should not be the minimum paths under certain circumstances. Thus, if three vehicles are to be dispatched to an emergency -- for example, two pumpers and one ladder to a fire -- the vector $\underline{T}$ would contain 3 elements. Under our assumptions, $\underline{T}$ follows a multivariate Normal distribution,

$$\underline{T} \sim \text{MVN} (\underline{\mu}_T, \underline{\Sigma}_T) \qquad (5a)$$

where $\underline{\mu}_T$ is a vector of mean path travel times whose elements are given by (2) and $\underline{\Sigma}_T$ is a variance-covariance matrix of path travel times whose diagonal elements are given by (3) and whose off-diagonal elements are given by (4).

As discussed above, the perceived quality of an emergency system is often related directly to the response time of the first unit on the scene after an emergency has been reported. While we recognize that the response time is composed of several components, including dispatching delays and possible delays incurred while waiting for an available vehicle [16], we focus on the travel time component of response time as this is the component most affected by vehicle location, dispatching and routing decisions. Let Y be a random variable equal to the travel time of the first unit on the scene. The cumulative distribution of Y is given by

$$\text{Prob} (Y \leq y) = \text{Prob} \left\{ \underset{i}{\text{MIN}} (T_i) \leq y \right\} \qquad (6a)$$

Alternatively, we may rewrite (5a) as

$$-\underline{T} \sim \text{MVN} (-\underline{\mu}_T, \underline{\Sigma}_T) \qquad (5b)$$

in which case

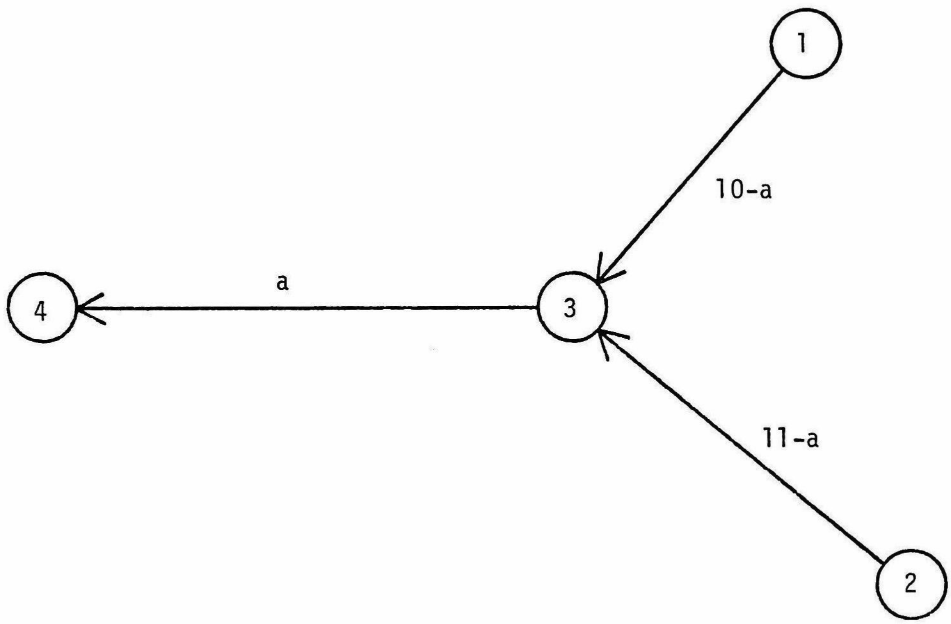
$$\text{Prob} (Y \leq y) = \text{Prob} \left\{ \underset{i}{\text{MAX}} (-T_i) \geq y \right\} \qquad (6b)$$

The transformation of equation (6a) into (6b) is useful since Clark [11] has developed a procedure for estimating the distribution of the maximum of a set of Normally distributed random variables. The result is that the maximum is distributed approximately Normally with a mean and variance that depend on $\underline{\mu}_T$ and $\underline{\Sigma}_T$. Thus, Y, the travel time of the first unit on the scene, is approximately Normally distributed. Daganzo [12] reports that Clark's approximation is satisfactory except when the means are similar and the variances are quite different. If assumption (1) holds -- that is, if the variances of link travel times are proportional to the means -- we will not encounter this problem, as the means could not be similar and the variances dissimilar under this assumption. Daganzo [12] also reports that positive correlations improve the accuracy of Clark's approximation. This effect accounts, in part, for the approximate nature of the results reported on the following section.

3. Application of the Normal Model

In this section we apply the model developed in Section 2 to the small example problem shown in Figure 1. A demand for service at node 4 requires a response by vehicles at both nodes 1 and 2. We assume the two vehicles are dispatched simultaneously and we will be interested in the probability that the first vehicle to arrive at node 4 has arrived by a given time. We refer to this as the probability node 4 is covered in the specified critical coverage time, T_C. The example allows us to explore the effect of changes in (i) the covariance of path travel times, (ii) the variance per unit travel time γ, and (iii) the critical coverage time, T_C, on the probability node 4 is covered. In addition, through this simple example we are able to develop additional insights into desirable dispatching and routing strategies.

FIG 1 - EXAMPLE NETWORK



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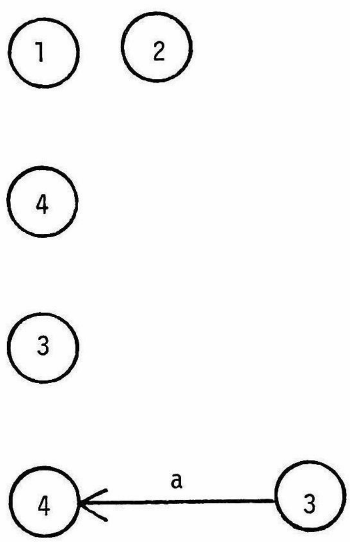


Vehicle Base Nodes

Demand Node

Intermediate Node

Expected Link Travel Time

For the example in Figure 1, we have

$$\underline{T} = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \qquad \underline{\mu}_T = \begin{pmatrix} 10 \\ 11 \end{pmatrix} \qquad \underline{\Sigma}_T = \gamma \begin{pmatrix} 10 & a \\ a & 11 \end{pmatrix} \tag{7}$$

Using Clark's formulae we find

$$E(Y) = E\{ \text{MIN } (T_1, T_2) \} = 11 - \Phi(\beta) - b\phi(\beta) \tag{8a}$$

$$E(Y^2) = 121 + 11\gamma - [21 + \gamma]\Phi(\beta) - 21b\phi(\beta) \tag{8b}$$

$$\text{Var}(Y) = E(Y^2) - E^2(Y)$$

$$= [11 - \Phi(\beta)]\gamma - [b\phi(\beta) + \Phi(\beta)]\ [b\phi(\beta) + \Phi(\beta) - 1] \tag{8c}$$

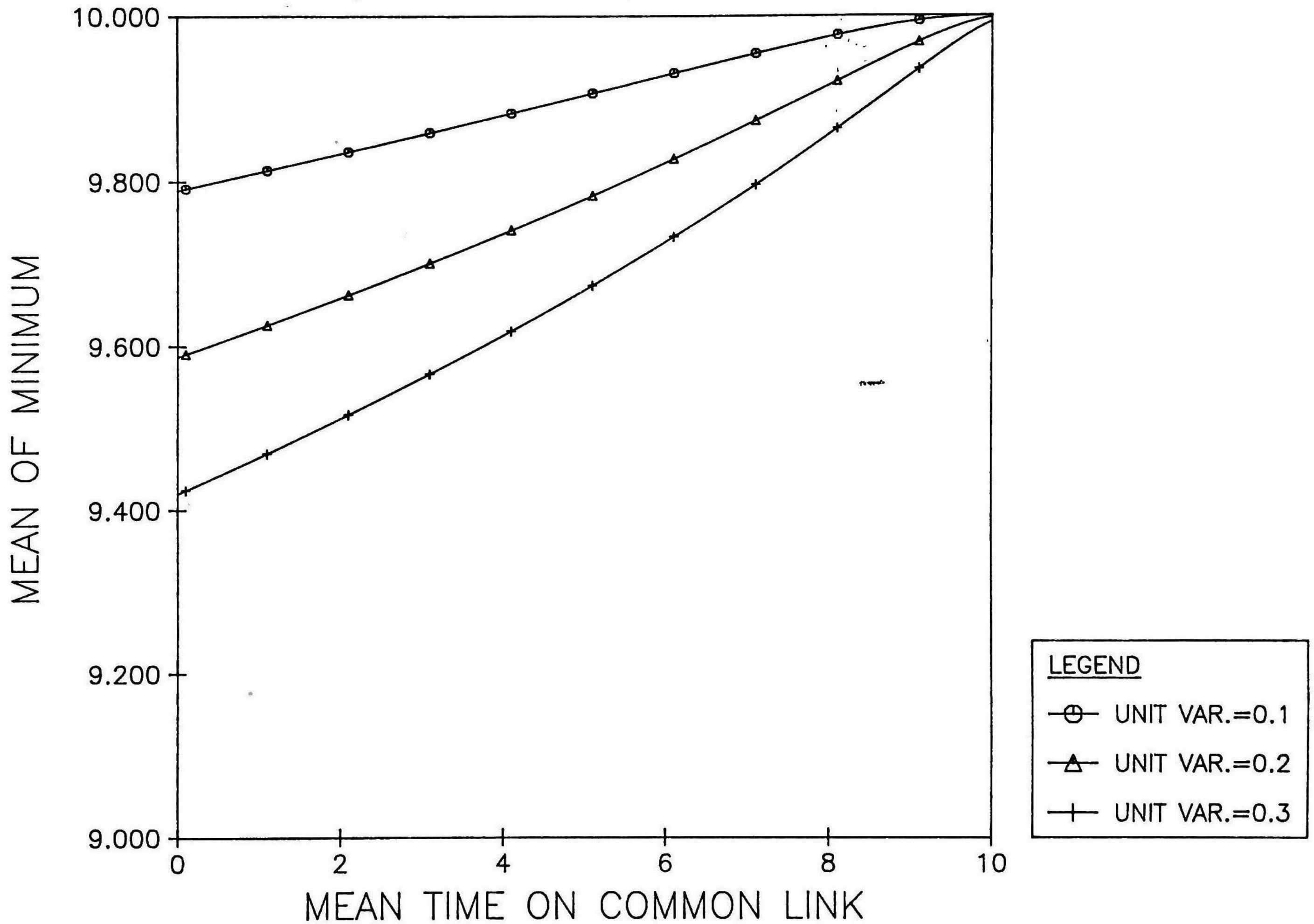
where $$b = [\ \gamma\ (21 - 2a)\ ]^{0.5} \tag{8d}$$

$$\beta = [\ \gamma\ (21 - 2a)\ ]^{-0.5} \tag{8e}$$

and $\phi(x)$ and $\Phi(x)$ are the Normal probability density and cumulative probability functions respectively.

Figures 2 and 3 plot the mean and standard deviation respectively of the minimum travel time or the time the first vehicle arrives at node 4 as a function of the mean travel time on the link the two vehicles travel in common, for values of the unit variance of travel time of 0.1, 0.2, and 0.3. The covariance of the two path times increases with the time on the common link. Both the mean and standard deviation of the minimum travel time

FIG 2—MEAN OF MINIMUM TIME



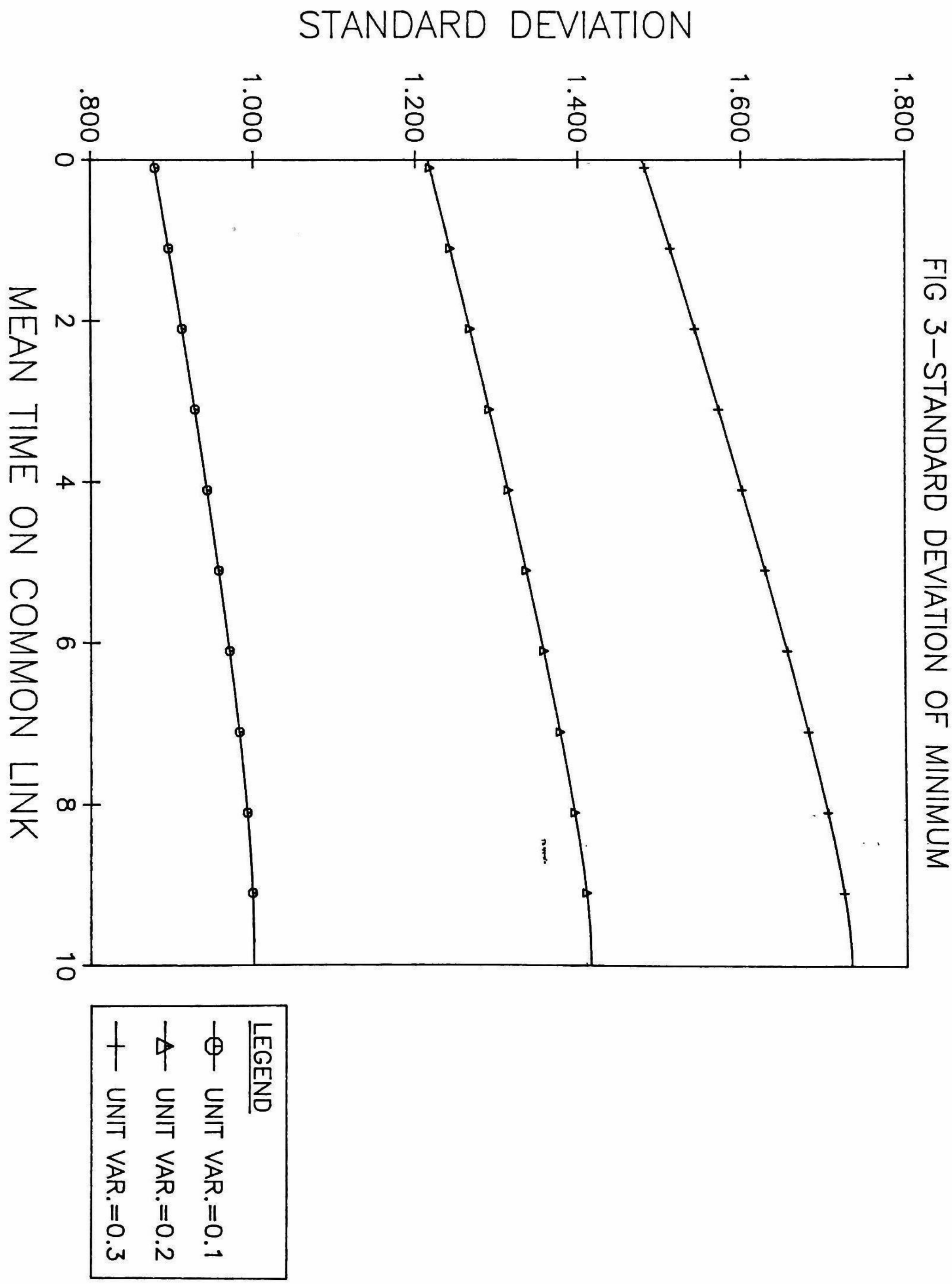


FIG 3—STANDARD DEVIATION OF MINIMUM

increase as the mean time on the common link increases. As the unit variance in travel time increases, holding the travel time on the common link fixed, the mean of the minimum travel time decreases while the standard deviation increases.

Figure 4 plots the probability node 4 is covered in a travel time of 11 units as a function of the travel time on the common link, for values of the unit variance of 0.1, 0.2, and 0.3. As expected, the probability node 4 is covered decreases as the travel time on the common link increases. This is a result of the increase in the mean of the minimum travel time with increased travel time on the common link (as shown in Figure 2). The probability node 4 is covered decreases with increases in the unit variance due to the larger standard deviation of the minimum travel time with larger unit variance (as shown in Figure 3). Figure 5 plots the probability node 4 is covered when the unit variance in travel time is 0.2 as a function of the travel time on the common link, for critical coverage times of 10.0, 11.0, and 12.0. Again, the probability node 4 is covered decreases with increases in the travel time on the common link. As expected, the probability node 4 is covered increases with increases in the critical travel time.

Figures 4 and 5 suggest that if the travel time on the common link is large, the probability that node 4 is covered may be increased by dispatching the two vehicles from node 1 and some node 5 which may be farther from the demand node 4 than is node 2, as long as the mean travel time on the paths in common for the two vehicles is small (as shown in Figure 6a). Alternatively, we may dispatch the two vehicles from nodes 1 and 2, routing the vehicle from 1 along the indicated path and the vehicle from 2 along an alternate path. Again the probability of node 4 being covered may be increased even if the mean time on the alternate path exceeds the 11 units shown for the path from 2 to 4 in Figure 1, as long as the travel time in common between the alternate path and the path taken by the node 1 vehicle is small. Figure 6b illustrates an independent alternate path with mean travel time Z for the vehicle located at node 2.

We can readily determine the mean travel time on the alternate independent path which makes the probability node 4 is covered equal to that obtained by routing the vehicle from 2 along the original path. Let Z_e be the equivalent mean travel time on the independent path. The probability that node 4 is covered using the original routing may be computed using the technique outlined in Section 2. Let this probability be P_c. The probability node 4 is covered using the independent path for the vehicle located at 2 may also be computed in the same manner. Alternatively, the probability node 4 is covered using the independent path is

$$\begin{aligned} P_c &= 1 - P(T_1 > T_c)\, P(Z_e > T_c) \\ &= 1 - \left[1 - \Phi\left(\frac{T_c - 10}{\sqrt{10\gamma}}\right)\right]\left[1 - \Phi\left(\frac{T_c - Z_e}{\sqrt{\gamma Z_e}}\right)\right] \end{aligned} \tag{9}$$

Solving for Z_e, we find

$$1 - \Phi\left(\frac{T_c - Z_e}{\sqrt{\gamma Z_e}}\right) = \frac{1 - P_c}{1 - \Phi\left(\frac{T_c - 10}{\sqrt{10\gamma}}\right)} \tag{10}$$

or

$$\frac{T_c - Z_e}{\sqrt{\gamma Z_e}} = \Phi^{-1}\left\{1 - \frac{1 - P_c}{1 - \Phi\left(\frac{T_c - 10}{\sqrt{10\gamma}}\right)}\right\} = \alpha \tag{11}$$

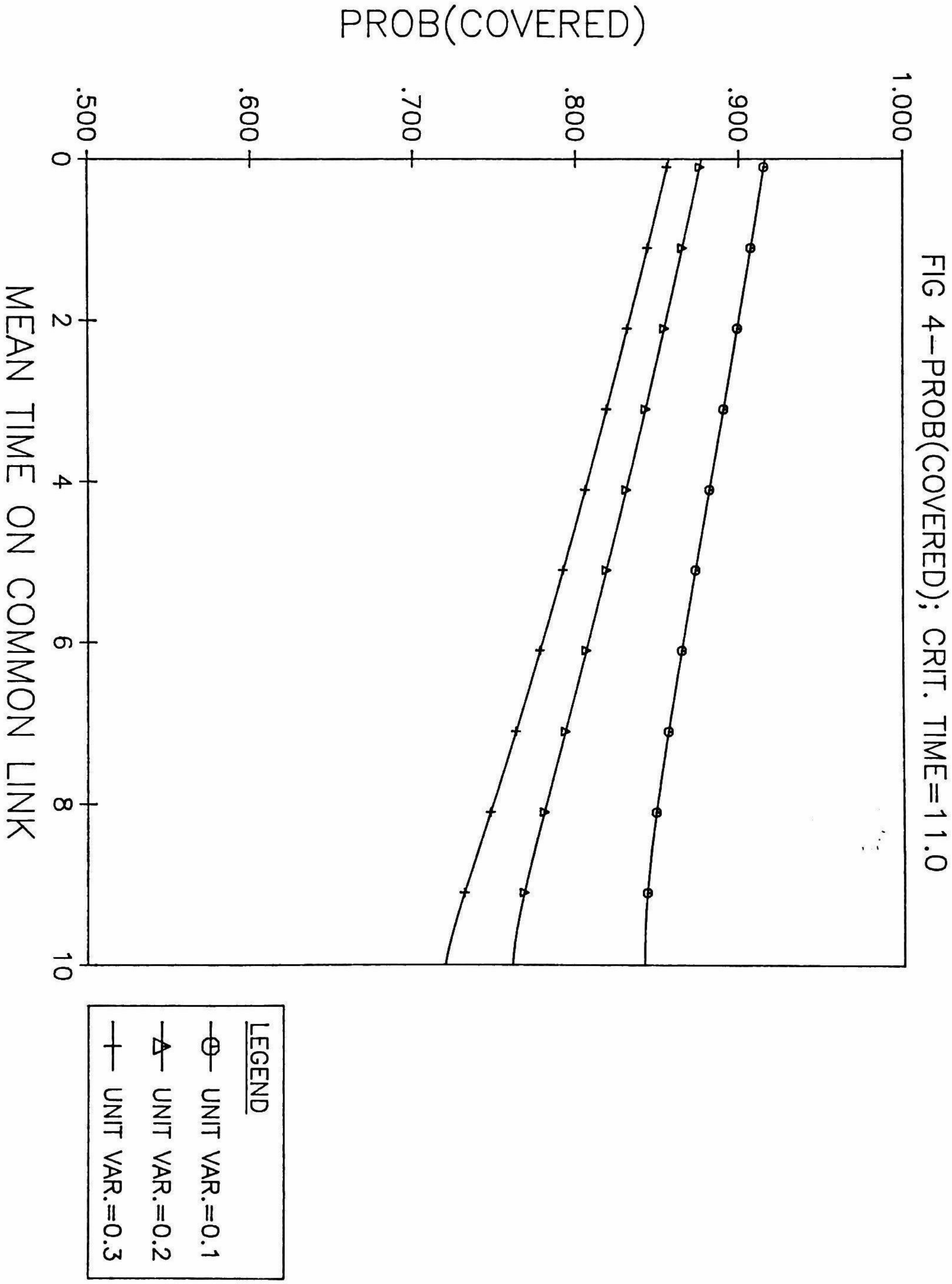


FIG 4--PROB(COVERED); CRIT. TIME=11.0
PROB(COVERED)
1.000
.900
.800
.700
.600
.500
0
2
4
6
8
10
MEAN TIME ON COMMON LINK
LEGEND
UNIT VAR.=0.1
UNIT VAR.=0.2
UNIT VAR.=0.3

FIG 5—PROB(COVERED); UNIT VAR.=0.2

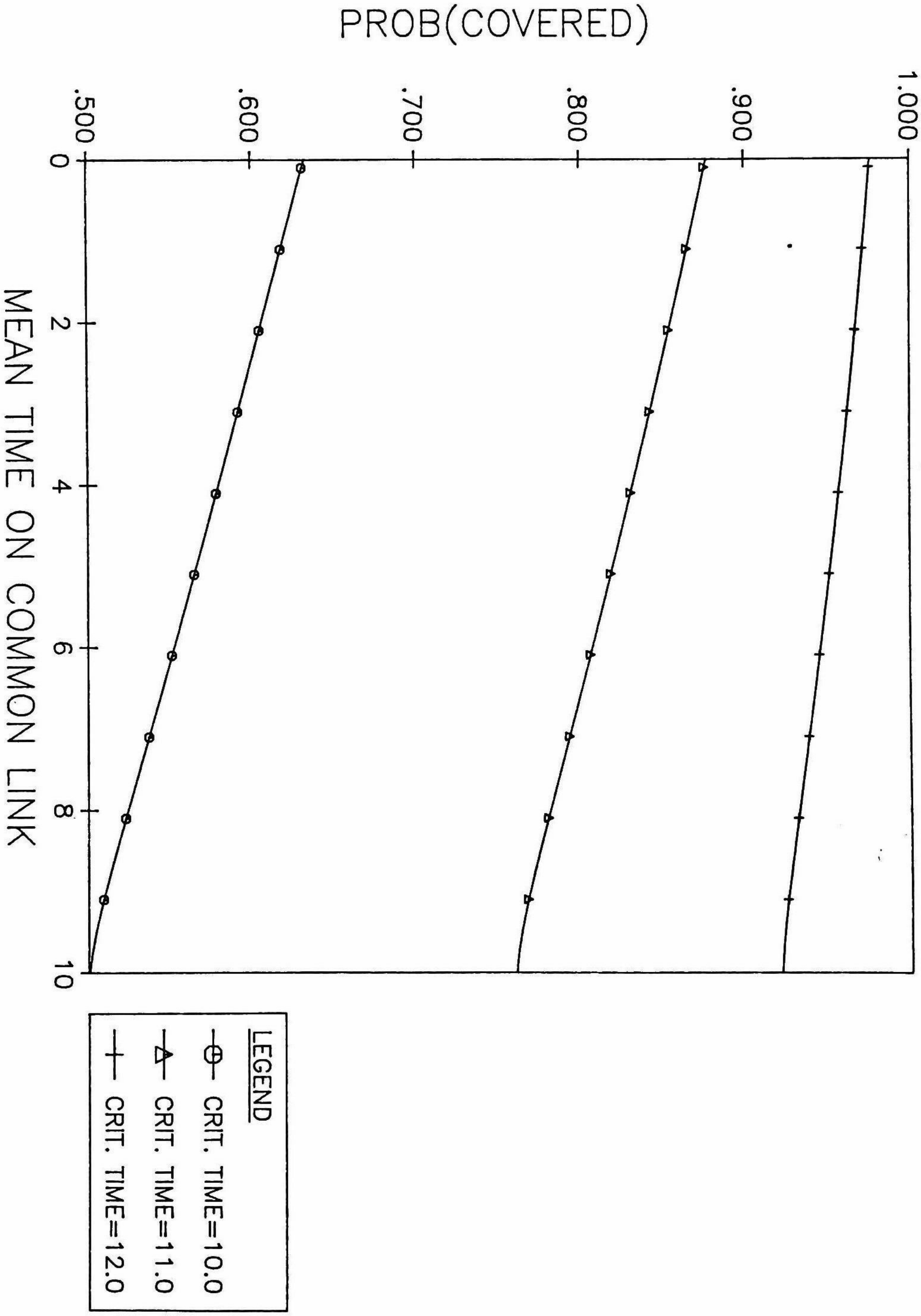
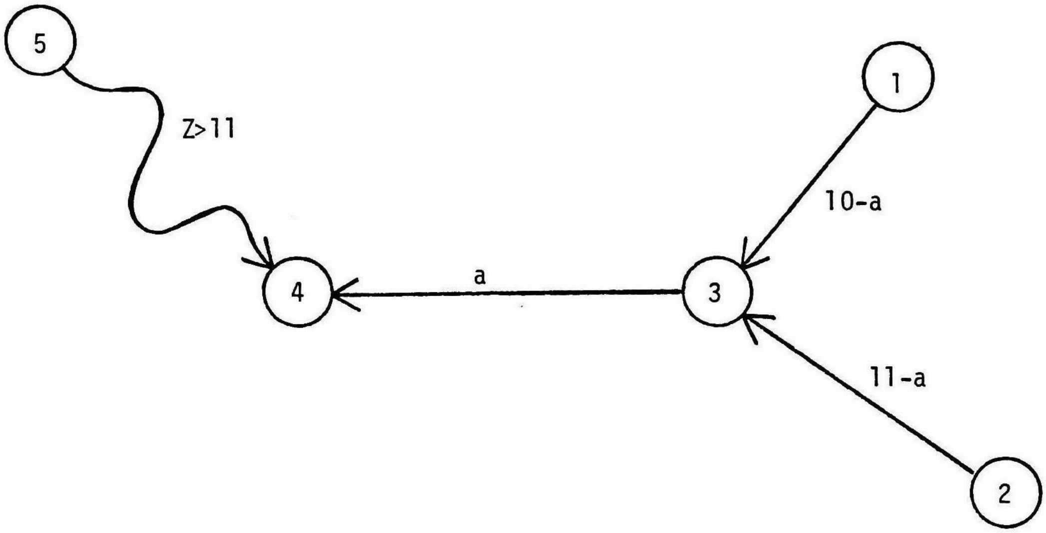
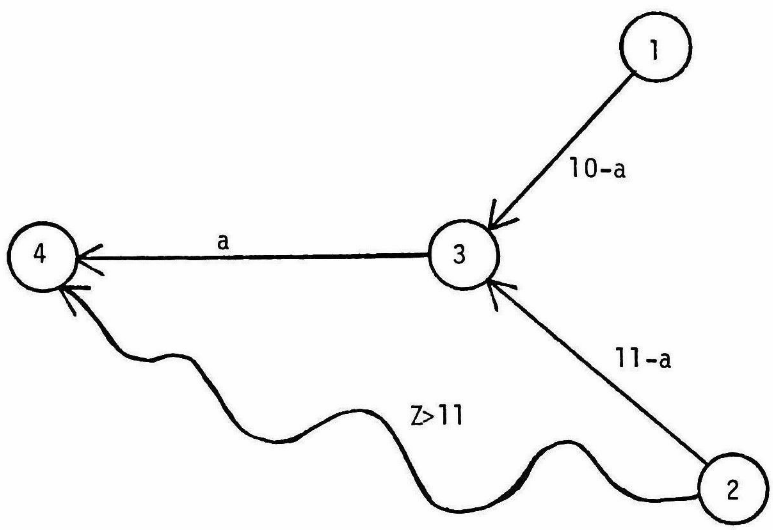


FIG 6 - ALTERNATE DISPATCHING AND ROUTING SCHEMES



Dispatching From A More Remote Node Along An Independent Path

(a)



An Independent Path For Vehicle Based At Node 2

(b)

where $\Phi^{-1}(x)$ is the inverse cumulative Normal distribution function. Squaring both sides of (11), we obtain a quadratic equation for Z_e

$$Z_e^2 - (2T_c + \alpha^2\gamma)\, Z_e + T_c^2 = 0 \qquad (12)$$

whose roots are

$$Z_e = \frac{(2T_c + \alpha^2\gamma) \pm \sqrt{[2T_c + \alpha^2\gamma]^2 - 4T_c^2}}{2} \qquad (13)$$

Since only one of the roots will actually satisfy equation (11), the correct root may be readily identified by substituting each into (11).

Figure 7 plots the equivalent mean travel time on the alternate independent path from 2 to 4 as a function of the travel time on the common link in the original routing scheme for unit variances of 0.1, 0.2, and 0.3. As the time on the link in common increases -- or as the covariance between the path travel times in the original routing scheme increases -- the equivalent mean travel time on the independent path increases. Thus, for example, if the mean time on the link in common is 8, the unit variance is 0.2, and the critical travel time is 11 units, the mean travel time on the alternate equivalent path is 13.189. In other words, we could increase the probability of covering node 4 in 11 time units by dispatching the vehicle from 2 along any path whose mean time was less than 13.189 units if the path was independent of the path to be taken by the vehicle from 1. Note that this means that even if we increase the mean travel time from node 2 to node 4 by up to 20 percent, we can improve the probability of coverage if we reduce the covariance in path travel times sufficiently. Alternatively, we could dispatch a vehicle further from the scene of the emergency than the vehicle located at 2 -- even up to 20 percent further -- and improve the probability that the first vehicle arrives at node 4 within 11 time units, as long as the two vehicles travel independent paths.

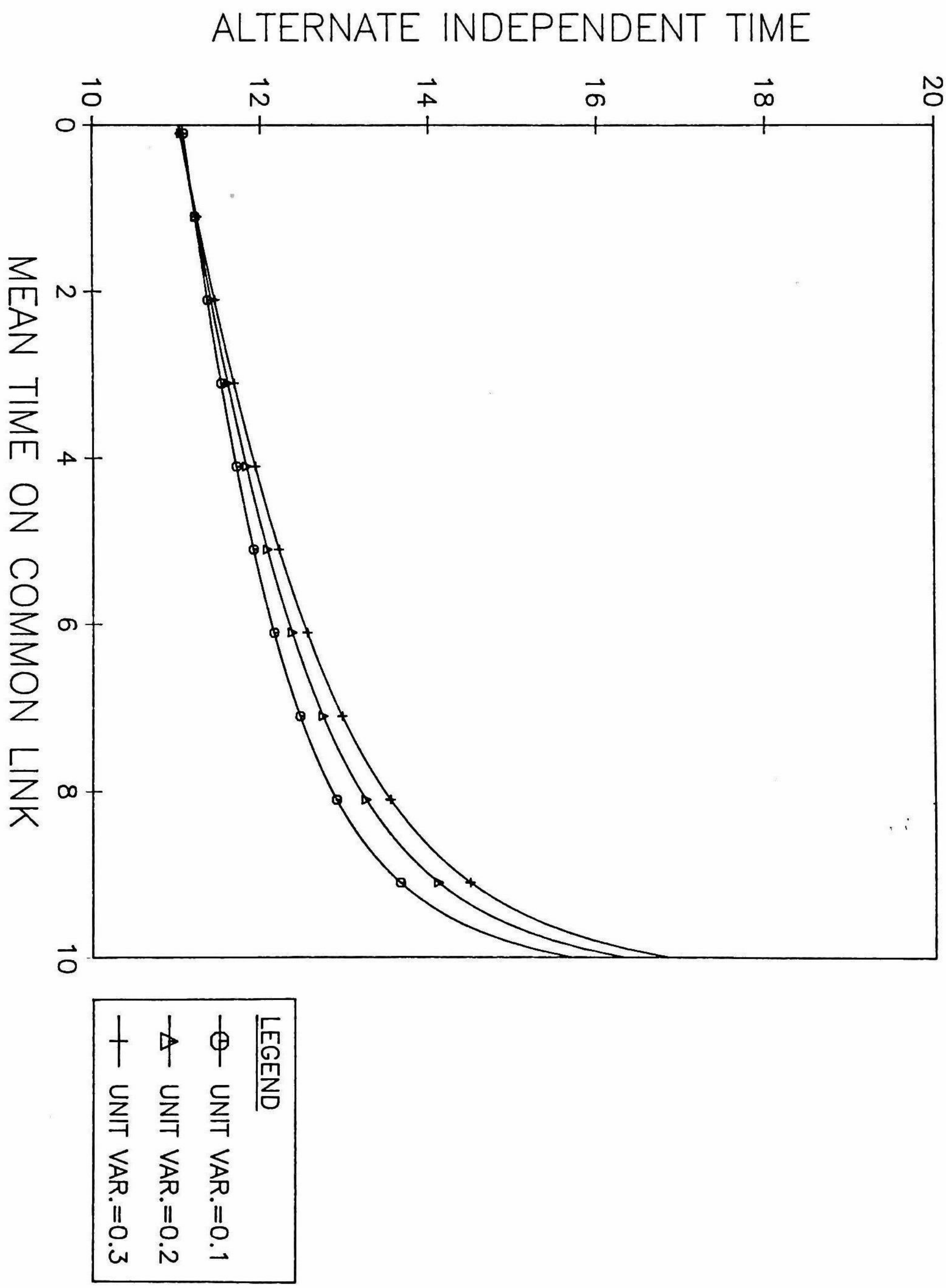


FIG 7—ALTERNATE INDEPENDENT TIME; CRIT. TIME=11.0

Figure 8 plots the equivalent mean travel time on the independent path as a function of the mean time on the link in common under the original routing scheme for critical times of 10.0, 11.0 and 12.0 units. Again, the equivalent mean time on the independent path increases with the covariance in path travel times and with the critical coverage time. For example, if the mean time on the common link in the original routing scheme is 8 units, the equivalent mean time on an independent path increases from 12.654 to 13.774 as the critical coverage time increases from 10 to 12 units.

Finally, we can gain insight into the magnitude of the errors introduced by using Clark's approximation to the maximum of two independent Normal variables, as well as the errors caused by using polynomial approximations [1] to the cumulative Normal and inverse Normal functions, by considering the results shown in Figure 8 for the case in which the mean time on the link in common is 0.0. At this point, the length of the alternate independent path should be 11.0, for all values of the critical travel time, since the path from 2 to 4 is independent of the route from 1 to 4 under the original routing scheme; therefore, nothing can be gained by using an alternate path from 2 to 4 of length greater than 11.0 units. However, as shown in Figure 8, our approximations result in estimates of 10.924, 11.043 and 11.216 for the time on the alternate path from node 2 to node 4 for critical travel times of 10.0, 11.0 and 12.0 respectively. We note that the largest of these three errors is less than 2 percent. Also, Clark's approximation improves as the covariance in route travel times increases [12] which is just when we are most interested in the results from a practical perspective. In the next section we consider the effects of another source of error, the assumption of Normally distributed link travel times.

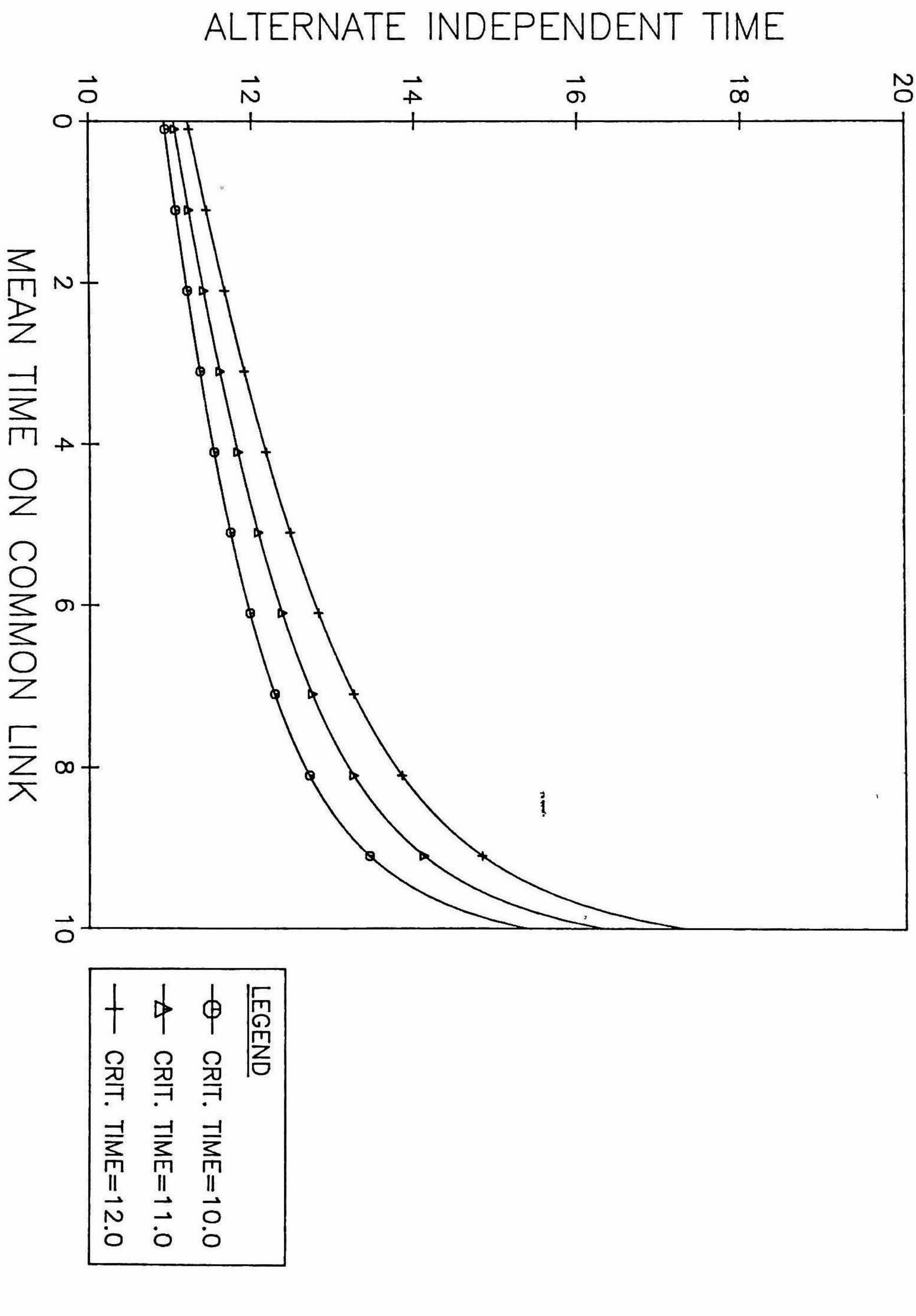


FIG 8–ALTERNATE INDEPENDENT TIME; UNIT VAR.=0.2

4. A Model with Erlang Distributed Travel Times

The assumption of Normally distributed travel times is subject to two criticisms: first, it admits the possibility of negative travel times; second, it implies that travel times on a link are distributed symmetrically about the mean link travel time when, in fact, the travel time distribution is likely to be skewed to the right. In this section, we relax the assumption of Normally distributed travel times, by assuming that X, the link travel time, follows an Erlang-k distribution,

$$f_X(x) = \frac{\theta(\theta x)^{k-1} e^{-\theta x}}{(k-1)!} \tag{14a}$$

with mean and variance given by

$$E(X) = \frac{k}{\theta} \tag{14b}$$

$$Var(X) = \frac{k}{\theta^2} \tag{14c}$$

For the mean of the Erlang distribution to equal the mean of the Normal distribution used in Section 2, we require

$$k = \theta E(X) = \theta\mu_X \tag{15a}$$

If we also retain assumption (1) that the variance is proportional to the expected time, we further require

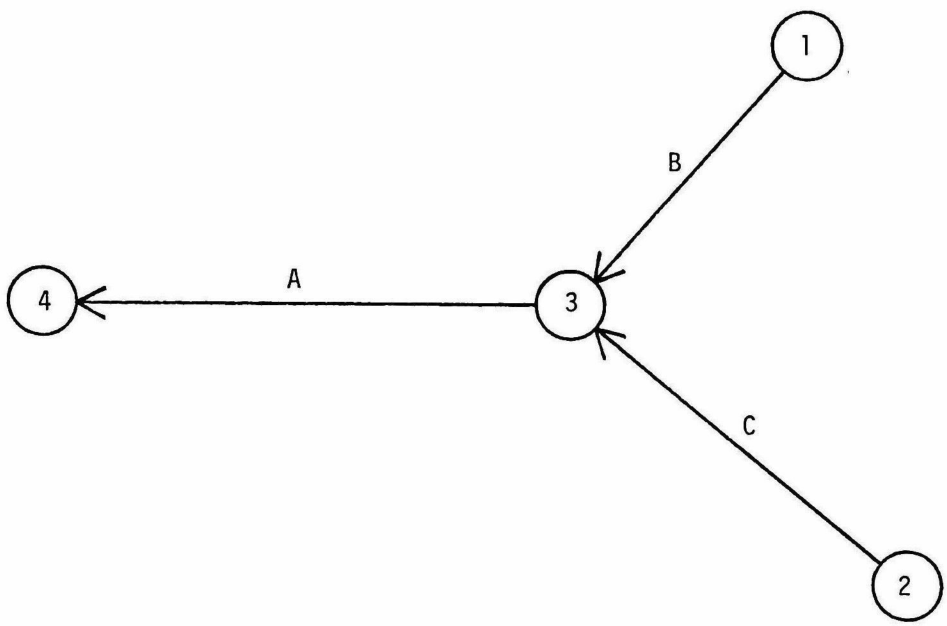
$$\theta = \frac{1}{\gamma} \tag{15b}$$

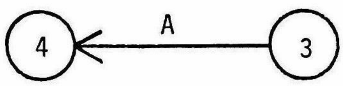
While equations (15) may be used to equate the means and variances for the Normal and Erlang approximations to the distribution of link travel times, the relation between the underlying assumptions for the two distributions should be made explicit. The Normal approximation assumes that the travel time on a link of unit distance is distributed Normally with mean 1 and variance γ. The Erlang approximation assumes that the travel time on a link γ distance units long is exponentially distributed, or equivalently (for integer values of θ) that the travel time on a link of unit distance follows an Erlang-θ distribution. Both approximations assume that travel times on non-overlapping segments of a link, or on different links, are independent and that the covariance of travel time for paths that share links in common is proportional to the expected travel time on the common links. Finally, we note that while the Erlang distribution may be a more realistic distribution of travel times since it does not allow negative times and is skewed to the right, the distribution requires k to be integer. However, for our purposes this is not a serious limitation as we will be using the Erlang distribution only to assess the approximate magnitude of the errors introduced by using the more flexible and more tractable Normal distribution.

To determine the probability that a demand is covered when two vehicles are dispatched to the emergency, consider Figure 9 with vehicles again located at nodes 1 and 2 and a demand at node 4. We assume that travel time on the link from 3 to 4 follows an Erlang-k_A distribution and let A be the random variable denoting this travel time. Similar assumptions hold for the other links. Again, let T_i be the travel time to the demand for the vehicle based at node i. The probability the demand is covered in time T_c is

$$P(\text{covered}) = 1 - P(T_1 > T_c)\, P(T_2 > T_c)$$

FIG 9 - NETWORK FOR ERLANG AND NORMAL DISTRIBUTION COMPARISONS





Link Travel Time Variable

$$= 1 - P(A + B > T_c)\, P(A + C > T_c)$$

$$= 1 - \left\{ \int_0^{T_c} f_A(a)\, P(B > T_c - a)\, P(C > T_c - a)\, da + P(A > T_c) \right\} \qquad (16)$$

From the cumulative Erlang-k_A distribution we we have

$$P(A > T_c) = \sum_{j=0}^{k_A - 1} \frac{(\theta T_c)^j\, e^{-(\theta T_c)}}{j!} \qquad (17)$$

Substituting (17) and similar expressions for $P(B > T_c - a)$ and $P(C > T_c - a)$ into (16) we find

$$P(\text{Covered}) = 1 - \sum_{j=0}^{k_A - 1} \frac{(\theta T_c)^j\, e^{-(\theta T_c)}}{j!}$$

$$- \int_0^{T_c} \frac{\theta(\theta a)^{k_A - 1}\, e^{-(\theta a)}}{(k_A - a)!} \left[\sum_{m=0}^{k_B - 1} \frac{\{\theta(T_c - a)\}^m\, e^{-\{\theta(T_c - a)\}}}{m!} \right] \bullet \left[\sum_{n=0}^{k_C - 1} \frac{\{\theta(T_c - a)\}^n\, e^{-\{\theta(T_c - a)\}}}{n!} \right] da \qquad (18a)$$

Reversing the order of summation and integration and collecting terms, we obtain

$$P(\text{covered}) = 1 - \sum_{j=0}^{k_A - 1} \frac{(\theta T_c)^j\, e^{-(\theta T_c)}}{j!}$$

$$- \frac{\theta^{k_A}\, e^{-(2\theta T_c)}}{(k_A - 1)!} \sum_{m=0}^{k_B - 1} \sum_{n=0}^{k_C - 1} \int_0^{T_c} a^{k_A - 1} (T_c - a)^{m+n}\, e^{\theta a}\, da \qquad (18b)$$

To evaluate the integral in (18b), let us write it in a general form as

$$I(\alpha,\beta) = \int_0^{T_c} y^{\alpha} (T_c - y)^{\beta} e^{\theta y} dy \tag{19}$$

where α and β are non-negative integers. Integrating by parts, we obtain

$$I(\alpha,\beta) = \frac{\beta}{\theta} I(\alpha,\beta-1) - \frac{\alpha}{\theta} I(\alpha-1,\beta) \qquad \alpha \geq 1,\ \beta \geq 1 \tag{20a}$$

$$I(\alpha,0) = \frac{T_c^{\alpha} e^{(\theta T_c)}}{\theta} - \frac{\alpha}{\theta} I(\alpha-1,0) \qquad \alpha \geq 1,\ \beta = 0 \tag{20b}$$

$$I(0,\beta) = -\frac{D^{\beta}}{\theta} + \frac{\beta}{\theta} I(0,\beta-1) \qquad \alpha = 0,\ \beta \geq 1 \tag{20c}$$

$$I(0,0) = \frac{e^{(\theta T_c)} - 1}{\theta} \qquad \alpha = 0,\ \beta = 0 \tag{20d}$$

Equations (20) provide a recursive method for evaluating the integral in (18b).

Figure 10 compares the probability that node 4 is covered using the Normal and Erlang distributions as a function of the critical travel time for the parameters shown in Table 1. As can be seen, the two approximations yield almost identical results. The maximum absolute difference in the probability node 4 is covered is only 0.0170, while the maximum absolute difference for probabilities in excess of 0.75 -- the range in which we are most interested -- is 0.0088. The Normal approximation slightly overestimates the coverage probability at the extremes of the distribution and underestimates the coverage probability in the midrange when compared with the Erlang distribution. The results are not surprising since, for the parameter values shown in Table 1, the Normal and Erlang distributions involved are almost identical.

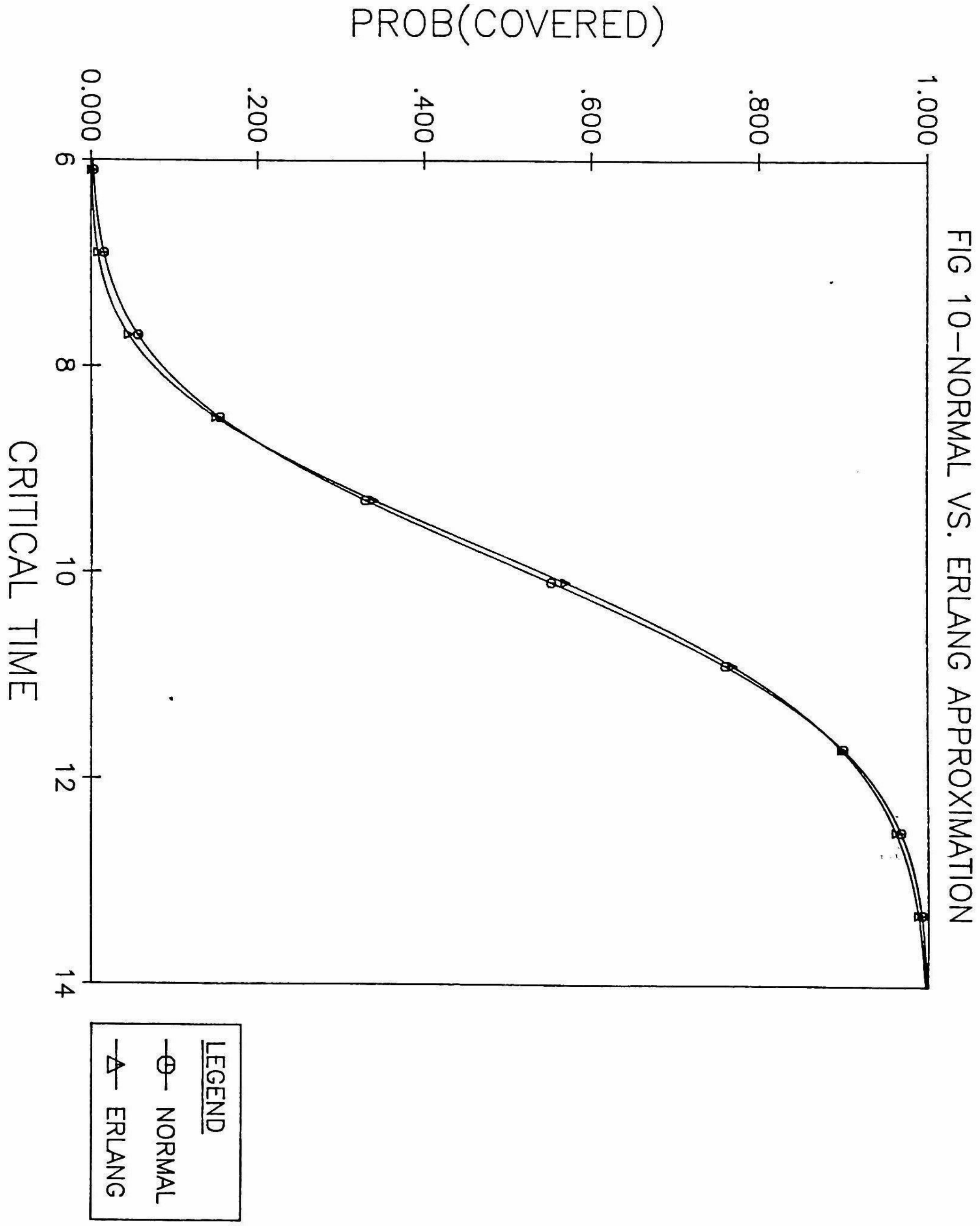


FIG 10—NORMAL VS. ERLANG APPROXIMATION

Table 1 - Parameters for Figure 10 Comparison

Normal		Erlang	
δ	$= 0.2$	θ	$= 5$
μ_{X_A}	$= 8$	k_A	$= 40$
μ_{X_B}	$= 2$	k_B	$= 10$
μ_{X_C}	$= 3$	k_C	$= 15$

Figure 11 compares the Normal and Erlang approximations using the parameter values shown in Table 2 for which the Normal and Erlang distributions are rather dissimilar. Note, for example that using the Normal approximation, the probability of obtaining a negative travel time on link B is 0.079 for the values shown in Table 2 but only 0.00078 for the values shown in Table 1. Even in the extreme case shown in Figure 11, the results for the model with Normally distributed travel times follow those of the model with Erlang distributed times quite well. The maximum absolute difference in coverage probability is 0.0471 while the maximum absolute difference for probabilities in excess of 0.75 is only 0.0323.

While the Erlang distribution may be a more realistic representation of link travel times than the Normal distribution, the computational burden associated with using the Erlang distribution is considerably greater. All the computations needed to evaluate the Normal model may be readily performed on a good programmable calculator; to evaluate the integrals needed in the Erlang model we were forced to use double precision arithmetic on a CDC CYBER 170/730 computer to avoid excessive roundoff errors. Also, as noted above, the Erlang model is restricted to cases in which the parameter k is an integer for all links. Finally, while in theory the Erlang model may be extended to the case of more than two vehicles, the computational burden involved in such an extension is likely to be prohibitive; there are no such difficulties in extending the Normal model to include cases in which three or more vehicles are dispatched to an emergency.

Table 2 - Parameters for Figure 11 Comparison

Normal		Erlang	
γ =	0.5	θ =	2
μ_{X_A} =	2	k_A =	4
μ_{X_B} =	1	k_B =	2
μ_{X_C} =	2	k_C =	4

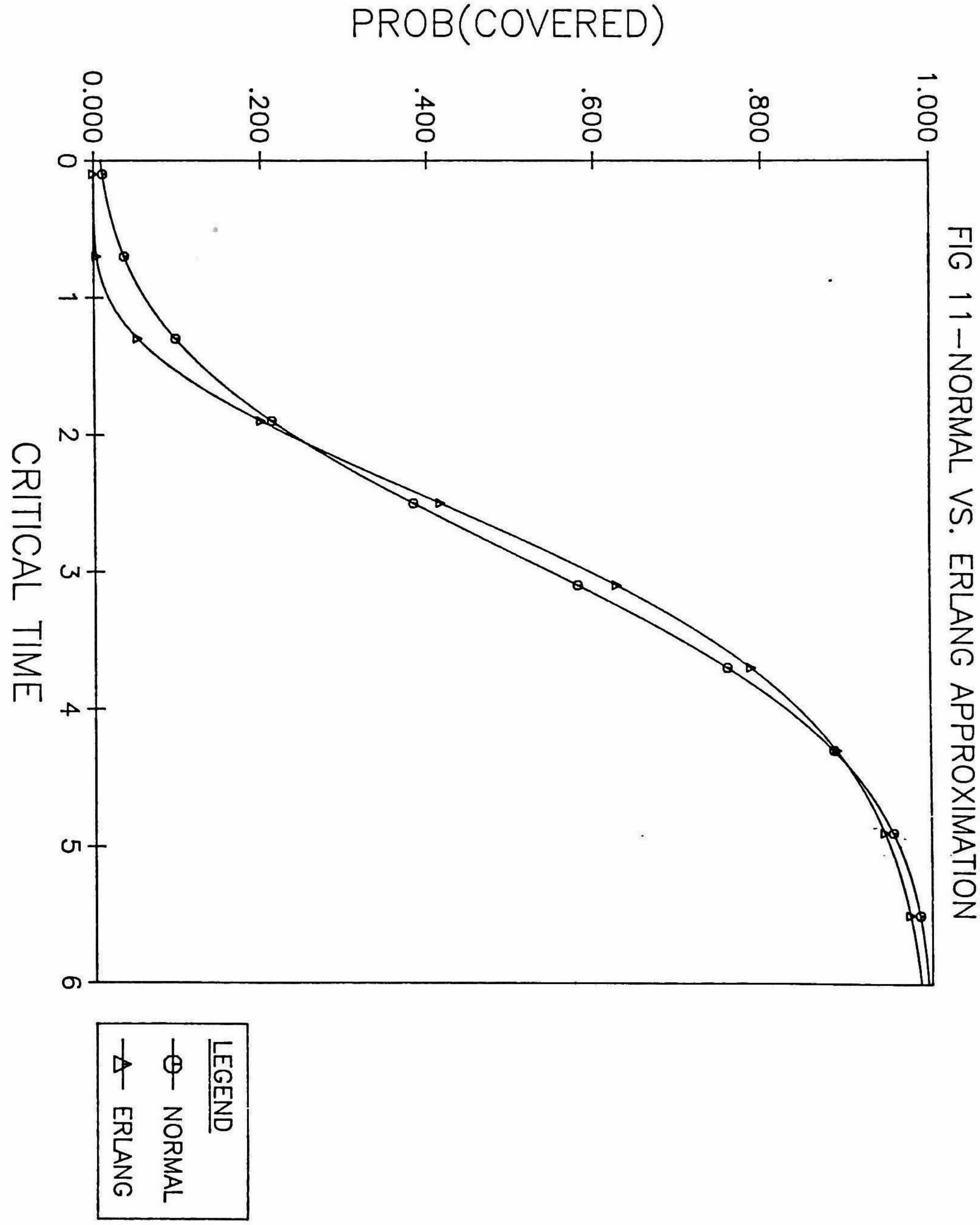


FIG 11—NORMAL VS. ERLANG APPROXIMATION

In light of the greater flexibility afforded by the Normal model, its computational simplicity, and the similarity of the results between it and the Erlang model as shown in Figures 10 and 11, we recommend using the Normal model instead of the Erlang model, despite its somewhat less realistic underlying assumptions.

5. Summary and Recommendations

In this paper, we have developed a model that estimates the distribution of time required for the first of n vehicles to arrive at an emergency scene assuming all vehicles are dispatched at the same time. The model accounts for travel time variability on links of the network as well as the covariance of travel times between paths for different vehicles. The model suggests that positive covariances reduce the coverage probability that the first vehicle will arrive within a given time. The model also suggests that the coverage probability may be increased by reducing the covariance in path times through the selection of alternate paths, or by dispatching alternate vehicles even if doing so necessitates increasing the mean travel time for some vehicles.

The model assumes that link travel times are distributed Normally. To test the sensitivity of this assumption, we also developed a model that assumes Erlang distributed link times. The Erlang model precludes the possibility of negative travel times which are permitted in the model based on Normally distributed travel times; also, the Erlang model relaxes the implicit assumption of the Normal model that travel times are distributed symmetrically about the mean. The primary drawbacks to the Erlang model are its computational complexity, the limitations it places on the selection of parameter values and the anticipated difficulties associated with extending the model to include more than two vehicles. The results of the Normal and

Erlang models are almost identical; therefore, we suggest using the more flexible and more computationally tractable model of Section 2 based on the assumption of Normally distributed link travel times.

The models developed in this study should be validated by collecting data on the travel times of emergency vehicles. This would enable us to determine both the shape of the travel time distribution and its parameter values. Assuming the times are approximately Normally distributed, the models developed above may be used to predict the distribution of the arrival time of the first of several vehicles dispatched to an emergency. The recommendations of the research regarding vehicle routing and dispatching should then be tested on a small scale before they are adopted more generally. In testing these recommendations, we should monitor not only changes in the mean arrival time of the first vehicle, but also changes in the disruption of local traffic caused by routing emergency vehicles along multiple parallel paths as well as changes in the number of traffic accidents caused by the emergency vehicles themselves. These latter two points are likely to be mitigating factors that will drive the desired policy back toward one of routing all vehicles along a common path.

Despite this tone of caution regarding the implementation of our theoretically-based findings, we conclude by noting that the routing and dispatching suggestions make intuitive sense as they are really saying, "Don't put all your eggs in one basket." Current practice apparently violates this guideline as one frequently observes several vehicles following each other on the same street en route to an emergency. The model suggests that they take different but parallel paths so that if one vehicle encounters an unexpected delay, other vehicles are not delayed with it and at least one vehicle will arrive expeditiously at the scene.

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